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Empirical approach for modelling time-dependent seepage line and improving slope stability predictions of homogeneous river levees

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ABSTRACT

When analysing the slope stability of levees, the soil mechanical parameters of the saturated and unsaturated cross-sectional areas are the key influencing factors. Thus, prediction of the seepage line is necessary. In practice, steady-state boundary conditions are typically assumed, where the seepage line reaches the maximum elevation within the levee. This scenario leads to conservative, potentially oversimplified safety assessments, reflecting the awareness that the actual level of safety is not fully known and acknowledges the inherently dynamic rather than steady-state nature of flood events. This study proposes a novel approach to enhance the safety assessment of homogeneous levees by accounting for the dynamic evolution of phreatic lines over time. By considering transient seepage flow and incorporating key flood hydrograph parameters, such as rising or peak duration, and using an analytical estimate of the transient seepage line, this method allows the practical application of risk-based design in flood protection contexts. An equation grounded in numerical principles was developed to empirically derive the transient behaviour of the seepage line, significantly reducing the computational effort compared with numerical methods. This equation can be subjected to a probabilistically supported assessment of homogeneous levees, contributing to an improved understanding of transient geohydraulic conditions.

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1. Introduction

Seepage can significantly affect the stability of river levees. In practice, this phenomenon is often analysed under steady-state or quasi-steady-state assumptions, where the seepage line reaches its maximum height within the levee, representing the critical design case (Heyer, 2009; Schwiersch et al., 2021; Van Woerkom et al., 2023).

However, studies have indicated that steady-state conditions are achieved only in cases with long-lasting peak water levels or in levees with high permeability. Although steady-state assumptions based on analytical solutions (see, e.g. Lendering et al., 2018; US Army Corps of Engineers, 2003) provide conservative safety estimates, they neglect the transient nature of floods and the variability of flood wave shapes (Hegnauer et al., 2014; Van Woerkom et al., 2023). Time-dependent seepage analyses offer a

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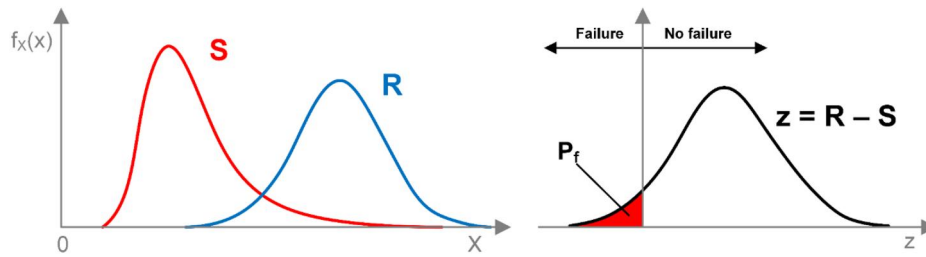


Figure 1. Definition of the probability of failure P_f (red) based on the limit state function $z = R - S$.

more realistic representation of the phreatic line but are computationally intensive relative to the current practice and are therefore rarely applied in a risk-based design framework (Haselsteiner, 2007; Kool et al., 2020; Scheuermann, 2005).

Numerical methods are necessary to accurately predict the transient behaviour of seepage lines within levees. Specifically, finite-element method (FEM) simulations provide a detailed representation of the transient seepage process, capturing dynamic changes in the phreatic line over time. However, these numerical approaches require significantly greater computational effort than steady-state analyses and often restrict the practical applicability of such methods (Nam et al., 2014).

This challenge becomes even more pronounced when probabilistic analyses are conducted in order to provide a more nuanced understanding of the risks involved, because many iterations are necessary to comprehensively account for the variability of the input parameters and their influence on the results (Bachmann et al., 2013; Lesny et al., 2023; Möllmann, 2009; Redaelli, 2012; Schwiersch & Stamm, 2022).

In a fully probabilistic analysis, for example, the system resistance R and prevailing loads S are described as random variables represented as distributions. This allows the assignment of exceedance or non-exceedance probabilities to their values.

Consequently, the limit state function $z = R - S$ is a random variable, as illustrated in Figure 1.

Given that the structure is considered to have failed in the case that $z < 0$, the probability of failure P_f can be determined through probabilistic analysis according to Eq. 1 (Bachmann et al., 2013; Braml et al., 2010; DWA-M 520, 2023; Kortenhaus, 2003).

$$P_f = P(z < 0) = \int_{-\infty}^0 f(z) dz \quad (1)$$

However, probabilistic assessments of levees present practical challenges. Simplified assumptions, such as steady-state seepage lines, can result in overly conservative and potentially unrealistic estimates of safety, whereas detailed transient seepage analyses, typically derived through the FEM, are computationally expensive and thus impractical for widespread probabilistic implementation.

To address these challenges, this study proposes an analytical surrogate model that efficiently captures the transient behaviour of seepage lines. By analytically estimating transient seepage lines according to characteristic flood hydrograph parameters, this method significantly reduces the computational effort compared with numerical methods. Consequently, probabilistic approaches for levee safety assessments have become practically feasible, directly supporting recommendations from recent guidelines, such as DWA-M 520, (2003) and the European Commission: Joint Research Centre et al. (2024) guidelines on reliability-based verification of geotechnical limit states.

The primary objective of this study is to develop an empirical equation capturing the time-dependent behaviour of the seepage line that can be readily applied in engineering practice, facilitating a broader and more accurate adoption of risk-based design methods in flood protection contexts.

Previous research by Van Woerkom et al. (2023) demonstrated the use of a transient probabilistic model for a levee design based on different input conditions. They investigated the influence of various flood hydrographs on two reference systems and a real cross section. However, the scope of the study was limited to only 50 computationally intensive events. Curran (2020) incorporated transient input variables into a probabilistically supported assessment. In this approach, the peak duration of a flood event was integrated into different failure mechanisms using adjustment factors, similar to the partial safety factor concept (DIN EN 1997-3:2022, 2022), which can be used to estimate trends.

