Examples and Counterexamples 8 (2025) 100198

Contents lists available at ScienceDirect

Examples and Counterexamples

journal homepage: www.elsevier.com/locate/exco

Weak and very weak solutions of the Laplace equation and the Stokes system with prescribed regularity[☆]

Thomas Apel[a], Katharina Lorenz[a,*], Serge Nicaise[b]

[a] *Universität der Bundeswehr München, Institute of Mathematics and Computer-Based Simulation, Werner-Heisenberg-Weg 39, Neubiberg, 85577, Germany*
[b] *Université Polytechnique Hauts-de-France, Campus Mont Houy, Valenciennes, 59313, France*

ARTICLE INFO

Communicated by F. Bertrand

Keywords:
Laplace equation
Stokes system
Non-homogeneous boundary conditions
Finite element approximation

ABSTRACT

To verify theoretical results it is sometimes important to use a numerical example where the solution has a particular regularity. The paper describes one approach to construct such examples. It is based on the regularity theory for elliptic boundary value problems.

1. Introduction

The design of numerical examples is often a delicate task. The test problem should confirm the theory but the solution should not be more regular than assumed in the theory. We formulate here two types of examples for the Laplace and the Stokes problems. In Sections 2.1 and 3.1 we formulate families of solutions of the homogeneous differential equations which solve problems determined by the non-homogeneous boundary data. The solutions are smooth except in the vicinity of one boundary point, and a parameter controls their regularity. In Sections 2.2 and 3.2 the parameters are restricted to achieve homogeneous boundary conditions, at least in the vicinity of the singular boundary point. The solution is then determined by the boundary datum away from the singularity or, by using a cut-off function, by a smooth right hand side of the differential equation.

In the case of the Laplace operator, these examples are widely known but not so much for the Stokes problem. In order to explain the ideas we start, however, with the Laplace equation. We assume that the domain Ω is two-dimensional and polygonal, and comment on the three-dimensional case in Sections 2.5 and 3.4.

2. The Laplace equation

In Sections 2.1 to 2.4 we assume that Ω is a bounded polygon with a Lipschitz boundary.

2.1. Fundamental solutions

Let (r,θ) be polar coordinates centered in the corner of $\Omega \subset \mathbb{R}^2$ with maximal interior angle ω such that the edges of this corner are described by $\theta = 0$ and $\theta = \omega$. Then the function

$$u(r,\theta) = r^\lambda \Phi(\theta)$$

solves the Laplace equation

$$-\Delta u = 0 \quad \text{in } \Omega \tag{2.1}$$

iff $\Phi'' + \lambda^2\Phi = 0$, i. e., iff

$$\Phi(\theta) = \begin{cases} c_1 \cos\lambda\theta + c_2 \sin\lambda\theta & \text{if } \lambda \neq 0, \\ c_1 + c_2\theta & \text{if } \lambda = 0, \end{cases}$$

such that the solution is

$$u(r,\theta) = \begin{cases} r^\lambda(c_1 \cos\lambda\theta + c_2 \sin\lambda\theta) & \text{if } \lambda \neq 0, \\ c_1 + c_2\theta & \text{if } \lambda = 0. \end{cases} \tag{2.2}$$

This solution satisfies

$$u \in H^s(\Omega) \quad \forall s < 1 + \lambda$$

such that the choice of λ can be used to develop test examples of the form

$$-\Delta u = 0 \quad \text{in } \Omega, \tag{2.3}$$

$$u = g \quad \text{on } \Gamma := \partial\Omega, \tag{2.4}$$

[☆] This article is part of a Special issue entitled: 'EXCO_Numerical Methods 2025' published in Examples and Counterexamples.
[*] Corresponding author.
E-mail address: katharina.lorenz@unibw.de (K. Lorenz).

https://doi.org/10.1016/j.exco.2025.100198
Received 19 March 2025; Received in revised form 28 July 2025; Accepted 9 September 2025

or with other types of boundary conditions, with solutions of prescribed regularity.

The case $\lambda = 0$ is of interest when the boundary data should have a jump at $r = 0$.

