Copula-Based Single Loss Development Model for the Pricing of Excess of Loss Reinsurance Motor Third Party Liability Claims

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Kurzfassung

Die genaue Schätzung eines Endschadenstandes ist für Rückversicherer von entscheidender Bedeutung für die Preisgestaltung und das Risikomanagement. Hierfür werden häufig das Chain Ladder oder Cape Cod Modell verwendet. Die Nachteile dieser Modelle bestehen darin, dass die Informationen eines Einzelschadens nicht vollständig berücksichtigt werden, die Merkmale des Rückversicherungsvertrags nicht korrekt angewendet werden können und eine Annahme über die Schadenverteilung getroffen werden muss. Da die derzeitigen Einzelschadenmodelle (SLD) nicht in der Lage sind, diese Nachteile zu überwinden, wird ein neues SLD-Modell basierend auf realen Kraftfahrzeughaftpflicht-Schadendaten (MTPL) entwickelt.

Die verfügbaren Schadendaten werden zunächst auf mögliche Einflüsse untersucht, um die Homogenität des Marktdatensatzes zu bewerten. Alle Einzelschäden des Marktes werden dann als Punktwolke bezüglich der Schadenständen, dem Zahlungsstand, und dem Entwicklungsfaktor (LDF) betrachtet. Hierbei wird die Zeitkomponente, die in den meisten aggregierten und SLD-Modellen berücksichtigt wird, implizit durch diese Parameter abgedeckt. Um ähnliche Schäden für die spätere Entwicklung zu finden, wird eine Clusteranalyse auf diese Punktwolke angewandt und die entsprechenden LDFs durch eine Gitternetz oder Bernstein-Copula modelliert. Der nächste Entwicklungsschritt berücksichtigt dann, basierend auf einer LDF-Oberfläche, den maximal möglichen LDF und die historische Schadenentwicklung gegeben durch ein Sprungmuster, um die Schadenprojektion zu verbessern. Bei diesem copula basierten SLD-Modell wird der nächster Entwicklungsschritt eines Einzelschadens aus der jeweiligen Copula, unter Berücksichtigung der beiden Bedingungen, gezogen.

Für die Bewertung werden verschiedene Tests in Bezug auf die Laufzeit, die Anzahl der Simulationen und die Plausibilität durchgeführt. Außerdem wird das copula basierte SLD-Modell auf reale MTPL Schadendaten angewendet, die von einem der führenden Rückversicherer zur Verfügung gestellt werden. Dabei wird ein Vergleich der Ergebnisse mit denen des Chain Ladder Modells, Munich Chain Ladder Modells und dem Cape Cod Modells durchgeführt. Zudem werden die simulierten und tatsächlichen Schadenverteilungen in einem Backtesting verglichen, da diese von großer Bedeutung für die Preisgestaltung in der Rückversicherung sind. Hierfür wird das copula basierte SLD-Modell auf den deutschen MTPL Marktdaten angewandt und die Auswirkungen der einzelnen Modellkomponenten analysiert. Außerdem wird die Modellgüte, bezogen auf unterschiedliche Marktbesonderheiten, untersucht, indem das copula basierte SLD-Modell auf den maltesischen, italienischen, schwedischen und dänischen MTPL Marktdaten angewandt wird.

Die Modellierung der Einzelschäden als Punktwolke ist ein vielversprechender Ansatz, der in Kombination mit der Gitternetz oder der Bernstein-Copula gute Ergebnisse liefert. Allerdings schränkt die Verwendung des Sprungmusters die möglichen Entwicklungen eines Einzelschadens ein. Darüber hinaus sorgt die Clusterung der Daten in Kombination mit der LDF-Oberfläche für zu viel Komplexität bezogen auf den Kompromiss zwischen Modellverbesserungen und Rechenaufwand. Daher sollte das derzeitige copula basierte SLD-Modell nicht in der Praxis verwendet werden, sondern dient vielmehr als neuer und vielversprechender Modellansatz für die Entwicklung von Einzelschäden, der weiter untersucht werden kann.

Abstract

The accurate estimation of ultimate claims is of vital importance for reinsurance companies in respect of the pricing and risk management. Among others, models like Chain Ladder or Cape Cod are widely used in practice. Disadvantages of these models are that information of each single loss is not fully taken into account, reinsurance contract features cannot be applied correctly, and an assumption about the ultimate loss distribution has to be done. Since current single loss development (SLD) models are not capable of overcoming these disadvantages, a new SLD model is developed based on real motor third party liability (MTPL) claims data.

The available claims data is analysed for possible influential effects to answer the question of homogeneity for the market data. All single claims of the market dataset are considered as point cloud focusing on the incurred values, the payment ratio, and the related loss development factor (LDF). Hereby, the time component that is usually considered for most of the aggregated and SLD models is implicitly covered by these parameters. For this point cloud as new model framework, a cluster analysis is performed to find similar groups for a later development. Based on each cluster, the related developments are then modelled by a grid-type or Bernstein copula and restricted by two conditions. The first condition is limiting the maximal possible LDF with an LDF surface while the second condition is focusing on the jump pattern, e.g. the historical development path of a claim, to improve the future developments. A next development step of a single claim is then drawn from the respective copula under these two conditions resulting in the new copula-based SLD model.

For the evaluation, several tests with respect to the runtime, number of simulations, and plausibility are done. Furthermore, the copula-based SLD model is applied to real MTPL claims data provided by a leading reinsurer. Hereby, a comparison of the results with the Chain Ladder model, Munich Chain Ladder model, and Cape Cod model is done. Moreover, a backtesting is performed to compare the simulated and original ultimate loss distribution which is of major importance for reinsurance pricing. Hereby, the copula-based SLD model is applied to the German MTPL market for which the impact of each model component is analysed. The full copula-based SLD model is also applied to the Maltese, Italian, Swedish, and Danish MTPL market to test its performance based on different market idiosyncrasies.

The model framework considering the point cloud is a promising approach working well in combination with the grid-type and Bernstein copula. However, the jump pattern is not working correctly, limiting the possible developments of a single claim. Furthermore, the clustering of the data combined with the LDF surface adds too much complexity to the model focusing on the trade-off between model improvements and computing efforts. Thus, the current copula-based SLD model should not be used in practice but serves as a new and promising model framework for the development of single claims which can be developed further.

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AD	Average Distance	68
ADM	Average Distance Between Means	68
APN	Average Proportion of Non-Overlap	68
a.s.	Almost Surely	79
CLARA	Clustering Large Applications	63
CLARANS	Clustering Large Applications Based on Randomized Search	66
CPI	Consumer Price Index	27
FOM	Figure of Merit	68
IBNER	Incurred But Not Enough Reserved	4
IBNR	Incurred But Not Reported	7
IBNYR	Incurred But Not Yet Reported	4
LDF	Loss Development Factor	24
MSEP	Mean Squared Error of Prediction	115
MTPL	Motor Third Party Liability	2
NP	Non-Proportional	2
P	Proportional	2
PAM	Partition Around Medoids	66
P&C	Property and Casualty	20
RAM	Random Access Memory	57
RBNP	Reported But Not Paid	15
RBNS	Reported But Not Settled	15
SLD	Single Loss Development	3
w.l.o.g.	Without Loss of Generality	28
XL	Excess of Loss	3

Nomenclature

α	Significance level for a statistical test.
a_i	The i-th mesh grit point related to the incurred values to create the grit for the LDF surface.
$a_k^\star$	Mesh grit point for the related LDF $LDF_k^\star$ used for the splin interpolation.
$a(x_i^g)$	Average dissimilarity of the point $x_i^g \in S_g$ to all other objects of its cluster S_g.
β_l	A constant that represents the loss development and could be identified with the Chain Ladder development factors f_l.
b_i	The i-th mesh grit point related to the payment ratio values to create the grit for the LDF surface.
$b(x_i^g)$	Average dissimilarity of point $x_i^g \in S_g$ to all objects of its nearest cluster neighbour S_{g^*}, $g \neq g^*$.
B_i	The i-th reduced Cluster sample for $i = 1,...,k^*$, and k^* being the number of cluster samples.
$B_j(a^\star)$	Normalized B-spline basis function with coefficients e_j.
χ^2	Test statistic of the χ^2 test of independence.
$c^\phi(y_1,y_2)$	Copula density with arguments y_1,y_2 based on the function ϕ defining the type of the copula.
$C^\phi(y_1,y_2)$	Copula with arguments y_1,y_2 based on the function ϕ defining the type of the copula.
$C_{i,l}$	Aggregated claims amount for the i-th accident and l-th development year.
$C_N^{emp}(y_1,y_2)$	Empirical copula-based on N points and arguments y_1 and y_2.
$C_{i,l}^{Inc}$	Aggregated incurred amounts for the i-th accident and l-th development year.
$C_{i,l}^{Pa}$	Aggregated paid amounts for the i-th accident and l-th development year.
CAR_i	Closing adjustment rate for the number of IBNYR for accident year i.
CC_i^t	Cluster centre for the i-th cluster in the t-th iteration.