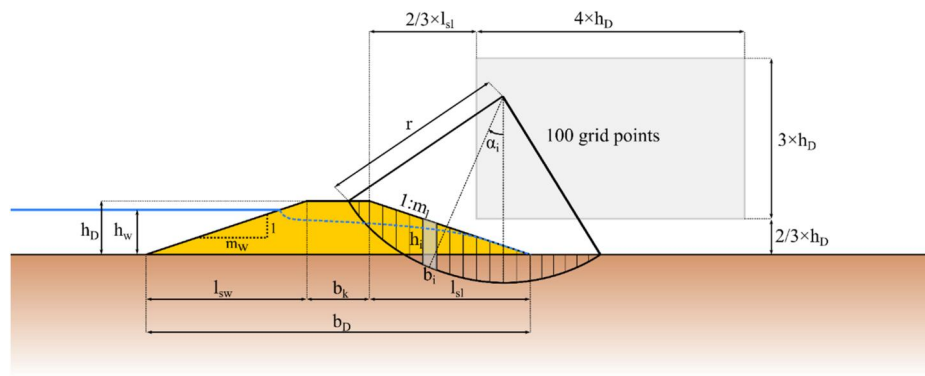


Figure 2. Parameterisation of a homogeneous levee cross section on cohesive subsoil for stability analysis (Schwiersch & Stamm, 2022).

In contrast to these studies, we seek to develop an approach which offers comprehensive consideration of the temporal variability of prevailing loads, allowing more accurate and efficient prediction of transient seepage lines for homogeneous river levees. This is accomplished by using numerical simulations to parameterise the transient seepage line based on key flood hydrograph variables (Öttl et al., 2023b), including the rising duration t_r [d], peak duration t_p [d], and water level h_w [m]. Further, the impoundment ratio s_w [-], as defined by Eq. 2, is also considered. Here, h_D [m] represents the levee height [m].

$$s_w = \frac{h_w}{h_D} \quad (2)$$

2. Standardised levee model for seepage analysis

This study considers standardised homogeneous levee systems on an impermeable substrate as the investigation model, as illustrated in Figure 2. For the numerical implementation, this impermeable substrate was represented by a no-flow boundary condition. Schwiersch and Stamm (2022) established a model to systematically assess the probability of landside slope failure, considering the uncertainties in soil parameters and levee geometry, which included the following geometric parameters: levee width b_D [m], levee height h_D [m], crest width b_k [m], and slope gradients on the water side m_w and land side m_l [-].

Building on this, the model developed in the study considers typical parameter ranges of 15–35 m for b_D , 2–5 m for h_D , and 1–5 m for b_k , and m_w and m_l are between 1:3.3 and 1:2.5, accounting for the minimum cross section for homogeneous levees based on German standards such as DIN 19712, (2013) and DWA-M 507, (2011).

Figure 2 depicts a potential slip circle to illustrate the subsequent integration of the proposed seepage-line modelling approach into probabilistic slope stability assessments (Öttl et al., 2026). However, slope stability analyses are not part of the present study, which focuses exclusively on the hydraulic modelling framework.

Various soil types with hydraulic conductivities k_s ranging from $5.10\text{E-}08 \text{ ms}^{-1}$ to $2.66\text{E-}04 \text{ ms}^{-1}$, have been investigated independently as fill materials. Further investigations beyond these application limits revealed that soils with low permeability do not lead to steady-state conditions under realistic flood conditions early, whereas soils with increasing sand content can exhibit quasi-steady-state conditions at the beginning of the flood hydrograph depending on the geometric constraints.

In the stability assessment, the actions on the levee resulted from the hydrograph, which is defined by s_w between 0 and 1, varying t_r (1, 5, 10, and 20 d), and a constant t_p (20 d) as the initial input. Section 4 details the implementation of these hydrographs within the numerical model.

3. Seepage flow in homogeneous levees

The location of the seepage line in the transient case is affected by the geometric and geotechnical properties of the levee as well as the characteristics of the flood hydrograph (Schwiersch et al., 2021).

Accordingly, the temporal position of the seepage line is crucial for reliably assessing levee stability (Curran, 2020; Haselsteiner, 2007; Scheuermann, 2005; Weißmann, 2014) and can be analysed through geohydraulic numerical simulations.

Steady-state conditions refer to hydraulic states in which water content, pressure head, and saturation do not change with time. In realistic flood scenarios, such conditions are only asymptotically approached if boundary conditions remain constant over a sufficiently long period. Under time-varying boundary conditions, the flow regime is transient (Kinzelbach and Rausch, 1995 and Walshire, 2024).

Darcy's law (Darcy, 1856) applies to laminar flow conditions, where the hydraulic gradient i , representing the head loss (Δh) over the flow-path length L , is linearly related to the Darcy velocity v_D through the soil's hydraulic conductivity k_s . The corresponding flow velocity v_D can be expressed as:

$$v_D = k_s \times i \quad (3)$$

Darcy's law and the continuity condition provide the fundamental principles for the geohydraulic numerical simulation of water flow within the levee system, as expressed by Equation 4, where θ represents the volumetric water content (Scheffer et al., 1984).

$$\frac{\partial}{\partial x} \left(k_s \times \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_s \times \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_s \times \frac{\partial h}{\partial z} \right) = \frac{\partial \theta}{\partial t} \quad (4)$$

Furthermore, in the unsaturated zone, a capillary fringe can form above the seepage line, particularly in fine-grained soils, where full saturation exists. This potential Ψ_h corresponds to the work the pore water performs against gravity at the atmospheric pressure due to capillary suction (Davidenkoff, 1957; Richards, 1931; Van Genuchten, 1980; Walshire, 2024). Therefore, it is crucial to incorporate these conditions into the analyses to approximate the seepage lines in the dam accurately.

This study utilized GeoStudio for geohydraulic modeling, employing the finite element method (GeoSlope International Ltd, 2015). Transient seepage analyses were performed using the SEEP/W module, explicitly considering unsaturated soil conditions. Soil-water retention behaviour and hydraulic conductivity relationships were implemented in SEEP/W through tabulated functions, defined individually for each soil class. The tabulated soil-water retention curves describe the relationship between matric suction and volumetric water content, while the corresponding hydraulic conductivity functions define the reduction of hydraulic conductivity with increasing suction. The functions used for the five soil classes are provided in Appendix A (Figure A1), ensuring reproducibility of the transient seepage simulations. For each material, the saturated hydraulic conductivity k_s and both the residual and saturated volumetric water contents (θ_r and θ_s) were specified. Material compressibility m_v was incorporated using a uniform volumetric compressibility of 0.0005 kPa^{-1} for all soil classes. Transient analyses were initiated with a groundwater distribution representing steady hydraulic conditions prior to the flood event, defined by a horizontal piezometric surface at the base of the model domain ($y = 0 \text{ m}$). Below this surface, a hydrostatic pore-water pressure distribution was assumed. Above the piezometric surface, the levee body was represented as an unsaturated medium, with pore-water pressures derived consistently from the specified piezometric surface and the soil-water retention function implemented in SEEP/W. Consequently, the initial degree of saturation and hydraulic conductivity in the unsaturated zone are defined by the corresponding material functions, ensuring a physically consistent initial condition. The simulations were conducted on a regular FEM grid ($0.25 \text{ m} \times 0.25 \text{ m}$ element size) using daily time steps.