2.2. Boundary conditions

The freedom of choosing λ, c_1, and c_2 can also be used to satisfy homogeneous boundary conditions, e. g.,

$c_1 = 0$, $\lambda = k\frac{\pi}{\omega}$, $k = 1, 2, \ldots$, leads to $u(r,0) = u(r,\omega) = 0$,
$c_2 = 0$, $\lambda = k\frac{\pi}{\omega}$, $k = 0, 1, 2, \ldots$, leads to $\partial_n u(r,0) = \partial_n u(r,\omega) = 0$,
$c_1 = 0$, $\lambda = (k - \frac{1}{2})\frac{\pi}{\omega}$, $k = 1, 2, \ldots$, leads to $u(r,0) = \partial_n u(r,\omega) = 0$.

The remaining constant is still free. The function does not vanish at boundary parts which are not adjacent to the point with $r = 0$. If homogeneous boundary conditions should be satisfied on the whole boundary of the domain, the functions could be multiplied by a smooth cut-off function $\eta : \mathbb{R}_+ \to \mathbb{R}$ with

$$\eta(r) = \begin{cases} 1 & \text{if } r < r_0, \\ 0 & \text{if } r > r_1 > r_0, \end{cases} \tag{2.5}$$

and appropriately chosen positive constants $r_0, r_1 \in \mathbb{R}$. Then $f = -\Delta u$ is zero only for $r \le r_0$ and $r \ge r_1$ but smooth in the region $r_0 < r < r_1$.

Note that for a given domain Ω the regularity can be influenced only in discrete steps with this approach. But by adjusting the interior angle of Ω at $r = 0$, any desired regularity is adjustable.

2.3. Weak and very weak solutions

For $\lambda > 0$, the function u from (2.2) is a weak solution of (2.1), in the sense that it belongs to $H^1(\Omega)$ and satisfies

$$(\nabla u, \nabla v) = 0 \quad \forall v \in H_0^1(\Omega),$$

and for $-\min(1,\xi) < \lambda \le 0$, $\xi := \frac{\pi}{\omega}$, it is a very weak solution, in the sense that it belongs to $L^2(\Omega)$ and satisfies

$$(u, \Delta v) = \langle u, \partial_n v\rangle_\Gamma \quad \forall v \in V = \{v \in H_0^1(\Omega) : \Delta v \in L^2(\Omega)\},$$

see also [1].

If $\lambda \le -1$, then the function u is not a very weak solution since $u \notin L^2(\Omega)$ such that $(u, \Delta v)$ is not well defined for all $v \in V$.

If the domain Ω is non-convex and $-1 < \lambda < -\xi$, then for $v = \eta(r) r^{\xi} \sin\xi\theta \in V$ and for $\Gamma_0 = \{x \in \Gamma : r(x) < r_0\}$ the dual bracket $\langle u, \partial_n v\rangle_{\Gamma_0}$ is meaningless because the product $u\partial_n v$ is not Lebesgue integrable. Indeed, on $\theta = 0$, we have

$$|u\partial_n v| = |\Phi(0)|\, \eta(r)\, r^{\lambda+\xi-1},$$

which is not integrable except if $\Phi(0) = 0$. Hence $u\partial_n v$ is integrable on Γ_0 if and only if $\Phi(0) = \Phi(\omega) = 0$, which is not possible since $\lambda \ne -k\xi$ for all $k = 1, 2, \ldots$.

Let us finally consider the limit case $u^*(r,\theta) = r^{-\xi}\sin\xi\theta$. Note first that $\Delta u^* = 0$ in Ω and $u^* \in H^t(\Omega)$ for all $t < 1 - \xi$.

However, we can show that

$$(u^*, \Delta v) - \langle u^*, \partial_n v\rangle_\Gamma \ne 0$$

for $v = \eta(r) r^{\xi}\sin\xi\theta \in V$. Indeed, for any $\varepsilon > 0$, by setting $\Omega_\varepsilon = \Omega \setminus \bar{B}(0,\varepsilon)$, where $\bar{B}(0,\varepsilon)$ is the closed ball centered at the origin with radius $\epsilon > 0$, we may write

$$(u^*, \Delta v) = \lim_{\varepsilon \to 0+} \int_{\Omega_\varepsilon} u\Delta v.$$

Now since u^* and v are smooth in Ω_ε, we can use the Green formula to show that

$$(u^*, \Delta v) = \lim_{\varepsilon \to 0+} \int_{\partial\Omega_\varepsilon} (u^*\partial_n v - v\partial_n u^*).$$