CT	Number of required transitions to count the transition probabilites as meaningful.
$\mathbf{C}(y_1, y_2)$	Two-dimensional copula function with arguments y_1 and y_2.
$\hat{C}_{i,n}$	Estimator of the ultimate claim amount for the i-th accident year in the understanding of the Chain Ladder model.
$\mathbb{C}_l^{Pa}$	Set of available information for the paid values up to the l-th development year.
$\mathbb{C}_l^{Inc}$	Set of available information for the incurred values up to the l-th development year.
δ_i	Margin used for the stabilisation, $i = 1$ for the franchise clause and $i = 2$ for the SIC clause.
d	Residual distance $d = LDF_k^\star - h_\psi\left(a_k^\star\right)$ used to estimate the 'fidelity' to the data.
D	Priority or deductable of an excess of loss contract.
$D_{i,j,l}^{stab}$	Stabilised priority of the j-th claim in the i-th accident year and the l-th development year.
$\eta_{i,j}$	Adjusted relative frequencies of a contingency table $\zeta_{i,j}$ so that the margins are uniformly distributed.
ε	An additional incurred amount to increase the maximal incurred value Inc^{surf} for the loss development surface grid.
ε_ψ	A check function which evaluates the residual distance d by applying a weight function w_ψ.
ε^{fit}	Termination condition in respect of the residual distance for the LDF surface fit.
e_j	Coefficients of the B-spline basis function.
$\hat{e}_j$	Respective coefficients for the B-spline representation.
E_i	The exposure value related to the i-th accident year.
f_l	Development factor in the understanding of the Chain Ladder model from development year $l-1$ to l.
f_l^{Inc}	Development factor based on the incurred values from development year $l-1$ to l.
f_l^{Pa}	Development factor based on the paid values from development year $l-1$ to l.
f_l^{stab}	Adjustment factor for the l-th development year.

$F_{i,l}$	True development factor in the understanding of the Chain Ladder model for the i-th accident year from year $l-1$ to l.
$F_{i,j,l}^{stab}$	Stabilisation factor for $i=1,...,n,\ j=1,...,k_i,\ l=2,...,n$.
$\mathbf{F}$	Distribution function.
$\hat{\mathbf{F}}_k^i$	Two-dimensional $k=1,2$ pseudo observations based on the empirical distributions $\mathbf{F}_k^N(X_k^i)$ based on N points evaluated at the i-th data point X_k^i.
$\mathbb{F}$	Discrete random vector $\mathbb{F}=(\mathbf{F}_1,\mathbf{F}_2)$ with uniform margins over the $T_i=\{0,1,...m_i-1\}$.
γ	Minimal value in a contingency table used to adjust $\eta_{i,j}$ so that all entries are positive.
g	Iterator for the number of clusters in a point cloud.
g_{opt}	Optimal number of clusters.
$\mathbf{g}_{\mathscr{X}}$	Density function of the ultimate claim size.
G	Maximum number of clusters in the point cloud.
$Gap_{\tilde{n}}(g^*)$	Gap statistic based on all points $\tilde{n}$ defining the point cloud and the cluster number g^*.
$\mathbf{G}_{\mathscr{X}}$	Distribution function of the ultimate claim size.
h_ψ	ψ-th conditional quantile function with.
$\hat{h}_{\psi,L_p}(a^\star)$	ψ-th L_p quantile smoothing spline as a non-parametric estimator for $h_\psi(a^\star)$.
$\mathbf{H}$	Two-dimensional distribution function.
i	Iterator.
I_l	Index for stabilisation for the l-th development year. If $l=B$, then the index refers to the basis year of the stabilisation.
I_{p_1,p_2}	Subintervals to create a two-dimensional grid with $(p_1,p_2)\in T_1\times T_2$, $T_i=\{0,1,...m_i-1\}$ and a grid size $m_1,m_2\in\mathbb{N}$.
$IBNYR_{i,l}^{\#}$	Number of IBNYR claims per accident year i and development year l where a claim exceeded the priority for the first time.
$IBNYR_i^{\#}$	Expected number of IBNYR claims per accident year $i=2,...,n$.
$ID_{i,j}$	Claims identification number for the j-th claim in the i-th accident year.

$Inc_{i,j,l}$	The total claim size of the j-th claim in the i-th accident year in its l-th development year.
Inc_i^{PC}	The incurred value of the i-th claim in the point cloud.
Inc^{surf}	The maximal incurred value $\max_{i=1,\ldots,\tilde{n}} \left(Inc_i^{PC}\right)$ of the point cloud.
$\overline{IBNYR_l}$	Average number of IBNYR claims per exposure for the development year l.
j	Iterator.
J	Number of Monte Carlo simulations.
κ	A constant that represents the average loss ratio in the Cape Cod method.
$\hat{\kappa}$	Unbiased and robustified estimator for κ in the Cape Cod method.
k_i	Number of single claims for the i-th accident year.
k^*	Number of cluster centre samples due to a data reduction.
λ^{Inc}	Slope of the regression lines in the residual plots based on the incurred values.
λ^{Pa}	Slope of the regression lines in the residual plots based on the paid values.
λ^{Spline}	Smoothing parameter to control the trade-off between 'fidelity' to the data and 'roughness' to the fit.
l	Iterator.
L	Limit of an excess of loss contract.
$L_{i,j,l}^{stab}$	Stabilised limit of the j-th claim in the i-th accident year and the l-th development year.
$LDF_{i,j,l}^{Inc,add}$	Additive approach of estimating the individual loss development factors of the j-th claim in the i-th accident year from development year l to $l+1$ based on the incurred values.
$LDF_{i,j,l}^{Inc,mul}$	Multiplicative approach of estimating the individual loss development factors of the j-th claim in the i-th accident year from development year l to $l+1$ based on the incurred values.
LDF_{new}^{Inc}	New development factor for the incurred values.
$LDF_{a_i,b_j}^{margins}$	The marginal maximal one-year development factor in the mesh grid $(a_i,a_{i+1}] \times (b_j,b_{j+1}]$ under the monotonously increasing assumption.

$LDF^{max}_{a_i,b_j}$	The maximal one-year development factor in the mesh grid $(a_i, a_{i+1}] \times (b_j, b_{j+1}]$ under the monotonously increasing assumption.
LDF^{min}_i	The smallest loss development factor than can be obtained from development year i to $i+1$ based on the monotonicity assumption.
$LDF^{PR,add}_{i,j,l}$	Additive approach of estimating the individual loss development factors of the j-th claim in the i-th accident year from development year l to $l+1$ based on the payment ratio values.
$LDF^{PR,mul}_{i,j,l}$	Multiplicative approach of estimating the individual loss development factors of the j-th claim in the i-th accident year from development year l to $l+1$ based on the payment ratio values.
LDF^{PR}_{new}	New development factor for the payment ratio.
LDF^{PC}_i	The loss development factor of the i-th claim in the point cloud.
$LDF^{PC,Inc}_i$	The one-year development factors from the point cloud related to the incurred values of the i-th data point. For simplicity, this is further written as LDF^{Inc}_i.
$LDF^{PC,PR}_i$	The one-year development factors from the point cloud related to the payment ratio values of the i-th data point. For simplicity, this is further written as LDF^{PR}_i.
$LDF^{surf}_{a_i,b_j}$	The maximal one-year development factor in the mesh grid $(a_i, a_{i+1}] \times (b_j, b_{j+1}]$.
$LDF^{\star}_k$	The LDFs used for the splin interpolation based on $LDF^{margins}_{a_i,b_j}$.
μ_i	m equidistant intervals on $[0,1]$ for $i = 0,...,m$.
m_i	Grid size of the i-th dimension to set up the Bernstein and grid-type copula.
m^{surf}_i	Grid size of the i-th dimension for the loss development factor surface.
$\hat{m}_{\lambda^{Spline},L}$	Fitted B-spline curve for L_1 or L_∞.
M_i	Number of classes in a contingency table for the i-th dimension.
M^z_i	Reporting threshold for the i-th accident year for cedant z.
ν_i	m equidistant intervals on $[0,1]$ for $i = 0,...,m$.
n	Number of known accident and development years.
$\tilde{n}$	Total number of observations in a point cloud.

$\hat{n}_{i,j}$	Total number of points $LDF^{margins}_{a_i,b_j} \neq Na$ for the two-dimensional case.
$n^{\star}$	Number of single claims resulting from of the claims ID $ID_{i,j}$.
N	Number of internal knots for the spline interpolation.
$\mathcal{N}$	Random variable for the claim frequency.
ω_i	Several thresholds for $i = 1,..,6$ to distinguish different jump areas.
$Out_{i,j,l}$	The reserve of the j-th claim in the i-th accident year in its l-th development year.
ψ	Scalar with $\psi \in [0,1]$ used for the conditional quantile function h_{ψ}.
$\phi\left(m_i, p_i, y_i\right)$	Function defining either the Bernstein polynomials or the indicator function used for the grid-type copula-based on the grid size m_i, a point on the grid p_i, and the function argument y_i.
$Paid_{i,j,l}$	The payment of the j-th claim in the i-th accident year in its l-th development year.
$PR_{i,j,l}$	The payment ratio of the j-th claim in the i-th accident year in its l-th development year, denoting the ratio of the already paid amount.
PR_i^{PC}	The payment ratio value of the i-th claim in the point cloud.
q	Order of the spline.
q_l	A drawn sample point in $[1,...,m]$ that is used to simulate copula outputs for $l = 1,2$.
$\mathbf{Q}_k$	Quantile function of the empirical distribution function $\mathbf{F}_k^N$ with $k = 1,2$.
ρ	Spearman's rank correlation coefficient.
r	Pearson correlation coefficient.
$\hat{r}_{i,l}$	Standardised residuals of the Chain Ladder model for the i-th accident year and the l-th development year.
$R(D)$	Relief function depending on the priority of an excess of loss contract.
σ_l	True variance in the understanding of the Chain Ladder model from development year $l-1$ to l.