The material parameters used in the simulations were assigned based on representative values for typical soil groups. Table 1 summarises these soil classes together with representative hydraulic and mechanical properties, as derived by Schwiersch (2023) according to established literature (e.g. Albert, 2014; DIN 18196, 2023; Schwiersch et al., 2022; Vismann, 2018).

4. Introducing parameter κ for evaluating seepage-line dynamics

The seepage line represents a one-dimensional projection of the surface between the saturated and unsaturated regions within the levee. This is defined as the locus of zero pore water pressure. While the steady-state seepage line corresponds to the critical loading scenario with the maximum water level and

Table 1. Representative hydraulic soil properties in the transient seepage simulations.

Soil classes (DIN 18196)	Unit weight γ [kNm ⁻³]	Hydraulic conductivity k_s [ms ⁻¹]	Residual volumetric water content θ_r [-]	Saturated volumetric water content θ_s [-]
Clay, low plasticity (TL)	20.2	5.10E-08	0.100	0.50
Silt, medium and highly plastic (UM, UA)	18.2	4.44E-07	0.059	0.47
Silt, slightly plastic (UL)	18.2	3.74E-06	0.043	0.43
Gravel, sandy, clayey, silty (GU, GT)	20.8	1.63E-05	0.034	0.42
Gravel, sandy, few finds (GW, GI)	19.6	2.66E-04	0.018	0.35

geometric properties, transient analyses must also consider the geotechnical properties of the levee, as well as the characteristics of the flood hydrograph.

Building on established mathematical and physical principles along with the defined boundary conditions, the finite-element model incorporates different hydrographs, considering s_w values ranging from 0 to 1. Each hydrograph exhibits a linear rising stage with the previously defined rising durations t_r and a constant peak duration of $t_p = 20$ d, representing the maximum hydraulic load. The descending branch was not simulated, as the focus of this study is on the maximum geohydraulic stress and its impact on the transient phreatic line development. For each applied hydrograph, the pore water pressure profile was extracted for a selected timestep. Subsequently, linear interpolation was used to describe the location of the seepage line at each point along the abscissa between the positive and negative pore water pressure regions.

For a comparative assessment between the steady-state and transient characteristics of the seepage line, a new variable κ is defined. Specifically, κ represents the difference between the angles α_{steady} and α_{trans} , which define the inclination of the regression lines describing the seepage line under steady-state and transient conditions, respectively.

In Figure 3, the solid blue line represents the course of the seepage line according to the analytical calculation of Kozeny and Casagrande (Erb, 1965). This line serves as the standard reference, aligning with the FEM solution under steady-state conditions. In contrast, the red line illustrates the transient seepage line at the respective time of observation.

The mathematical description of κ requires two cases to be distinguished depending on the seepage-line exit point (Equation 5 to 7). Thus, for the calculation of α_{trans} , the area below the transient seepage line is divided into multiple sub-triangles (Figure 4). Here, m_x and m_y represent the slope gradient of the water side with $m_w = m_y \times m_x^{-1}$. A_{steady} and A_{trans} denote the respective cross-sectional areas beneath the sketched seepage lines. $L_{1,2}$ represents the distance between the point of water entry at h_w and the bottom of the levee. Once the transient seepage line exits the landward slope of the levee, Equation 6 applies to both the steady-state and transient cases.

The quantities A_{steady} and A_{trans} are not purely geometric but instead reflect the seepage-line configuration under steady-state and transient hydraulic conditions. Consequently, hydraulic conductivity k_s , rising duration t_r , and peak duration t_p do not appear explicitly in Equations 5 to 7. Their influence is implicitly included in the transient seepage development governing A_{trans} , and thus κ , as expressed in Equation 8. Given the geometry shown in Figure 4, the two cases can be defined as follows:

- Seepage-line exit point extending from the landward bottom to the landward toe of the levee

$$\alpha_{trans} = \tan^{-1} \left(\frac{L_1}{h_w} \right) = \tan^{-1} \left(\frac{\frac{2 \times A_{trans}}{h_w} - h_w \times \frac{m_x}{m_y}}{h_w} \right) = \tan^{-1} \left(\frac{2 \times A_{trans}}{h_w^2} - \frac{m_x}{m_y} \right) \quad (5)$$

- Seepage-line exit point at the landward slope above the toe of the levee

$$L_2 = b_D - h_w \times \frac{m_x}{m_y} \quad (6)$$

$$\alpha_{steady/trans} = \tan^{-1} \left(\frac{L_2}{h_w} \right) + \tan^{-1} \left\{ \frac{2 \times \left(A_{steady/trans} - \frac{h_w \times b_D}{2} \right)}{h_w^2 + L_2^2 - \tan \left[\beta - \tan^{-1} \left(\frac{h_w}{L_2} \right) \right] \times h_w \times L_2} \right\} \quad (7)$$