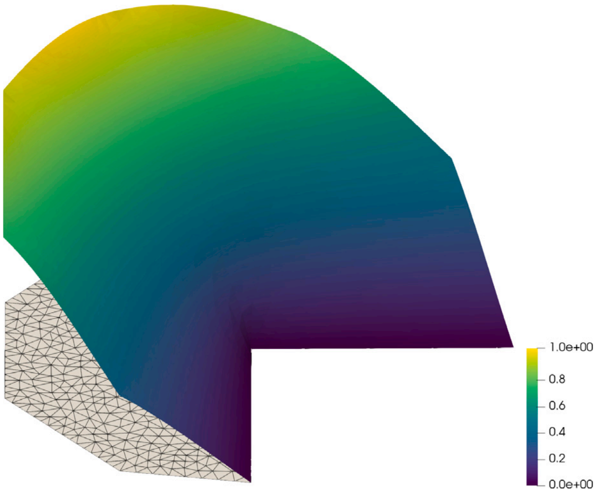

Fig. 1. Illustration of u^*.

Splitting the integral in $\partial\Omega_\varepsilon$ into the integral into the boundary of the disc $B(0,\varepsilon)$ and the remainder part, and using the form of u and v, we get

$$\begin{aligned}(u^*, \Delta v) - \langle u^*, \partial_n v\rangle_\Gamma &= \lim_{\varepsilon \to 0+} \int_0^\omega (u^*\partial_n v - v\partial_n u^*)\varepsilon \,\mathrm{d}\theta = -2\xi \int_0^\omega \sin^2(\xi\theta)\,\mathrm{d}\theta \\ &= -\pi,\end{aligned}$$

which proves the assertion. Alternatively, one could show that the problem (2.4) with $g = u^*$ on Γ has a weak solution $u \ne u^*$ which also means that u^* is not a weak or very weak solution, see Section 2.4.

2.4. Numerical test for the limit case

The numerical test is carried out using the method described in [1, Section III.B] which proved to be able to approximate very weak solutions, even with pole, see [1, Section IV]. For the illustration here, we consider a polygonal domain $\Omega \subset \mathbb{R}^2$ with an interior angle $\omega = \frac{3}{2}\pi$. Accordingly, we set $\xi = \frac{2}{3}$, and the boundary data g is given by

$$g(r,\theta) = r^{-\frac{2}{3}} \sin(\tfrac{2}{3}\theta).$$

It fulfills $g(r,0) = g(r, \frac{3}{2}\pi) = 0$ and is not zero at the boundary parts which are not adjacent to $(0,0)$. However, the obtained solution $u(r,\theta)$ is not $r^{-\frac{2}{3}}\sin(\frac{2}{3}\theta)$. In particular, it has no pole, see the illustration in Fig. 1. The explanation is that the boundary data is piecewise smooth and continuous, hence at least in $H^{\frac{1}{2}}(\Gamma)$ such that a unique weak solution $u \in H^1(\Omega)$ exists. In conclusion, this numerical test confirms that we are in the limit case, where $r^{-\frac{2}{3}}\sin(\frac{2}{3}\theta) \in L^2(\Omega)$ is not a very weak solution although is harmonic and solves the Laplace equation.

2.5. Three-dimensional case

The boundary of a polyhedral domain has edges and corners. Near an edge, the typical behavior of a solution of the Poisson equation is

$$cr^{\lambda}\Phi(\theta) \in H^s(\Omega), \quad s < 1 + \lambda,$$

with some function c, where we used polar coordinates (r,θ) perpendicular to the edge. Boundary conditions can be satisfied as in the two-dimensional case described in Section 2.2.

A second family of fundamental solutions has to be considered near corners. Using spherical coordinates (R,ϑ,θ), they can be described by

$$R^{\nu}\Phi_c(\vartheta,\theta) \in H^s(\Omega), \quad s < \tfrac{3}{2} + \nu$$

if Φ_c is smooth enough. Boundary conditions can be satisfied when Φ_c is an eigenfunction of the Laplace–Beltrami operator defined on the intersection of Ω with a sphere centered at the corner; however this can be done analytically only in very special cases.

3. The Stokes system

In Sections 3.1 to 3.3 we assume that Ω is a bounded polygon with a Lipschitz boundary.