σ_l^{Inc}	True variance based on the incurred values from development year $l-1$ to l.
σ_l^{Pa}	True variance based on the paid values from development year $l-1$ to l.
$\hat{\sigma}_l$	Estimator for the variance in the understanding of the Chain Ladder model from development year $l-1$ to l.
$s\left(x^g\right)$	Silhouette of the g-th cluster.
$s(a^\star)$	B-spline representation.
$\bar{s}\left(x^g\right)$	Average silhouette width for the points lying in the g-th cluster.
S_g	The set of all points of the g-th cluster.
S_N^{CvM}	Cramer-von-Mises test statistic value to test the goodness of a copula fit.
τ	Kendall rank correlation coefficient.
θ	Vector with B-Spline coefficients to be minimized.
$\theta_i^l\left(\omega\right)$	Classification function for the l-th claim with the i-th development step and thresholds ω.
θ^{JP}	A function that groups the one-year development factors into several jump areas.
T	Test statistic for association between paired samples.
T_i	The i-th step for $T_i = \{0, 1, ...m_i - 1\}$ with a grid size $m_1, m_2 \in \mathbb{N}$.
T^{knots}	General knot mesh to set up the B-spline representation of a smooth function.
TH	Transition history based on a series of the classification function $\theta_i^l\left(\omega\right)$.
$TP_{i,j}^k$	Transition probabilities based on the time horizon k, the current case i, and the consecutive case j.
Ult_i^{Agg}	Aggregated ultimate value for accident year i.
$Ult_{i,j}^l$	Ultimate values for $i = 1, ..., n,\ j = 1, ..., k_i$ for $l = 1, ..., J$ simulations.
$Ult_{i,j}^{l,XL}$	Layered ultimate values for $i = 1, ..., n,\ j = 1, ..., k_i$ for $l = 1, ..., J$ simulations.
v	Inflation.

$w_{\psi}(d)$	Weight function used to derive the 'fidelity' to the data based on the residual distance d.
W_{g^*}	Average intracluster differences based on g^* clusters.
WSS_{g^*}	Within-cluster sum of squares.
ξ_i	The euclidean distance of the loss development factor changes for the incurred and payment ratios of the i-th data point in the point cloud.
ξ^{new}	Threshold for which the claim frequency can be estimated based on an already known claim frequency at ξ^{old} and the claim size distribution.
ξ^{old}	Threshold for which the claim frequency was estimated.
x_i	The i-th data point among all data points of a point cloud consisting of $\tilde{n}$ data points.
x_i^g	The i-th data point lying in the g-th cluster.
$\bar{x}_g$	Mean of all points x_i^g lying in cluster S_g.
X	Random variable describing the mesh grit points used for the fitting of Qualitatively Constrained Quantile smoothing B-splines.
X_k^i	Incremental changes of the incurred value and payment ratios rewritten for simplicity reason for creating the empirical copula for two-dimensions $k = 1,2$ and single points $i = 1,...,N$, where N denotes the number of allowable development factors.
$\hat{X}_l$	A sample point drawn from a Beta distribution utilizing the sample q_l for $l = 1,2$.
$\mathscr{X}_{i,j}$	Random variable for the claim size of the j-th claim in the i-th accident year.
$\bar{y}_{i,j}$	Adjusted contingency table of the relative frequencies obtained in $\eta_{i,j}$ so that the margins are uniformly distributed and all entries are positive.
Y	Random variable describing the loss development factors used for the fitting of Qualitatively Constrained Quantile smoothing B-splines.
$\zeta_{i,j}$	Relative frequency of data points in a contingency table build on a grid with size m so that $i,j = 1,...,m$.
Z	A MTPL contract from the group of possible cedants.

1. Introduction

1.1. Concept of Reinsurance and Problem Definition

At the beginning of the 2020s, the existence of reinsurance is essential and hardly known to anyone [197]. While many different definitions of reinsurance exist, the following puts a possible description in a nutshell:

'Reinsurance is basically insurance for insurance companies.' [104]

Among others, the field of reinsurance evolved from the larger family of risk management [104]. As such, a primary insurance company transfers a part of their risk to the reinsurer, who receives a part of the premium in return [141] and reimburses the primary insurer for the expenses for internal and personal costs, which are included in the corresponding insurance premium. In doing so, reinsurance allows the reinsured parties to reduce their risk exposure and own capital requirements [197]. Additionally, it smoothes the volatility in an insurance company's earnings and protects their balance sheet [197]. Due to the global presence of reinsurers, the risks that are taken over are diversified all over the world[1]. This has many benefits, not only for primary insurance companies, but also for society [197]. Due to the effect of diversification, a reinsurer needs to hold less capital to cover the risks compared to an insurance company, that can spent this free capital to write more business, enabling innovation and economic growth [197]. Furthermore, reinsurers provide advisory expertise, helping insurers to underwrite and process risks and new business. Focusing on society, reinsurance stabilises the domestic insurance markets, making insurance available more broadly for an affordable price. Since many risks could not be insured without reinsurance, this encourages commerce and trade, unlocking the full potential of insurance as a catalyst for economic growth [197]. Additionally, reinsurers provide long-term capital to the economy, thus supporting the production and provision of goods and services since most investments are usually made with a long-term perspective. Moreover, reinsurers are improving the capital allocation by constantly scanning the horizon for indications of emerging and future risks to put a price on them [197].

Reinsurance can be divided into non-life and life reinsurance. The former covers several lines of business such as property, automobile, liability, engineering, marine, agriculture, and aviation, while life reinsurance focuses on mortality, morbidity, critical

[1] A good example are natural catastrophes. Losses caused by a hurricane in America can be offset by premiums in Europe if no natural catastrophe occurred there.

illness, disability, long term care, medical expenses, and longevity. With this distinction, the diversification of risk coming from the reinsurance industry is well reflected in the premium ratios. For primary insurance companies, the global non-life premium in 2018 was USD 2400 billion and the global life premium USD 2900 billion. In comparison to that, the reinsurance premium in 2018 was only 4.9% of their premium volume split into USD 180 billion for non-life and USD 75 billion for life reinsurance [221], which is justified by the diversification effect and the reinsurance of mainly larger and more substantial risks. Nevertheless, the insured risks have to be accessed properly in order to calculate a reasonable premium, which is usually done using actuarial methods. On the reinsurance side, a distinction has to be made between proportional (P) and non-proportional (NP) treaties, as well as between short-tail[2] and long-tail[3] lines of business. While the premium and claims for P treaties are shared between primary insurer and reinsurer according to a predefined ratio [197], only claims exceeding the insurers' deductible up to an agreed cover limit are reinsured in the case of NP treaties [197].

This work focuses on Motor Third Party Liability (MTPL) claims, which are usually insured on a NP basis and count as a long-tail line of business within the non-life segment. It covers the insured's legal liability for death or disability of third-party losses or damages to third-party property, e.g. injuries due to road traffic accidents. It has to be noted that automobile data in general can be distinguished into motor hull, meaning just material damage, and MTPL claims. Since the MTPL claim part is usually larger than the motor hull part, it drives the risk for an insurer. Furthermore, claims of this type happen often and have a major impact on the individual and on society [196]. This line of business is chosen since MTPL insurance is compulsory in most countries and is one of the dominating lines within the non-life market in terms of premium volume [111]. Furthermore, MTPL risks comprise a high uncertainty and risk due to their long-tail character, which makes the pricing challenging.

Therefore, aggregated models are predominantly used [151] for modelling and reserving in the reinsurance industry[4] and are based on the principle of the law of large numbers. Hereby, all losses are aggregated according to their corresponding accident year and are then used for the prediction of the final settlement value. The best known method for this is the Chain Ladder method, which will be described in Section 2.8.1. However, aggregated models have disadvantages that are particularly challenging for the pricing and reserving of NP long tail reinsurance contracts [225] such as MTPL

[2]Claims for short-tail lines of businesses are usually settled within a short time horrizon, e.g. property business [41].

[3]For long-tail lines of business, the claim settlement takes many years over which the potential costs continue to rise through inflation. Those lines incorporate a huge uncertainty in their final settlement value [41], e.g. motor third party liability business and liability lines to some extent.

[4]Usually, pricing and reserving use similar techniques based on different data. Reserving focuses on the reinsurance loss data while a pricing is based on the insurance loss data.

treaties. The main downside is that it is not possible to simulate a severity distribution [157] or to apply the reinsurance contract structure correctly. Besides that, a reinsurer has to face the difficulty of sparse claims data [74, 157, 240], which is disrupting the convergence of the law of large numbers and, thus, the main principle of aggregated models. Consequently, the estimated price might not be accurate or requires a larger workload for actuaries due to manual adjustments of the models and the risk of human error. Hence, the uncertainty related to the long-tail character of MTPL claims results in a higher risk premium and, thus, a higher price for an Excess of Loss (XL) treaty.