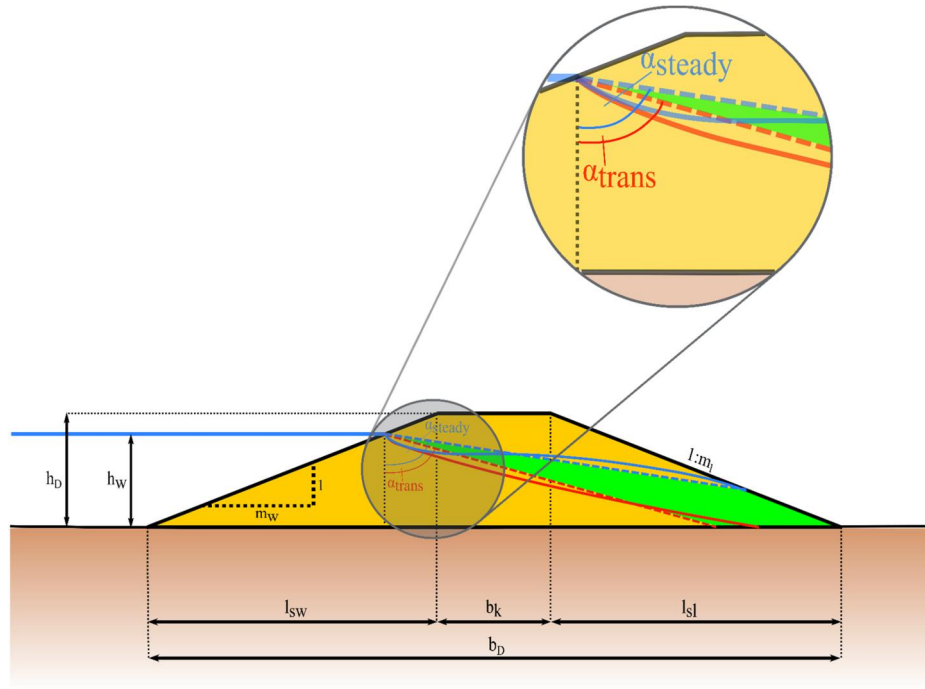


Figure 3. Definition of κ (dashed line in blue: simplified steady-state assumption; dashed line in red: simplified transient assumption).

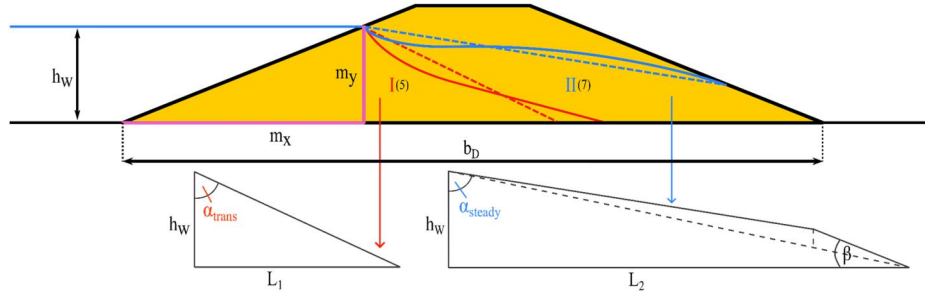


Figure 4. Mathematical representation of κ , divided into several sub-triangles.

The deviation between the steady-state and transient courses of the seepage line is expressed using the variable $\kappa = \alpha_{steady} - \alpha_{trans}$ [°], which depends on the respective time point t_i of the flood hydrograph. An automated approach allows the calculation of κ for each system state. As the duration of the peak water level increases, κ converges to zero, indicating the attainment of steady-state conditions.

To illustrate the variation of κ , Figure 5 presents its dependence on the peak duration t_p for various soil types listed in Table 1, and different rising durations t_r . As an example, the κ evolution is analysed for a homogeneous reference levee, with calculations exemplified for $s_W = 0.25, 0.50$, and 0.80 . The full results of the κ approximation, derived from 6000 simulation runs covering transient input conditions and different soil mechanical properties for the standardised homogeneous levee systems defined in Chapter 2, informed the regression analysis used to develop the function for κ .

The findings from the simulation runs yield the following general insights:

- Steady-state conditions ($\kappa = 0^\circ$) are achieved only if t_p is sufficiently long or in soils with significant sand content and thus high hydraulic conductivity. In the latter case, quasi-steady-state conditions may exist at the start of the peak. In contrast, soils with low hydraulic conductivity do not reach steady-state conditions under realistic flood conditions. This is attributed to the directly proportional relationship between the soil-dependent k_s and the hydraulic gradient i , as described by Darcy's law (1856).

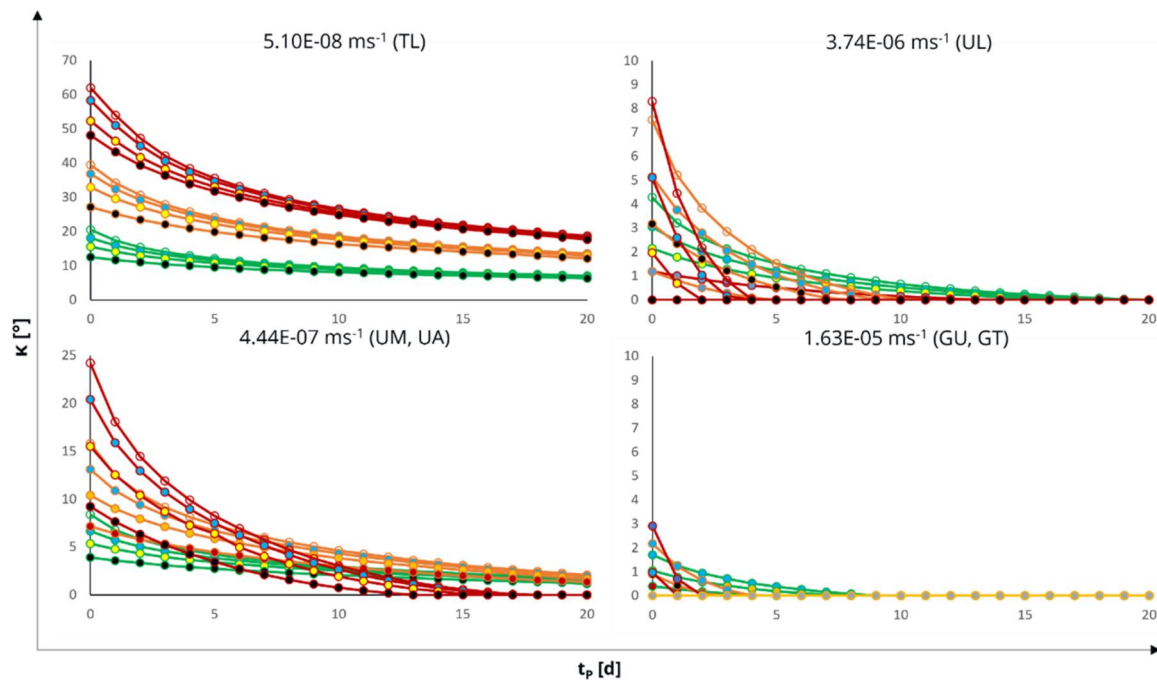


Figure 5. Results for the selected reference system ($b_D = 15$ m, $h_D = 2$ m, $b_k = 3$ m, and $m_{l,w} = 1:3$) for $s_W = 0.25$ (green), 0.50 (orange), and 0.80 (red) and $t_r = 1$ (colourless circle), 5 (blue circle), 10 (yellow circle), and 20 d (black circle).