3.1. Fundamental solutions

We describe now solutions of the Stokes system of the form

$$u(r,\theta) = r^{\lambda} U(\theta), \quad p(r,\theta) = r^{\lambda-1} P(\theta), \tag{3.1}$$

where again (r,θ) with $r \in \mathbb{R}_+$, $\theta \in (0,\omega)$ are polar coordinates centered in the corner with maximal interior angle, such that

$$\begin{aligned} -\Delta u + \nabla p &= 0 \quad \text{in } \Omega \subset \mathbb{R}^2, \\ \nabla \cdot u &= 0 \quad \text{in } \Omega. \end{aligned}$$

It is now important to write u in polar components, $u = u_r e_r + u_\theta e_\theta$ with $e_r = \cos\theta\, e_1 + \sin\theta\, e_2$, $e_\theta = -\sin\theta\, e_1 + \cos\theta\, e_2$, such that $\|e_r\| = \|e_\theta\| = 1$. Note that the components u_r and u_θ are related to the Cartesian components of u via a rotation computed by

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u_r \\ u_\theta \end{pmatrix}.$$

We follow the derivation in [2, Sect. 5.1] and [3, Sect. 9.3.2]. Inserting the ansatz (3.1) into the equations, the variable r cancels out, and it remains a system of ordinary differential equations for U and P. The four solutions of this system are for $\theta \neq 0$

$$\begin{aligned}
&U_r^{(1)} = \cos(1+\lambda)\theta, && U_\theta^{(1)} = -\sin(1+\lambda)\theta, && P^{(1)} = 0, \\
&U_r^{(2)} = \sin(1+\lambda)\theta, && U_\theta^{(2)} = \cos(1+\lambda)\theta, && P^{(2)} = 0, \\
&U_r^{(3)} = (1-\lambda)\cos(1-\lambda)\theta, && U_\theta^{(3)} = -(1+\lambda)\sin(1-\lambda)\theta, && P^{(3)} = -4\lambda\cos(1-\lambda)\theta, \\
&U_r^{(4)} = (1-\lambda)\sin(1-\lambda)\theta, && U_\theta^{(4)} = (1+\lambda)\cos(1-\lambda)\theta, && P^{(4)} = -4\lambda\sin(1-\lambda)\theta.
\end{aligned}$$

For the convenience of the reader, we convert the velocities to Cartesian components in Appendix. In the case $\lambda = 0$, the functions $(U^{(i)}, P^{(i)})$, $i = 1,2$, remain the same (with simplifications), but the others have to be replaced by

$$\begin{aligned}
&U_r^{(3)} = -\cos\theta + 2\theta\sin\theta && U_\theta^{(3)} = -\sin\theta + 2\theta\cos\theta, && P^{(3)} = -4\cos\theta, \\
&U_r^{(4)} = -\sin\theta - 2\theta\cos\theta && U_\theta^{(4)} = \cos\theta + 2\theta\sin\theta, && P^{(4)} = -4\sin\theta.
\end{aligned}$$

The solutions u, p, from (3.1) can now be concluded with

$$U(\theta) = \sum_{i=1}^{4} c_i U^{(i)}(\theta), \quad P(\theta) = \sum_{i=1}^{4} c_i P^{(i)}(\theta),$$

with arbitrary λ and arbitrary coefficients c_i. They satisfy

$$u \in H^s(\Omega), \quad p \in H^{s-1}(\Omega) \quad \forall s < 1 + \lambda,$$

and the parameter λ can be chosen such that the test example has the desired regularity.

3.2. Boundary conditions

As in Section 2.2, the coefficients c_i and the parameter λ can be used to satisfy homogeneous boundary conditions. Two boundary conditions for both $\theta = 0$ and $\theta = \omega$ give a homogeneous linear system of 4 equations which has a non-trivial solution iff the determinant vanishes. This condition is used to find again a countable number of values of λ. Let us sketch this approach for the case of Dirichlet boundary conditions and $\lambda \neq 0$.