One way to avoid this is to use Single Loss Development (SLD) models[5]. Since all claims are developed on their own, this offsets many shortcomings of aggregate models and allows a better assessment of the risk. Consequently, this would allow to price business more accurately resulting in a competitive advantage. Despite the advantages of such models, they are rarely used in practice on the reinsurance side. This is due to the fact that many of the individual loss models developed so far are not suitable for practical use. They pose the problem of model calibration when only sparse data is available and in some cases exhibit large fluctuations and deviations. As a result, acceptance of individual loss models is limited in the reinsurance sector and only some market leaders use them for specific contracts and tasks.

Hence, this thesis aims to develop a SLD model using data from the MTPL business for the pricing of NP long-tail reinsurance contracts, which does not only provide realistic estimations but is also applicable in practice. Therefore, extensive testing and thorough analysing have to be done to verify the goodness of the model and to justify a later usage.

1.2. Introduction to Reinsurance Pricing

For P reinsurance treaties, a reference between the premium income of a portfolio and the related split between the primary and reinsurance company exists. However, this is not possible in the case of NP reinsurance contracts. Here, an independent calculation is required to determine the price for a specific layer. The layer defines the range for which claims are paid by the reinsurer and consists of a priority D and the limit L.

Notation: L *xs* D, e.g. EUR 2 million *xs* EUR 1 million

[5] Depending on the scientific literature, such models are also called: individual, claim-by-claim, micro-level, granular, or single loss development models.

Without focusing on specific terms and conditions in a reinsurance contract[6], the reinsurer pays everything that exceeds the priority up to the limit for each single claim of the reinsured portfolio.

$$\text{Amount paid by reinsurer} = min\left(max\left(\text{claim size} - D, 0\right), L\right)$$

Since loss-free periods are rather normal than exceptional, the pricing of such contracts is challenging [74]. Depending on the existing loss history and the structure of the reinsurance layer, several different procedures can be used to determine a price as described in Liebwein [141]. Based on a good data basis, a burning cost quotation can be done by setting the time adjusted layer losses in relation to the time adjusted premium volume. If there is little or no claims experience, purely model-based procedures working with assumptions about the loss distribution can be applied. Both methods can also be combined in the extrapolation procedure. Usually, the loss experience is too sparse to get a meaningful result for the burning cost quotation but can be used to calibrate the loss distribution assumptions from the model-based procedure better. If there is no experience data given, an exposure rating could be applied. Hereby, only the composite and covered portfolio with the related premium is taken into account. The last method is the pay back method which uses the claims frequency and the period within which the reinsurer has compensated a total loss to determine the expected loss and price [141]. It should be clear that the more experience is available, the more accurate is the calculation of the necessary price. Since the burning cost or extrapolation procedures as mixture of the burning cost and model-based procedure are used in practice for long-tail NP pricings, this is also focused on in this work. For the determination of the layer damage, several components have to be taken into account. Hereby, the loss experience of a cedant is usually given in the form of a claim triangle as shown in Figure 1.1.

The claims triangle consists of the already known claims which have to be projected to their ultimate value. Hereby, those claims are usually named Incurred But Not Enough Reserved (IBNER) claims since they are already known but not settled, which leaves the possibility for further development[7]. Besides those already known claims, also claims that occurred and have not been reported yet as well as future claims falling in the coverage period have to be considered and are known as Incurred But Not Yet Reported (IBNYR) claims[8]. Additionally, a possible tail development of the claims has to be taken into account since the usual projection ends with the oldest known

[6]Sometimes reinstatements are defined for the layers meaning that if the layer damage has been paid once, it is refilled for the number of reinstatements, limiting the amount that has to be paid in total by the reinsurer. Other features would be annual deductibles on the side of the primary insurer or aggregate limits. The default case is that every claim is paid.

[7]This development can reduce or increase the claim sizes.

[8]Those naming conventions are not unique in the scientific literature and are further described in Section 1.3.4.

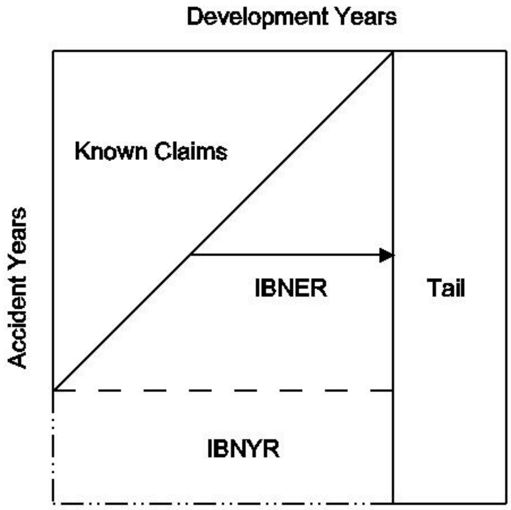


Figure 1.1.: Schematic Representation of a Claims Triangle.

development year of the triangle. In order to estimate the ultimate value of the IBNER claims and to incorporate the IBNYR claims, a variety of actuarial methods and models are available.

These methods have to be divided into aggregated and individual loss models [76]. While aggregated methods accumulate all claims of an accident year to estimate the respective total ultimate value for this accident year[9], all single claims are projected on their own in the case of individual loss models [225]. Furthermore, a distinction between deterministic and stochastic models can be done. Deterministic models calculate a point estimate of the future reserves without using random variables making it easy to interpret and communicate the results [76]. In comparison, stochastic models allow for random variations of future payments, they are explicit about the assumptions used, and in general, they allow using tools from risk theory [76]. However, this makes such models more complex and allows for criticsm that the model assumptions are too simple and unrealistic [76].

Due to their simple application and adequate results, aggregated models such as the Chain Ladder method are still predominantly used in practice at end of the 2020's. Especially for large datasets, these models provide stable results due to the law of large numbers. Nevertheless, those methods also have many disadvantages. Due to the aggregation of claims per accident year, the information about single claim development is lost. This can limit the predictive power of a pricing model and is problematic for the application of reinsurance contract features that are based on single claims basis such as the XL structure or the stabilisation[10] of claims [58]. Additionally, the most recent paid claims have a large impact on the extrapolation of the loss triangles and outliers might lead to a huge overestimation or underestimation of the overall reserve [241].

[9]The best known and most frequently used method for this is the Chain Ladder method.
[10]This feature will be discussed in Section 4.6.2.

This is aggravated by the fact that the lowest number of claims is available in the most recent accident years since many claims have not yet been reported at that time. Overall, this leads to the necessity of manually smoothing the development to avoid unreasonable projections for the most recent accident years in practice. Furthermore, so far known IBNYR claims are implicitly developed within those methods making the control and traceability of the related impact difficult. A further aspect is that the adequacy of these models is getting worse with a decreasing number of claims as it is the case for reinsurers [90, 157]. This has to be offset by making some strong assumptions about the loss or frequency distributions and cannot be avoided by adding stochastic elements to the aggregated methods. Overall, the input and adjustment from actuaries is required to offset those shortcomings, which makes the assessment of a risk adequate price difficult and open for a human bias.

The usage of SLD models can offset these disadvantages to a certain extent. Since SLD models simulate the ultimate values for each single claim, the full available information of the claim history can be accessed and a case by case estimation can be done. In comparison to aggregated models, this would allow for a better estimation of the risk adequate price for each single claim. Additionally, they are less susceptible to the number of individual claims per accident year since they are not based on the law of large numbers. This is an advantage for a reinsurance pricing where usually only sparse claims data is available. Nevertheless, SLD models need to be calibrated for practice as well, which requires a solid data basis. On top of that, no assumption of the ultimate single loss distribution has to be done and IBNYR claims as well as extreme scenarios can be calculated explicitly and incorporated in the simulations. Focusing on reinsurance structures, e.g. applying the layer structure of an XL treaty or applying the stabilization to the claims[11], these features can be applied on a single claim basis without the usage of further assumptions.

However, modelling single claims is more challenging and the above named advantages come at the price of more complexity. Additionally, single claims data cannot be adjusted for possible trend effects in the data, which will be further discussed in Section 2.4. This is a large disadvantage compared to commonly used aggregated models for which several methods exist to deal with trend effects. In summary, SLD models are more complex and prone to trend effects in the data but offer the possibility of outperforming commonly used aggregated models, especially in the case of reinsurance pricings. Hereby, their practical application has the potential to improve the pricing quality and to reduce manual work at the same time.

[11] See Section 4.6.2.

1.3. State of Research

To illustrate the motivation for this work, the requirements for aggregated and, in particular, for SLD models have to be discussed briefly. Therefore, it has to be mentioned that with regard to loss models, the usage of pricing and reserving models can be used interchangeably. Hereby, both fields, reserving and pricing, use the same models and methods but with a different data level and might interpret the output differently. While reserving is done on the reinsurer's data aiming at setting a correct reserve for all payments the reinsurer has to pay, pricing is based on cedant data to find the risk adequate price of the current portfolio. Both are subject to and depend on a certain level of optimism, realism, regulations, risk, and uncertainty. Due to the focus and aim of this thesis to develop a SLD model for pricing purpose, the term pricing models will be further used even if these models are referred to as reserving models in the scientific literature. A comprehensive overview of scientific literature about loss reserving in general can be found in Schmidt [199].