- Furthermore, an increased t_r due to the hydraulic conductivity can result in a greater hydraulic load and pre-saturation of the levee.

5. Developing a general function for κ

To facilitate practical application of the previous analysis results (Figure 5), this chapter's focus is deriving an empirical equation for parameter κ . The functional relationship between the transient input variables is determined via numerical analysis, in which all input variables (t_r , s_W , t_p and k_s) are varied with respect to the κ evaluation.

Regarding the selection of a suitable mathematical expression for κ , linear functions exhibit simplicity and only two unknown parameters, making them well-suited for highly permeable and low-permeable soils. However, they are inadequate for silty soils, as they fail to capture the nonlinear, convergent trend of κ identified in the analyses. More flexible mathematical models are needed to address these limitations. Exponential functions are more suitable because they can accurately describe nonlinear progressions and enable precise adjustments across the full range of fill materials.

In the initial step, an inverse exponential function of the form $A \times e^{-b \times x}$ was fitted to all the analysis results of the simulation runs. The fitting process was based on nonlinear regression of the simulated κ -values for each parameter set, providing case-specific values for A and b as intermediate calibration results.

Here, the variable x in the functional expression corresponds to the peak duration t_p shown on the horizontal axis in Figure 6. The red curve represents the κ -values obtained from the transient simulations, while the blue curve illustrates the fitted exponential function.

Furthermore, in relation to the steady-state design assumption of Kozeny and Casagrande, the geometric changes in the dependence on s_W and L were considered, where L represents the horizontal reference width of the levee, corresponding to either L_1 and L_2 depending on the geometric conditions illustrated in Figure 4. The resulting ratio, which directly links the hydraulic loading to the levee geometry allows for a relatively simple and comprehensive consideration of geometric changes (b_D , h_D , b_k , and $m_{w,l}$) as well as the changed geohydraulic boundary conditions with respect to s_W .

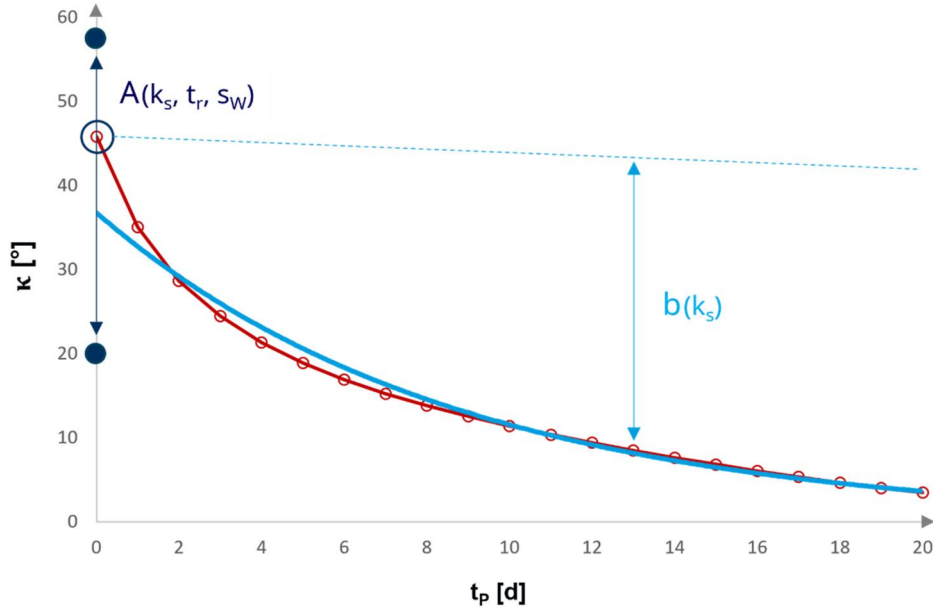


Figure 6. Influence of the input variables on the function parameters $A(k_s, t_r, \text{ and } s_W)$ and $b(k_s)$.

The parameters of the inverse exponential function are primarily determined by the control variables of the system. Parameter A represents the intersection of the graph with the ordinate, which corresponds to the end of t_r and beginning of a constant t_p for the geohydraulic load. It is determined by k_s , t_r , and s_W and is shifted along the ordinate according to the input variables. Parameter b influences the stretching and compression of the course of the κ curve and is related to k_s (Figure 6). According to Darcy's law (1856), an increase in k_s leads to a proportional increase in v_D and significant compression of the κ curve.

Furthermore, the investigations identified that the graphical representation of κ as a function over t_p for the same k_s and s_W generally exhibits a similar course but can be shifted by the same amount on the abscissa over t_p depending on t_r . As illustrated in Figure 5, varying t_r significantly affects the ordinate entry point.

To generalise this relationship, a comprehensive parametric search was applied (Cormen et al., 2009; Dasgupta et al., 2006), independent of geometric adjustments. The procedure systematically explores all relevant combinations of the transient input variables to determine their functional dependency on parameter A . In this process, t_p is expressed as a function of t_r , enabling the identification of the configuration that achieves the best agreement with the numerically derived κ -values.