The condition $U(0) = 0$ leads to

$$\begin{pmatrix} U_r(0) \\ U_\theta(0) \end{pmatrix} = \begin{pmatrix} c_1 + (1-\lambda)c_3 \\ c_2 + (1+\lambda)c_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad \text{i. e.} \quad \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -(1-\lambda)c_3 \\ -(1+\lambda)c_4 \end{pmatrix},$$

hence

$$U(\omega) = -(1-\lambda)c_3 U^{(1)}(\omega) - (1+\lambda)c_4 U^{(2)}(\omega) + c_3 U^{(3)}(\omega) + c_4 U^{(4)}(\omega).$$

The 2×2 linear system $U(\omega) = 0$ for the coefficients c_3 and c_4 has the determinant

$$4(\sin^2\lambda\omega - \lambda^2\sin^2\omega) = 4(\sin\lambda\omega - \lambda\sin\omega)(\sin\lambda\omega + \lambda\sin\omega). \tag{3.2}$$

This means that for given angle ω one gets the corresponding exponents $\lambda \in \mathbb{C}$ by solving (separately) the two transcendental, scalar equations $\sin\lambda\omega = \pm\lambda\sin\omega$. All values $\operatorname{Re}\lambda \in [\frac{1}{2}, 4]$ are given for $\omega_k = k\pi/10$, $k = 4, 5, \ldots, 20$, in [4].

3.3. Weak and very weak solutions

As in Section 2.3, the pair (u,p) is a weak solution for $\lambda > 0$ and a very weak solution for $-\min(1,\xi) < \lambda \le 0$, where

$$\xi = \min\{\operatorname{Re}\lambda > 0 : \lambda \text{ satisfies (3.2)}\}$$

in the case of Dirichlet boundary conditions. The weak solution $(u,p) \in H^1(\Omega) \times L_0^2(\Omega)$ is defined by

$$\begin{aligned} (\nabla u, \nabla v) - (\nabla \cdot v, p) &= 0 \quad \forall v \in H_0^1(\Omega), \\ (\nabla \cdot u, q) &= 0 \quad \forall q \in L_0^2(\Omega). \end{aligned}$$

The very weak solution $(u,p) \in L^2(\Omega) \times P'$ with P' the dual-space of $P = \{v \in H^1(\Omega) \cap L_0^2(\Omega) : r^{-1}v \in L^2(\Omega)\}$ is defined by

$$(u, -\Delta v + \nabla q) - (\nabla \cdot v, p) = \langle u, qn - \partial_n v\rangle_\Gamma \quad \forall (v,q) \in \mathcal{V}$$

where $\mathcal{V} := \{(v,q) \in H_0^1(\Omega) \times L_0^2(\Omega) : -\Delta v + \nabla q \in L^2(\Omega), \nabla \cdot v \in P\}$, see [5].

3.4. Three-dimensional case

The three-dimensional case is very similar to the Laplace case described in Section 2.5. The most interesting difference is that the function u is a vector function with different regularities of the components near edges. The components u_r and u_θ have the two-dimensional behavior as described in Sections 3.1 and 3.2. The component in the edge direction, however, has a behavior as for the Laplace operator. For more details, see e. g. [3, Chap. 9].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix. Velocities in Cartesian coordinates

Since most finite element packages work with Cartesian components of the vector functions, we convert here the velocities in the fundamental solutions of the Stokes system.

$$\begin{aligned}
\begin{pmatrix} U_1^{(1)} \\ U_2^{(1)} \end{pmatrix} &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos(1+\lambda)\theta \\ -\sin(1+\lambda)\theta \end{pmatrix} \\
&= \begin{pmatrix} \cos\theta\cos(1+\lambda)\theta + \sin\theta\sin(1+\lambda)\theta \\ \sin\theta\cos(1+\lambda)\theta - \cos\theta\sin(1+\lambda)\theta \end{pmatrix} = \begin{pmatrix} \cos\lambda\theta \\ -\sin\lambda\theta \end{pmatrix} \\
\begin{pmatrix} U_1^{(2)} \\ U_2^{(2)} \end{pmatrix} &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sin(1+\lambda)\theta \\ \cos(1+\lambda)\theta \end{pmatrix} \\
&= \begin{pmatrix} \cos\theta\sin(1+\lambda)\theta - \sin\theta\cos(1+\lambda)\theta \\ \sin\theta\sin(1+\lambda)\theta + \cos\theta\cos(1+\lambda)\theta \end{pmatrix} = \begin{pmatrix} \sin\lambda\theta \\ \cos\lambda\theta \end{pmatrix} \\
\begin{pmatrix} U_1^{(3)} \\ U_2^{(3)} \end{pmatrix} &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} (1-\lambda)\cos(1-\lambda)\theta \\ -(1+\lambda)\sin(1-\lambda)\theta \end{pmatrix} \\
&= \begin{pmatrix} (1-\lambda)\cos\theta\cos(1-\lambda)\theta + (1+\lambda)\sin\theta\sin(1-\lambda)\theta \\ (1-\lambda)\sin\theta\cos(1-\lambda)\theta - (1+\lambda)\cos\theta\sin(1-\lambda)\theta \end{pmatrix}
\end{aligned}$$