After a short discussion of the requirements, the current state of research for aggregated models as well as for individual loss models is discussed briefly. Therefore, also the separation of Incurred But Not Reported (IBNR) claims in IBNYR and IBNER claims is targeted since a separated development of both can be achieved with individual loss models.

1.3.1. Requirements for SLD Models in Theory, Science, and Practice

A model is a simplistic image of reality [37]. Therefore, a well-known and real scenario has to be understood, mathematically reproduced, and finally improved to predict unknown scenarios of the same kind [26, 37]. In order to achieve this, the qualitative and quantitative aspects of what has to be modelled, the related variables, and their impact have to be clarified. For the model development itself, it is necessary to clarify the relationship network of variables and find instruments to describe these interactions and dependencies [26, 37]. Consequently, the final model can be derived. Focusing on insurance and actuarial sciences, several requirements are placed on loss models, which are both theoretical and practice-oriented in nature.

Based on Bossel [26], Bungartz [37], and the Faculty and Institute of Actuaries of Great Britain [76] the following requirements are placed on and have to be verified by the model:

- Accuracy: This is related to the sharpness or predictability of the model with respect to the quality of the input data and the question that has to be answered.

If the input data is of poor quality, it cannot be assumed that the model results have much validity. Additionally, it has to be checked whether the results are sufficient enough to answer the research question the models should answer. In actuarial claim models, this is usually done by enabling an error calculation [143] as the root mean squared error which cannot be calculated for many models. However, this can also be achieved to some extend by cross-checking the results with the ones of conventional models.

- Competitiveness: The cost and benefit or price and performance ratio of the model has to be evaluated. This has to be done especially against the background of real-time requirements. From a company economic point of view, the development and implementation effort must also be proportionate to the benefit and result.
- Completeness: Availability of all features that were stated in the model requirements. These are mentioned in Chapter 3.
- Consistent and Reasonable: Hereby it has to be checked whether the model is contradiction-free and if the assumptions are reasonable.
- Correctness: The model has to work as expected and fulfill its purpose. In the case of a SLD model, the development of single claims up to their ultimate value is of essential importance leading to an individual ultimate loss distribution.
- Explainable: The model functionality and its results should be easy to explain and intuitive in order to facilitate communication with other departments such as underwriting or reserving. In the case of an SLD model, this is challenging since they are inherently complex [249].
- Flexibility: The model should allow for incorporating expert opinion and market knowledge or trends besides the pure data to make a smoothing of the results, adjustment of unrealistic results, or the consideration of future trends possible.
- Practicability: In order to be usable in practice, the model has to be analysed with regard to further processing, implementation efforts, required data quality, and the effort corresponding to simulated results. This has to be evaluated under the aspect that only sparse loss data is usually available for individual cedants in many insurance markets. Furthermore, the runtime of the model must be within reasonable limits, which can be critical in the case of SLD models [249]. Ideally, the maximum runtime would be a few minutes or less for the required number of necessary simulations.
- Realistic and plausible: The output of the model should be realistic compared to what was expected and what can be seen in practice. This includes an adequate and realistic projection or prediction of the claims' development and the ultimate

value. This is of major importance with regards to Solvency II regulations [72] and the consistency to model the run-off of old business and the payment patterns of future cashflows [243]. This has to be fulfilled despite the small amount of data that might be available.

- Robustness: The susceptibility of the model with respect to errors in the input data has to be analysed. In the case of claims data this is not limited necessarily to errors in the data but also to outliers and extreme values in the dataset [76].

- Sensitivity: Hereby, the behaviour of the model and its results with respect to changes in the input data have to be investigated. If small changes already lead to a different solution behaviour than in reality, the results of the model should be treated with caution.

- Simplification/Idealization: The model should capture the essential structure to solve the problem while avoiding an overfitting. There should be enough parameters to describe the characteristics of the data, while avoiding an overfitting and a limitation of the descriptive power, which goes hand in hand with a greater understanding of the underlying data [76]. Related to the explainability, the model should be as simple as possible while still fulfilling its purpose.

- Traceability: The model approach has to be well documented, comprehensible, and reproducible in its logic, assumptions, hypotheses, propositions and results. Here, special attention should be paid since this excludes black box models to enable communication with clients and supervisory authorities.

- Uniqueness: The results of the model should be unique. In the case of a SLD model, which is based on a computer-aided approach, this means that the model results should converge with an increasing number of simulations.

Hereby, it has to be noted that the 'correctness' of a model cannot be proven in principle [26] and that only a statement about the validity for the model purpose can be made. This can be examined by validating the applicability, behavioural, structural, and empirical validity [26]. Since models are often computer-aided, different approaches can be followed for this purpose. Firstly, a comparison with experiments could be performed which is not possible for the SLD model due to the complexity of the real system. Secondly, a-posteriori observations can be used which is already done for actuarial methods and which will be applied in this thesis as well in form of back-testing. Thirdly, plausibility checks, which are only focusing on a purely theoretical result in respect of valid and proven theory, can only be applied partially. While some distributions exist that are known to be loss distributions, it is not clear whether this also accounts for the ultimate distribution of MTPL claims and their future development under a reinsurance schema. For this, only assumptions exist so far. The fourth method is a comparison with other commonly used models which can only be applied partly in the case of aggregated models as it is the standard in scientific literature and practice [76]. However, a comparison with other single loss development models in particular is not possible since they are not commonly used in the reinsurance sector and currently published models are not acknowledged there as well.

Irrespective of the method chosen, the results should be treated with caution since there are numerous sources of errors such as bugs, programming errors, or logical mistakes that can distort and influence the validation results [37]. Finally, it has to be clear that it is difficult or even impossible to create a 'correct' model and that a model hierarchy is usually created [76] as an interplay between effort and accuracy where the balance has to be struck between what is required and what is feasible [37].

1.3.2. Aggregate Models

In order to capture the characteristics of data and use them for a loss prediction, aggregated methods that sum all losses according to their accident years are widely used in practice. Those models are easy to apply, understandable, and have proven to be appropriate for a long time. Driven by the law of large numbers, they provide good and stable overall results if the data basis is sufficient enough, which is usually the case in primary insurance companies.

The best known and most commonly used aggregated method is the Chain Ladder method, which provides an estimate for the ultimate claims by allowing for a general tail factor. In the beginning, this method was only developed as an algorithm which could be carried out easily and quickly by hand [270]. However, there is no definitive source in which this method is defined [251]. Among others, two papers, in which the method is explained, are from England [70] and Pfeifer [178]. Since the Chain Ladder method only provides a point estimate of the outstanding claims, several stochastic models were sought behind it [45, 153, 155, 187, 188, 229, 251, 253, 270]. The distribution-free approach of Mack [149, 150] is the best-known stochastic model behind the Chain Ladder procedure and has established itself ultimately in scientific literature as the background model[12]. In practice, this model is mainly used because it allows an assessment of the model's predictive power. Additionally, there is a good understanding among actuaries how the model works and how it can be controlled.

Since the Chain Ladder model has proven itself in practice, several refinements and improvements have been developed. These focus on the performance of the Chain Ladder model related to outliers [200, 241, 242], negative increments [251], smoothing of the development factors [70, 247], accident year variations [244], and the uncertainty of

[12] Besides that, connections between the Chain Ladder method and loglinear models [248, 250, 254], Poisson models [59], as well as classical statistical estimation principles [202], and generalized linear models [224] were done.

the results with respect to solvency purpose [100, 264, 267]. Besides these improvements, there are three other methods that are used in practice [27], even though they are not as commonly used as the Chain Ladder method itself. Firstly, there is the separation method [22, 208, 228, 240] allowing to better incorporate an influence of calendar year effects like inflation. Secondly, the Bornhutter-Ferguson method [25, 70, 151] which assumes an a-priori final loss ratio for all accident years in order to minimize the influence of outlier effects in the data. Therefore, the final loss ratio is the ratio of the sum of all claims at the end of the settlement to the premiums received. The third model is the Cape Cod model [34, 35, 89, 209, 265] which follows the same principle by assuming the same final loss ratio for all accident years. However, it establishes an influential effect between the claims data and the underlying exposure data[13] of the portfolio. In addition to these extensions, there were two additional central ideas that influenced the further development. On the one hand, the necessity to include additional information from outside such as expert knowledge [246], which was included following a bayesian approach [57, 97, 246, 252]. While this was not necessarily limited to the Chain Ladder method, independent models also evolved from this idea. On the other hand, there was the idea to separate the development of IBNYR and IBNER claims [143, 172, 203, 246]. This was done for count and incurred data [161, 249] as well as paid and incurred data which resulted in the Munich Chain Ladder model [120, 142, 183]. Nevertheless, these different models and improvements do not offset the shortcomings of the Chain Ladder model with respect to reinsurance contract features, sparse claims data, or the assumption about the loss and frequency distribution. Furthermore, manually adjustments from the actuary in respect of outliers and smoothing of the development factors based on expert opinion are necessary.

Regardless of the Chain Ladder method, there are other approaches to estimate loss reserves. However, these methods are not widely used in the reinsurance industry and will not be discussed in detail here. Therefore, interested readers are referred to the following publications[14]. Overall, good summaries of aggregated methods and some adjustments can be found in the following papers and books[15].