Building on the parameter relationships identified in the previous step, an optimisation procedure was implemented to refine the coefficients and minimise the residual error between simulated and predicted κ -values. To perform this optimisation, genetic algorithms (Schöneburg et al., 1981; Tanweer et al., 2020) were applied, guided by the control variables t_r , s_W , t_p , and k_s . Thus, the function parameter d is directly related to the hydraulic conductivity k_s . To improve the scaling of the geometric adjustment, a constant exponent is used, and a variable scaling factor y depending on k_s is applied. Here, L denotes the geometric levee width, as defined in Eq. 5 to 7, for its respective application. Finally, the resulting relationships among κ [defined in °] and t_r [s], s_W [-], t_p [s], and k_s [ms^{-1}] are described by the following empirical equation (defined as κ_F in °):

$$\kappa_F = e^{s_W} \times \text{abs}[\ln(k_s)] \times e^{-d \times \left[\frac{t_p + (t_r \times 0.2)}{86400} \right]} \times \text{abs} \left[\ln \left(\frac{s_W}{L^{-y}} \right)^{0.63} \right] \quad (8)$$

The function parameters d and y exhibited clear patterns in the results, which is attributed to Darcy's law (1856). As k_s increases, v_D increases proportionally, resulting in larger d values because of the increasing compression of the graphs under the quasi-steady-state assumption. Correspondingly, the scaling factor y decreases with increasing k_s as the influence of the hydraulic

Table 2. Coefficients for the κ -approximation.

k_s [ms^{-1}]	Small ($L < 12$ m)		Medium ($12 \text{ m} \leq L \leq 21$ m)		Large ($L > 21$ m)	
	d	y	d	y	d	y
5.10E-08 (TL)	0.054	1.036	0.044	1.109	0.027	1.128
4.44E-07 (UM, UA)	0.194	0.644	0.064	0.710	0.055	0.934
3.74E-06 (UL)	0.778	0.403	0.122	0.671	0.079	0.855
1.63E-05 (GU, GT)	1.071	0.112	0.411	0.387	0.081	0.775
2.66E-04 (GW, GI)	1.933	0.079	1.924	0.373	1.815	0.656

gradient decreases. Within the system limits of this study (k_s) and for the defined auxiliary coefficients (d and y), each application case in the present hydraulic-conductivity range can be calculated using linear interpolation. The optimised coefficients d and y obtained from this calibration were subsequently grouped according to levee-width classes, forming the basis for the parameter ranges presented in Table 2.

A classification approach is applied to ensure a structured evaluation. In alignment with German regulations, levees are categorised into three classes according to their structural height and damage potential (DIN 19712, 2013; DWA-M 507, 2011). This study adopted a similar approach to evaluate d and y , which were categorised and defined for three classes (small, medium, and large), in accordance with the available data for width L within the homogeneous levee model range studied. Table 2 presents the results of the study.

To avoid abrupt changes in the function parameters d and y at the defined class boundaries, a transition zone of ± 1 m is introduced, with linear interpolation applied within this zone. For example, at $L = 12$ m ($k_s = 4.44\text{E-}07 \text{ ms}^{-1}$), an interpolated value of $d = 0.129$ and $y = 0.677$ is used. This ensures smooth parameter progression across the classification boundaries and avoids discontinuities in the functional behaviour.

6. Comparative analysis results and evaluation of κ and κ_F

The results focused on two main aspects. First, the analysis results are presented through a selection of representative examples and are compared with the empirically calculated κ_F , derived from the developed Equation 8 (Section 6.1). Second, all the simulated κ values are evaluated in relation to the calculated κ_F based on the soil type. For this purpose, a parameter called the angular deviation σ is introduced, which represents the difference between κ_F and κ and is defined as a measure of the quality (Section 6.2). Finally, the proposed approach is discussed in the overall context of this study (Section 6.3).

6.1. Validation of seepage-line approximation with κ and κ_F

In Figure 7, four representative flood scenarios for a homogenous levee ($b_D = 15$ m, $b_k = 3$ m, $h_D = 2$ m, and $m_{w,l} = 1:3$) with the following representative boundary conditions are presented.

1. $s_W = 0.95$ (blue), $t_r = 5$ d, $k_s = 5.1\text{E-}08 \text{ ms}^{-1}$, $t_p = 4$ d, $\kappa = 45.47^\circ$ (green) and $\kappa_F = 47.16^\circ$ (black dashed line) with $L = 9.30$ m, $d = 0.054$ and $y = 1.036$ (angular deviation $\sigma = |\kappa_F - \kappa| = 1.69^\circ$)
2. $s_W = 0.875$ (blue), $t_r = 10$ d, $k_s = 4.44\text{E-}07 \text{ ms}^{-1}$, $t_p = 2$ d, $\kappa = 12.53^\circ$ (green) and $\kappa_F = 13.56^\circ$ (black dashed line) with $L = 9.75$ m, $d = 0.194$ and $y = 0.644$ ($\sigma = 1.03^\circ$)
3. $s_W = 0.95$ (blue), $t_r = 10$ d, $k_s = 3.74\text{E-}06 \text{ ms}^{-1}$, $t_p = 1$ d, $\kappa = 1.68^\circ$ (green) and $\kappa_F = 1.67^\circ$ (black dashed line) with $L = 9.30$ m, $d = 0.778$ and $y = 0.403$ ($\sigma = 0.01^\circ$)
4. $s_W = 0.95$ (blue), $t_r = 1$ d, $k_s = 3.74\text{E-}06 \text{ ms}^{-1}$, $t_p = 12$ d, $\kappa = 0^\circ$ (green) and $\kappa_F = 0^\circ$ (black dashed line) with $L = 9.30$ m, $d = 0.778$ and $y = 0.403$ ($\sigma = 0^\circ$)