$$
\begin{aligned}
&= \begin{pmatrix} \cos\lambda\theta - \lambda\cos(2-\lambda)\theta \\ -\sin\lambda\theta - \lambda\sin(2-\lambda)\theta \end{pmatrix} \\
\begin{pmatrix} U_1^{(4)} \\ U_2^{(4)} \end{pmatrix} &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} (1-\lambda)\sin(1-\lambda)\theta \\ (1+\lambda)\cos(1-\lambda)\theta \end{pmatrix} \\
&= \begin{pmatrix} (1-\lambda)\cos\theta\sin(1-\lambda)\theta - (1+\lambda)\sin\theta\cos(1-\lambda)\theta \\ (1-\lambda)\sin\theta\sin(1-\lambda)\theta + (1+\lambda)\cos\theta\cos(1+\lambda)\theta \end{pmatrix} \\
&= \begin{pmatrix} -\sin\lambda\theta - \lambda\sin(2-\lambda)\theta \\ -\cos\lambda\theta + \lambda\cos(2-\lambda)\theta \end{pmatrix}
\end{aligned}
$$

In the case $\lambda = 0$ we modify $U^{(3)}$ and $U^{(4)}$ to

$$
\begin{aligned}
\begin{pmatrix} U_1^{(3)} \\ U_2^{(3)} \end{pmatrix} &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} -\cos\theta + 2\theta\sin\theta \\ -\sin\theta + 2\theta\cos\theta \end{pmatrix} \\
&= \begin{pmatrix} \cos\theta(-\cos\theta + 2\theta\sin\theta) - \sin\theta(-\sin\theta + 2\theta\cos\theta) \\ \sin\theta(-\cos\theta + 2\theta\sin\theta) + \cos\theta(-\sin\theta + 2\theta\cos\theta) \end{pmatrix} \\
&= \begin{pmatrix} -\cos 2\theta \\ 2\theta + \sin 2\theta \end{pmatrix} \\
\begin{pmatrix} U_1^{(4)} \\ U_2^{(4)} \end{pmatrix} &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} -\sin\theta - 2\theta\cos\theta \\ \cos\theta + 2\theta\sin\theta \end{pmatrix} \\
&= \begin{pmatrix} \cos\theta(-\sin\theta - 2\theta\cos\theta) - \sin\theta(\cos\theta + 2\theta\sin\theta) \\ \sin\theta(-\sin\theta - 2\theta\cos\theta) + \cos\theta(\cos\theta + 2\theta\sin\theta) \end{pmatrix} \\
&= \begin{pmatrix} -2\theta - \sin 2\theta \\ \cos 2\theta \end{pmatrix}
\end{aligned}
$$

Data availability

No data was used for the research described in the article.

References

[1] T. Apel, S. Nicaise, J. Pfefferer, Discretization of the Poisson equation with non-smooth data and emphasis on non-convex domains, Numer. Methods Partial Differential Equations 32 (5) (2016) 1433–1454, http://dx.doi.org/10.1002/num.22057.

[2] V.A. Kozlov, V.G. Maz'ya, J. Rossmann, Spectral problems associated with corner singularities of solutions to elliptic equations, Mathematical Surveys and Monographs, vol. 85, American Mathematical Society, Providence, RI, 2001, http://dx.doi.org/10.1090/surv/085.

[3] V. Mazya, J. Rossmann, Elliptic equations in polyhedral domains, Mathematical Surveys and Monographs, vol. 162, American Mathematical Society, Providence, RI, 2010, http://dx.doi.org/10.1090/surv/162.

[4] M. Dauge, Stationary Stokes and Navier-Stokes systems on two- or three-dimensional domains with corners. I. Linearized equations, SIAM J. Math. Anal. 20 (1) (1989) 74–97, http://dx.doi.org/10.1137/0520006.

[5] T. Apel, K. Lorenz, J. Pfefferer, Numerical Analysis for the Very Weak Solution of the Stokes Equations in Polygonal and Polyhedral Domains, Technical Report, 2025, in preparation.