In summary, the common feature of aggregated procedures is the exploitation of the law of large numbers. The more individual losses are aggregated, the more stable the results become. This aspect is challenging in particular with regard to reinsurance pricing due to the limited number of individual claims. Additionally, the reporting times of unpaid claims, empty or negative cells in the triangle, the number of currently open claims, and separate frequency and severity information [130] are problematic. This leads to some unrealistic assumptions and inaccurate projections of the ultimates, e.g. due to few cells in the upper triangle part [138], independence of the reported claim

[13] For MTPL this can be the premium or number of vehicles per accident year.

[14] [5, 50, 63, 86, 132, 134, 148, 160, 163, 193, 222, 225, 239, 245, 262, 263, 274].

[15] [27, 82, 85, 96, 144, 201, 226, 265, 266].

numbers between early and late development years [102], or independence between the aggregate incremental values [138]. Additionally, a usually small number of observations in the triangle, few observations for the recent accident years and the sensitivity to the most recent paid claims are shortcomings of aggregated models [90, 146, 240]. Hence, aggregated models are usually adjusted a lot to get a reasonable projection of the ultimate values and can be volatile in their output. With focus on reinsurance, it is furthermore necessary to properly include individual properties such as the limit and priority of XL contracts [157, 275], the stabilisation of single claims, and to have an approach that does not depend too much on the law of large numbers. While different aggregated models and improvements usually tackle only one critical aspect, all the other shortcomings of these models cannot be offset. Since it is well known that a lot of information is discarded when using aggregated models [81, 121, 122, 130, 159], the usage of SLD models could overcome those shortcomings and offer an enormous advantages for reinsurers in particular, resulting in a long term competitive advantage due to a better risk assessment.

1.3.3. Individual Loss Models

Individual loss models do not represent the mainstream in the reserving field [146]. The idea of projecting every single claim individually dates back to the late 80s and early 90s and can be divided into models coming from a theoretical or practical point of view. They evolved from the idea of incorporating additional information of single claims to improve the projection of the aggregated results compared to the Chain Ladder model and only a few can directly simulate individual claims themselves. Former models are mentioned here since these are widely spread in the scientific literature and can provide ideas that can be transferred for SLD models in the case of pricing XL reinsurance treaties. Hereby, theoretical models can be divided into a group using stochastic processes and all others.

In the case of stochastic processes, especially the market poisson process provides a comprehensive and probabilistic framework for general computation of claim reserves [5, 93, 121, 122, 168, 170, 265]. This is the basis for many adaptations and extensions [93, 138], as well as model approaches following other stochastic processes[16] such as a dirichlet process [169]. Other approaches not following the idea of stochastic processes use a variety of ideas. This includes a model handling the frequency component of pure IBNYR by modeling claim emergence times directly [174], a transition matrix model based on markov chains [157], or an approach providing consistent data-driven predictions of the future claims together with full probabilistic distribution

[16] [4, 17, 28, 108, 168, 227, 272, 273].

by using archimedian or vine copulas [146]. Besides the previously mentioned approaches, some other models and ideas have been published[17]. However, all these theoretical models have disadvantages making an application in practice difficult. Firstly, they assume a fixed and parametric structural form which is not flexible enough, difficult to estimate, and complex. This accounts for the assumptions made about the used distribution functions in particular, which are usually build so that they fit into the theoretical model[18]. However, the different distributions are usually unknown in practice and an estimation of possible parameters depends on each cedant, the number of available claims, and might vary over time due to portfolio changes. Secondly, some of these models have a continuous time framework while a claims' update happens once a year in practice which would require a discrete time model. Additionally, many of these models usually stay within the framework of an aggregated projection view by incorporating more information from single claims such as reporting delays or aggregated development pattern to improve the Chain Ladder aggregate results. Thus, the projection of individual claims incorporating reinsurance contract features is not targeted here. Furthermore, these models are usually constructed under several model assumptions without a cross-check to real data, which makes it difficult to understand these model and to justify an application in practice.

Besides the theoretical approaches, models evolving from a practical point of view, which goes hand in hand with discrete time models, have been developed as well. One of these models uses a discrete stochastic structure to model the occurrence times, the reporting delays, the first payment delays, the number of payments for each claim and the number of periods between two subsequent payments and combines this with the Chain Ladder model [179, 180]. This idea is based on a similar approach where the claims' path is tried to be projected [181, 190, 258], including additional aspects like a closing mechanism for claims or a distinction between the development of smaller and larger claims [64, 164]. These models consider the empirical distribution of development factors[19] similar as it is done by the next neighbour method for single claims [61, 62, 147]. Another approach assumes a parametric distribution instead of the empirical for choosing individual development factors [103, 181] or an approach using a machine learning approach on granular data to get similar claim development paths focusing on paid and outstanding values from a primary insurance company [40]. These models usually have in common that the claim development process is modelled in detail, e.g. the claim status, the occurrence times, reporting delays, payment times, and number of payments among others. However, the main issue of these kind of models is the calibration within the framework of a claims triangle. For earlier development years with enough data, the models perform well but they fall off for an increasing number of development years and less data for the calibration [204]. While this is not true for

[17][92, 130, 249].

[18]This includes exponential, poisson, or gamma distributions.

[19]Except for [181].

the model of Carrato [40], it has the disadvantage that it requires a lot of data for a good performance. Additionally, some of these models stay within the framework of predicting aggregated ultimate values which is not sufficient for a reinsurance pricing. However, the basic idea that is followed by Drieskens et al. [64, 164] as well as the idea of a next neighbour approach [61, 62, 147] are considered to be a good starting position since a combined model would have the possibility to project the individual claims even in a portfolio with sparse claims data in a clear and comprehensible way.

Overall, a main issue with current models is that many of them do not offer the possibility to incorporate reinsurance pricing features on single claim basis and to estimate the ultimate loss distribution, which is crucial for the pricing of XL reinsurance treaties. Hence, only some of the biggest five players [186] in reinsurance[20] are using individual loss models in practice. In these cases, the models are in-house developments or adjusted published methods. The calibration of those models is highly complicated, requires a lot of effort, and the usage is mostly restricted to special claims or cases. Thus, no practical model exists that works for the pricing and projection of claim triangles for a reinsurer in general. All single claims need to be projected to their ultimate value so that the reinsurance features can be applied. Afterwards, these values can be used to fit a severity distribution which can then be used further on. However, the models capable of doing this so far do not work sufficiently in practice. Hence, this thesis aims at the development of such a model which includes a separate modelling of IBNER and IBNYR claims or in general, of each individual loss component.

1.3.4. Individual Loss Components

In search for a more adequate estimate of the ultimates and in connection with the inclusion of additional information, e.g. from individual losses or experts, it is natural to separate the final loss levels into different components, which are considered separately [51, 172, 179]. The most common classifications and separation into individual loss components are shown in Figure 1.2.

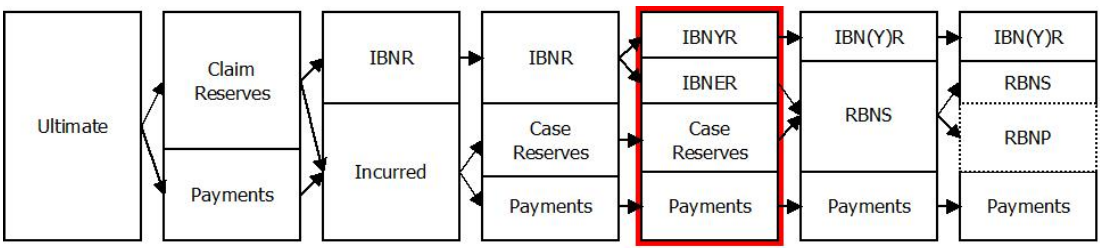


Figure 1.2.: Breakdown of Losses into Individual Loss Components.

First, the ultimate claims can be divided into the paid and outstanding claim amount. The loss reserve for outstanding claims can also be divided into the case reserve, fur-

[20]Munich Re, Swiss Re, Hannover Rück, Gen Re, SCOR SE (as of 2019).

ther refered to as outstanding value, and IBNR claims, whereby case reserves and paid values are often combined as incurred values. The currently most common further subdivision for the IBNR losses separates them into expected losses that already occurred but are not yet reported, the IBNYR losses, and the outstanding loss developments of already known claims, the IBNER claims. It should be noted here that the names are not uniformly used in current publications and the actuarial world. Frequently, IBNR is used as a substitute for IBNYR claims. This is because IBNER claims imply that only a negative claims' development exists[21] and is, thus, included in another group leaving only IBNYR claims, which are then only titled as IBNR claims. Thus, also the subdivision into Reported But Not Settled (RBNS) claims, which includes the settlement of all claims already reported, i.e. case reserves and subsequent positive or negative claims' settlement, and IBNR claims [180, 249]. Especially in this subdivision, IBNR claims refer to claims that are not reported yet and their further development, i.e. IBNYR claims. Furthermore, it can happen that the RBNS losses receive a subgroup with losses without previous loss payments, the Reported But Not Paid (RBNP) losses. However, this classification is rather unusual. In the further course of this thesis, the classification of IBNR claims into IBNYR and IBNER claims is used, whereby it is explicitly pointed out that the IBNER reserve can also be negative if a positive claims' settlement occurs. Furthermore, the term outstanding is used to describe the case reserve of a single claim[22].