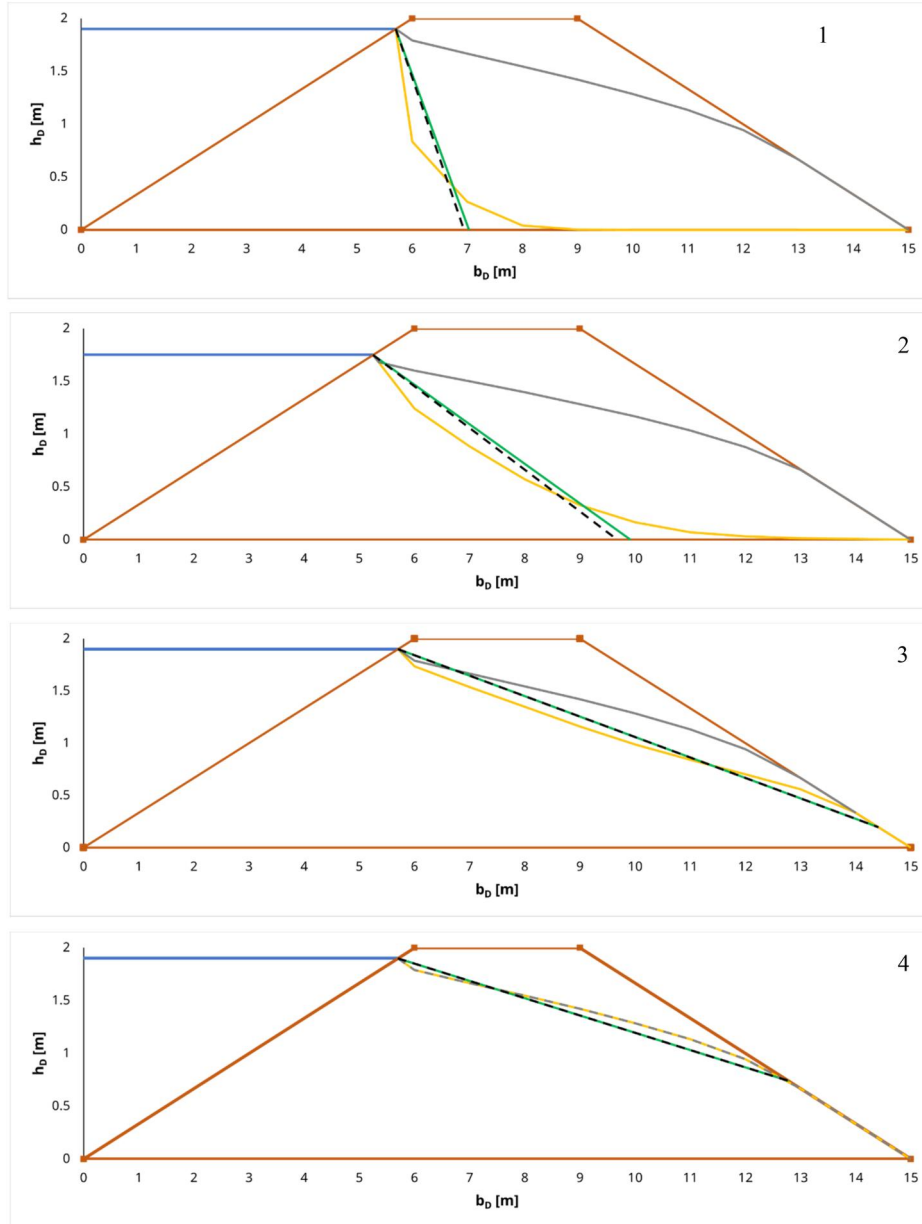


Figure 7. Comparative evaluation of the transient analysis based on the κ -approximation (green) and numerical calculations (yellow) as well as the steady-state seepage line (grey) and the κ_F -approximation (black dashed line) for the described boundary conditions (1) – (4).

A comparative analysis was conducted to demonstrate the goodness of fit between the transient flow analysis using numerical simulations (yellow) and the described approximation of κ in green (Chapter 4). The steady-state seepage line, according to Kozeny and Casagrande (Erb, 1965), is shown in grey for the respective flood conditions. For clarity, the simplified linear steady-state assumption is not shown in Figure 7.

Additionally, the value of κ_F is plotted as a black dashed line for each considered system state, to demonstrate the goodness of fit of the developed equation. The difference between κ and κ_F is indicated using angular deviation σ .

The results show a very good fit for the approximation of transient seepage lines using the developed empirical approach with the selected input variables. This is evidenced by the small angular deviations σ and low RMSE values observed in the comparative analysis (Figures 7 and 8).

When κ and κ_F are equal to 0 (boundary condition 4), the approximation closely matches the steady-state load case, and these findings can be extended to the other application cases. A slight difference is

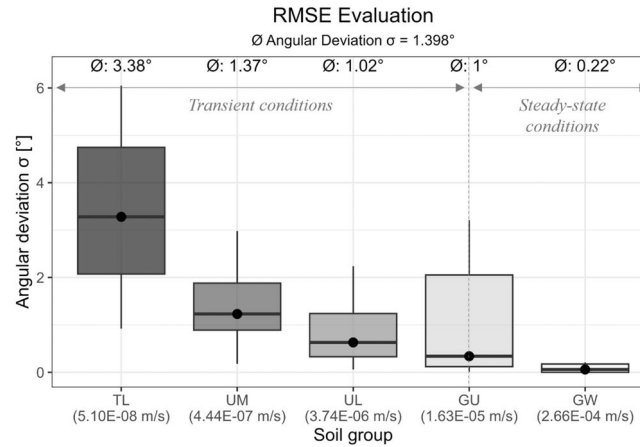


Figure 8. Box-whisker diagram of the angular deviation σ ; whisker-multiplier: 1.5-fold interquartile range, number of data points compared (κ_F to κ) per filling material: 1.200.

visible at the inlet and outlet of the seepage line owing to the quadratic parabola employed by Kozeny and Casagrande (Erb, 1965). However, it does not diminish the overall good fit of the developed methodology. Minor deviations in the approximation of the seepage line in soils where no threat to landside slope stability is expected can be considered negligible for the geotechnical assessment of system reliability.

The four boundary conditions shown in Figure 7 represent combined flood scenarios selected for validation and do not isolate single-parameter effects. A detailed sensitivity analysis, varying s_w , k_s and t_r , individually while keeping other parameters constant, is provided in the Appendix (Figure A2).

6.2. Statistical evaluation of seepage-line approximation with the angular deviation σ

Building on these examples, in Figure 7, all 6000 simulation runs are analysed in the same manner to determine the angular deviation σ , according to the actual seepage-line course. To this end, 240 reference hydrographs (s_w between 0 and 1) were used as transient input conditions for all examined systems. In the analysis ($n=60$ for each soil group), a constant t_p of 20 d was assumed to simulate the geohydraulic stress.

Figure 8 displays the σ values of all systems and soil groups in a box-whisker diagram. Box plots were used to clearly represent the angular deviation and make the central statistical parameters briefly visible (McGill et al., 1978). The quality of the approximation is evaluated using the root-mean-square error (RMSE) (Chai & Draxler, 2014; Hodson, 2022).

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (\kappa_{F,i} - \kappa_i)^2} \quad (9)$$

The results clearly indicate that σ tends to increase with a low k_s value. In engineering practice, soils with $k_s < 5.1 \times 10^{-8} \text{ ms}^{-1}$ (TL) are typically considered waterproof. Consequently, in most cases (TL), these events are well on the safe side compared with the assumption of a steady-state seepage line with $\kappa > 20^\circ$ in temporary flood events.

With a higher hydraulic conductivity of the system, the conditions more closely approach the quasi-steady-state load case, leading to lower κ , κ_F , and σ values. This trend is confirmed by the presented results, which demonstrate the effectiveness of the developed equation.

The larger scatter for the GU materials with $k_s = 1.63 \times 10^{-5} \text{ ms}^{-1}$ is noteworthy and is attributed to the dependence of the v_D dynamics on the geometric conditions of the levee. In smaller reference systems, quasi-steady-state conditions can already occur at the flood peak, whereas in larger systems, even after constant hydraulic loads, steady-state conditions are not achieved. These differences led to greater overall variability compared with the other soil groups. Nevertheless, the varying conditions of the developed models were reliably depicted.

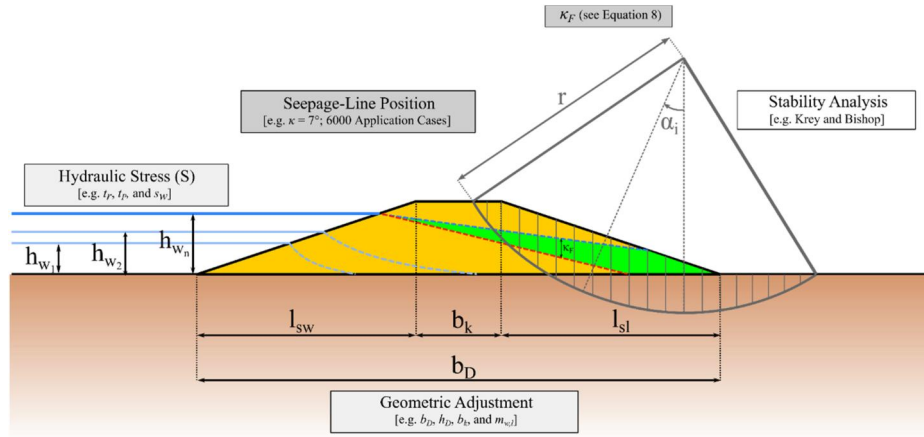


Figure 9. Relevant input parameters for stability assessment of a river levee system (grey: focus of the study).

To further verify the reliability of the developed method, the analysis was extended beyond the defined system boundaries. Both smaller and larger systems outside the application limits were examined. In addition, various k_s values were interpolated, and the results were validated. Comparative analyses demonstrated that the developed method maintained a good fit under these conditions, and the results were reliably reproduced.

6.3. Classification of developed methodology

The proposed approach allows the empirical derivation of the transient course of the seepage line for homogeneous levees and different soil types in the range considered in this study. This significantly reduces the computational demands compared with numerical simulations, providing an important tool that can be utilised, for example, to quickly and effectively assess the reliability of the entire system at the conceptual level during floods (Figure 9).

The developed functional solution (Eq. 8), which can be integrated into various modelling environments, is currently implemented in ProMalDes (*Protection Measures against Inundation Decision support*), a decision-support system for risk-based assessment of flood-protection infrastructure under transient hydraulic conditions (Bachmann et al., 2013).

By offering a computationally efficient representation of the seepage-line evolution, the approach provides a solid foundation for probabilistic analyses requiring the evaluation of numerous parameter variations. A key advantage is its numerical efficiency, as both steady-state and transient analyses require nearly identical computational effort. This enables large-scale Monte Carlo or reliability assessments by reducing runtimes by several orders of magnitude.

Consequently, approximating the empirical seepage line significantly reduces the time and computational effort compared to finite-element simulations, where each grid element must be recalculated for every parameter variation. This efficiency makes the proposed method particularly suitable for probabilistic stability assessments of homogeneous river levees, as demonstrated in this study. Its practical value for operational, rapid levee reliability assessment has already been confirmed through the probabilistic evaluation of the Rhine levees near Karlsruhe (Germany) presented in Öttl et al. (2026).

7. Conclusions and outlook

This study proposes a method for empirically evaluating transient seepage flow in homogeneous levees, with a focus on hydrological conditions and the influence of geometric and material parameters. Analysing a broad range of input variables, including the hydraulic conductivity k_s , peak duration t_p , rising duration t_r , and impoundment ratio s_w , of a flood hydrograph revealed that these factors significantly impact the transient location of the seepage line, which can be approximated from the typical steady-state assumption using the angle difference κ .

Building on this, a general function (κ_F) was developed to describe κ based on comprehensive simulations, with a high goodness of fit verified by the angular deviation σ . This enhances the applicability of the approach, by considering uncertain seepage lines and determining the system failure probabilities of the entire levee section at the conceptual level (for example in ProMalDes). In the context of flood risk management, it is important to prioritise flood protection measures and coordinate disaster responses when a flood occurs with a significant reduction in computational effort compared with conventional finite-element models.

To enhance the practical applicability in the medium-to-long term, the analysis of structured cross sections and other relevant failure mechanisms is recommended. Additionally, this approach can be applied to assess the stability of waterside slopes or to analyse the geotechnical failure modes of levee systems. The integration of artificial intelligence-based models may also offer new possibilities for refining and enhancing this empirical solution method, particularly for analysing structured cross sections and handling a comprehensive database.

These aspects constitute promising avenues for future research, which can lead to a more comprehensive understanding of the levee performance under dynamic hydrogeological conditions.

Practically, the proposed method contributes to a more reliable identification of critical seepage conditions, thereby supporting the prioritization of maintenance efforts, reducing the need for overly conservative designs, and ultimately enhancing flood protection while optimizing resource allocation. Furthermore, the approach can significantly improve decision-making in levee safety management, particularly in time-critical flood scenarios.

Author contributions

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Data availability statement

The values underlying Figures 5, 7 and 8 have been made openly available in a Zenodo repository under a CC-BY 4.0 licence: <https://doi.org/10.5281/zenodo.17976626> (Öttl, 2025). Raw GeoStudio simulation files (6000 transient seepage analyses) cannot be shared due to licensing restrictions; however, all processed data required to reproduce the figures and results reported in this manuscript have been provided.

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