Due to the nature of SLD models, IBNER and IBNYR claims can be simulated and projected separately. Thus, the focus will be set on the modelling of IBNER claims aiming at the projection of a claim to its ultimate value. Afterwards an IBNYR model fitting into the development scheme of the IBNER claims can be constructed by adding additional claims to the cedants data. However, a comparison with commonly used aggregated models is only possible if the IBNYR claims are simulated as well since these methods in their basic form do not allow an explicit consideration of IBNER and IBNYR claims [249]. Hence, a simple approach will be used here to concentrate on the modelling of IBNER claims. This approach is based on Schnieper [203] and has been developed further since [143, 172, 265]. The main idea is to split the data into two triangles with IBNYR and IBNER claims and to combine their developments for further predictions. However, this can also be used to estimate the expected number of additional IBNYR claims for the cedants' portfolio.

Focusing on the IBNER claims, the model needs to project the latest known incurred value of a claim to its ultimate settled value. Since this value is unknown, the projection of the claim should be based on a stochastic approach to create multiple outputs and a

[21] A positive claims' development results in a smaller final loss amount, whereas a negative claims' development results in a larger final loss amount.

[22] Sometimes the term outstanding is also used for the claims' reserve or even for the ultimates [51].

claim size distribution within a Monte Carlo simulation. This needs to be done for all single claims in a portfolio by considering the available information.

1.3.5. Starting Position

Based on the disadvantages of aggregated models and theoretical SLD models, the focus is set on SLD models following a practical point of view. Therefore, a discrete time approach is chosen since a reinsurer only gets data once per year for pricing purpose and considers reserving on a quarterly basis in order to update the balance sheet. Hence, a continuous time model seems to be inappropriate given the data basis of a reinsurer. Moreover, the prediction of the ultimate severity distribution of the single claims of a portfolio is the main aspect. Then, reinsurance features could be applied and different XL layers can be priced. In order to derive such a distribution, a stochastic approach is chosen since a deterministic approach could lead to unrealistic developments [227, 246]. This can be seen by assuming a small portfolio with a few claims showing high and market atypical developments due to 'bad luck' of these MTPL claims. Following a deterministic approach, these high values would be taken as basis for the prediction of all other claims which would lead to an unrealistic projection.

Furthermore, the main focus is set on the development and projection of RBNS claims and their related payments and not on IBNYR claims in a first step as motivated in Section 1.3.4. This is done since a functioning and adequate model for the RBNS must be created before a model for the IBNYR can be developed since their development would be logically based on the developments of the RBNS claims. Since the IBNYR development should fit to the RBNS framework, a simple approach is chosen which can be exchanged later. This is done in order to allow a cross-check against commonly used aggregated models which develop IBNYR claims implicitly. An exposure-based approach without distributional assumptions, which is only based on the available empirical data, is chosen for this. Therefore, it is flexible for changes, e.g. application for different countries, with the disadvantage that a sufficient amount of data has to be available [198]. In this approach, the frequency of the IBNYR claims is estimated first and then multiplied by the average claim costs. Besides the comparison with commonly used aggregated models, an evaluation on real data is done as well to evaluate the goodness of the SLD model. Here it should be noted that this is usually not done within scientific literature since these models only focus on the projected aggregated values in comparison with the Chain Ladder method and not on the prediction of an ultimate loss distribution.

In order to derive a suitable RBNS claims model, distributional assumptions as well as identically distributed claim frequencies, severities and delays should be avoided as much as possible since those are often unrealistic in practice [138]. This is due to seasonal effects and changes in portfolio size, mix of business, claim types, and empirical claim distributions [138]. Moreover, distributional assumptions might not be valid for different countries while fitting these distributions on a triangle basis is challenging at the same time. Related to this, fitting distributions, e.g. for the development factors per development year [103], or drawing from the empirical distribution [64] is problematic due to the sparse data on triangle basis [74]. This accounts for the most recent development years in particular. Thus, a flexible model based on the empirical data available for each country and cedant is required. Since the sparse claims data of a cedant can be problematic for the application of SLD models, it has to be decided whether only the cedant data or market data related to the same country should be used. The latter would require to analyse the homogeneity of the market data and if it is comparable to the data of the cedant. This is also related to the question whether the SLD model should be applicable for a new market where no market data is available or to a known market where a solid data basis is available.

The amount of data and data quality is also related to the disadvantage of SLD models to deal with trend effects in the underlying data. Here, it is widely known that claims data is subject to the influential impact of, e.g. economical inflation, legal or political changes, as well as changes of the business mix or the underwriting policy [75]. These influences can lead to trends in the data which need to be taken into account for a pricing. While several methods exist to adjust aggregated data for such trends, this is not possible in the case of single claims data in general [20, 73] which will be discussed further in Section 2.6. Thus, special attention has to be paid to data quality and the associated difficulties for a pricing.

Focusing on the claim developments that are observable in real data, the history of a claim might have an impact on the future development. If a large case estimate adjustment has been done in the previous year to account for an expected worse run-off in the future, the probability of seeing an additional adjustment in the same direction for the next year is smaller[23]. Thus, stochastic processes and other methods assuming the memorylessness for the claim paths are not an option.

The consideration of the history of a claim is also related to the key idea behind the next neighbour method. Here, a specific development should follow some kind of pattern that has been observed previously for other claims, e.g. bodily injury cases due to a car accident should have a similar medical and judicial process. However, these methods usually only target the incurred values based on the same development year which is too restrictive for an application in practice and, thus, requires further development.

[23]This is due to the information gap between the primary insurer and the reinsurance company for which not all information is available. This will be discussed further in the next chapter.

Following the points mentioned above and considering the requirements from Section 1.3.1, the first starting point is the idea by Drieskens et al. [64] to assume a different development of claims regarding their incurred values. Using a threshold to distinguish between the developments of larger and smaller claims is a promising approach in particular since it can also be observed for real data. Since Drieskens et al. [64] are not stating how such a threshold has to be determined, a distinction of claim developments according to similar pattern is targeted.

Additionally, the claims' development within a claims triangle for individual claims is scrutinized. For more recent accident years, only sparse individual claims' information is available and is usually projected based on the latest developments of the oldest accident years. Since it is questionable whether this information is trustworthy for the current year, a more flexible and triangle-free framework is targeted. Here, an approach similar to Mahon [157] is applied to capture the development paths of single claims more precisly. Overall, the available data has to be analysed for dependencies between different information which can then be utilized for a copula-based approach [146, 272, 273]. However, this is also related to the previously mentioned point of choosing between cedant, market data, or a mixture of both.

To summarize, a promising approach for the projection of RBNS claims considers a more flexible triangle-free framework for which the claims' projection considers different claim sizes, development of similar claims, the claim's history itself, and possible dependencies between the available data types. Hereby, the model is developed and tested based on real internal data provided by a leading reinsurance company.

1.4. Objective and Procedure

The aim of the present thesis is to develop a market-flexible individual loss model for reinsurance pricings, which allows an adequate estimation of ultimates, taking the most realistic claims' development pattern and settlement into account while avoiding unrealistic assumptions. In order to create such a model, the following research questions have to be answered:

- What kind of data exists; how can it be used; what is its quality and what factors influence the data?
- How can the data be used to create a framework that is as flexible as possible for pricing non-proportional XL treaties?
- If such a framework can be created, how can an adequate and realistic claims' development be simulated on this framework and what factors and influences have to be considered?

- What is the performance of the new model with regard to different markets, backtest simulations and commonly used models?

Since individual loss models allow a separate consideration of RBNS and IBNYR claims, a correspondingly separate modelling has to be available as well. However, an adequate model for the RBNS claims has to be created first before a suitable IBNYR model can be set up. Hence, the focus is set on a model for the projection of RBNS claims[24]. The procedure of this work to answer the main question is illustrated in Figure 1.3.:

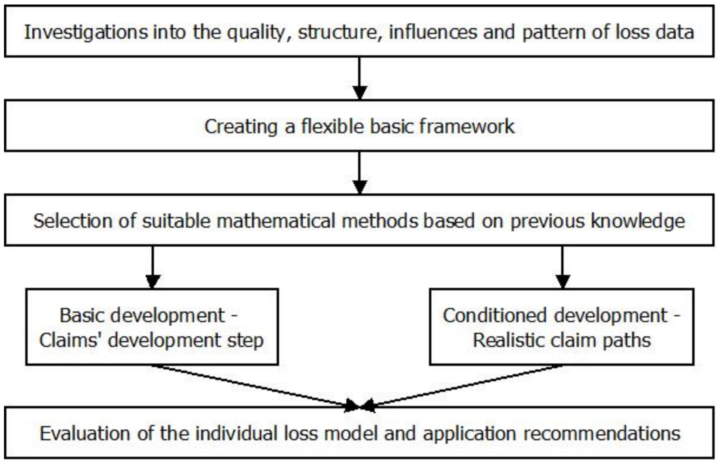


Figure 1.3.: Structure of the Thesis.

First, the focus is set on the existing claims data for a reinsurance company in respect to XL treaties. The specific characteristics, influencing factors and possible shortcomings are discussed briefly. Based on this and the requirements from Section 1.3.1, a framework is developed which serves as a basis for the claims' development. However, this leads to various mathematical approaches that can be considered for the claims' development. Based on preliminary investigations and the requirements of individual loss models, a promising approach is selected based on practical observations and used later on. In a further step, the claims' development is supplemented by conditions in order to guarantee a realistic claims' projection. Comparisons with currently used aggregated methods and backtest simulation results serve as evaluation criteria. Based on the assessments, recommendations are given for a usage in practice and for further research.

[24]See Section 1.3.4. and 1.3.5.

2. Data Basis and Classical Reserving Methods

Pricing methods use available claims data for a projection of future payments in order to assess the risk of a portfolio. Therefore, specific requirements are set for the data quality, which are stated in this chapter. Additionally, influencial effects on the data are discussed since they play an important role in the pricing of XL treaties. This chapter concludes with three commonly used reserving and pricing methods in Section 2.8. that are briefly summarized.

2.1. Data Preparation and Quality

A fundamental source of value for an insurance company is the data it can access and use for further processes. Therefore, they collect vast amounts of data and use them to make key decisions and more complex analysis of their business [81]. Thus, a huge data pool as well as good data quality is essential for insurance companies. The data can be available due to internal[1] or external sources[2]. In the case of a reinsurance company, data from an internal source could be market models or deeper insights into claims from the claims department whereas data from external sources are inflation indices or all data provided by a primary insurer, e.g. claims data, exposure values, or risk profiles. Since mostly external information sources are used for the purpose of pricing, the following fundamental classification of the data is common:

- Territory: The data can be divided by continents, countries, or regions. With focus on reinsurance pricings, the focus is usually set on country level to have a similar legislation and jurisdiction.
- Insurance segments: This includes the separation regarding the non-life insurance, e.g. property and casualty (P&C) insurance, and the life insurance, e.g. life and annuity insurance. As stated in Chapter 1, this thesis uses data from the P&C segment.

[1] An internal sources can be the same insurance segment, another insurance segment, or data that is available in the corporate group. Examples for possible data that can be accessed by this source are: rate features, premium and exposure values, payment addresses, acquisition costs, or the reason for termination.

[2] External sources can be the Federal Office of Statistics, natural catastrophe models, or for a reinsurer the data from a primary insurance company. Examples of such data are population growth, wage and price indices, car registrations, or catastrophe data.

- Line of business: The non-life and life segment have different business lines. Examples for the P&C segment are property, automobile, liability, engineering, marine, agriculture, or aviation. On the Life and Annuity side, examples are mortality, morbidity, critical illness, disability, long term care, medical expenses, or longevity. Additionally, these lines are commonly divided into short and long tail business, depending on the run-off period of the claims. A single loss development model is useful for MTPL claims in particular, which counts as a long-tail business line due to possible payments of annuities for a third-parties disability.

- Treaty type: Here the distinction is made between non-proportional business[3] and proportional[4]. MTPL claims are usually reinsured on a non-proportional basis.

- Treaty structure: Depending on the treaty type, commonly used treaty structures are XL, quota share, stop loss, or surplus. Here, the focus is set on an XL treaty structure.

- Additionally, there are some special features like natural catastrophe covers[5] or claims-made policies[6].

Furthermore, the available data has to fulfil particular requirements regarding data quality[7] in order to be appropriate for further analysis [173]. Therefore, the Insurance Data Management Association (IDMA) set up a certification model listing four components, validity, accuracy, reasonableness, and completeness, to assess the data quality [42, 81]. Hereby, validity refers to the values of the given data which has to be from a set of allowable data values. In the case of accuracy, the problem of incomplete or missing data is targeted by dividing it into 100% correct data, data with some imperfections that is still useable, and data that is inaccurate but consistent over time. Additionally, the data has to be reasonable compared to prior and current knowledge and should contain all necessary data that is also only used for the predefined processes. This certification model is the basis for a guidance to actuaries for the following data quality review in practice [27, 42]:

[3] For the non-proportional business, the risk transfer is based on the claims that occur. In case of a loss, the insurer takes an amount of the claim above a previously defined threshold.

[4] In this case, the risk transfer is defined by a previously fixed ratio between insured and insurer.

[5] For the life segment, this includes pandemic risks and on the P&C side floods, earthquakes, storms, hail showers, and wildfires, among others.

[6] Some examples are directors and officers, employment liability insurance, or errors and omissions.

[7] A definition of data qualitiy is given by Olson [173], p. 24: 'data has quality if it satisfies the requirements of its intended use. It lacks quality to the extent that it does not satisfy the requirement. In other words, data quality depends as much on the intended use as it does on the data itself. To satisfy the intended use, the data must be accurate, timely, relevant, complete, understood, and trusted'. This definition is pragmatic since the data has quality if it is sufficient enough for the required task. For a deeper insight on the data quality topic as well as negative effects due to poor-quality data, the reader is referred to Olson [173].

- Select the data which underlies the actuarial work product. This includes a classification of the accuracy of the data as above as well as selecting the specific type of what is required for the later methods. Hereby, the data quality depends on the respective cedant.
- Review data for accuracy:
  - Appropriateness: Information needed for the analysis; homogeneous to allow evaluation; consistent with the purpose of the study.
  - Reasonableness: Consistent with prior data or other information.
  - Comprehensiveness: Necessary records and data elements to do a proper analysis are available.
- Making appropriate disclosures, evaluating the results in light of the business knowledge and available external benchmarks.

Ensuring data quality is an important task in practice since it can delay delivering data to decision makers due to clarification requests [173]. If the claims data is of bad quality, the projections done using actuarial methods will not deliver reliable predictions. Hence, this requires more adjustments of the actuary, who will usually put a safety loading on top of the pricing to account for the bad data quality and the resulting uncertainties.

For further insights into the topic of data quality and data classification, the reader is referred to the following pulications [75, 80, 173, 237].

2.2. Motor Third Party Liability Data

The insurance automobile data can be divided into two major variables. The categorical variables containing qualitative information, e.g. gender or if the car is garaged, and quantitative variables, e.g. the driver's age or paid losses [81]. A comprehensive list of possible data elements for the automobile business is stated in Francis [81]. In practice, insurance companies consider some data to be more useful than others, e.g. claim sizes and paid values for an accident are usually more important than the injury type or return to work data [81]. Hence, data quality might also depend on the usefulness of the data from the insurer's point of view.

Since the available data for a primary insurance company differs a lot compared to the available data for a reinsurance company [20], a closer look has to be taken at the reinsurance specific data. In particular, since this is the data available for a later modelling of single claims.

2.3. Data Assessable for Reinsurance Companies

Based on the reinsurance treaty type and structure, different data is transfered from the primary insurer to the reinsurer. This is mainly limited by data privacy regulations and the specific coverage type. Based on habits and the historical usage of aggregated models, there is specific data that is usually passed and an extension cannot be achieved easily. This information asymmetry between the primary insurer and the reinsurer can hinder accurate pricing. First of all, a loss size distribution of claims is unknown and the data available for a reinsurer might be insufficient for a reliabel model selection and parameter estimation [73]. This is due to the fact that losses below the priority are not, or not completely, reported to the reinsurer [73]. This results from the reporting threshold that is stated in the claims notification clause of a reinsurance contract. Only claims exceeding the reporting threshold are sent to the reinsurer[8]. In practice and depending on the country, it is not unusual that even no claim or just a few are reported. Thus, the claims data from the primary insurance company is just available partly for a reinsurance company [46]. The reported claims are then usually stored in a single loss claim triangle [20, 249] or in already aggregated triangles that suppress heterogene effects of the data [92]. While the former can be used for a pricing of XL treaties, this is usually not the case for the aggregated triangles. Besides that, also some information on the underlying exposure, e.g. about the premium and number of vehicles or policies per accident year, is reported. The third table of information that is usually available is the latest portfolio split into different segments, e.g. cars, trucks, trailors, motorcycles. Even if this does allow some comparisons between clients to identify similar portfolio structures, this does not necessarily mean that the claims data is comparable as shown later in Section 2.4. Thus, this sort of information is not used in any pricing approach known to the author.

The data stated above can be seen as a minimal data requirement that is at least demanded by the reinsurer and does highly depend on the respective country. However, in some countries it is common that the primary insurer reports more information to the reinsurer, who has then a greater data basis for further analysis and pricings[9]. Thus, the amount of data depends on the size of the cedant and the habits in the respective country. It has to be noted here that requests for additional data are usually rejected and could worsen the relationship between primary insurer or broker[10] and the reinsurer in practice.

In the case of MTPL claims data following information shown in Table 2.1. is usually available for a reinsurance company and can be used for further analysis [20, 249].

[8] This threshold is usually 50% or 75% of the priority.

[9] Compared to the German market, more information about the claims is reported in France or the UK, for example [56, 74, 75].

[10] In some countries, the requests for a reinsurance cover are managed by brokers as intermediary.

Basic Information			Development Year								
Accident Year	Exposure	Claims	$Paid_1$	Out_1	$Paid_2$	Out_2	$\dots$	$Paid_{n-1}$	Out_{n-1}	$Paid_n$	Out_n
		$ID_{1,1}$	$Paid_{1,1,1}$	$Out_{1,1,1}$	$Paid_{1,1,2}$	$Out_{1,1,2}$	$\dots$	$Paid_{1,1,n-1}$	$Out_{1,1,n-1}$	$Paid_{1,1,n}$	$Out_{1,1,n}$
1	E_1	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
		ID_{1,k_1}	$Paid_{1,k_1,1}$	$Out_{1,k_1,1}$	$Paid_{1,k_1,2}$	$Out_{1,k_1,2}$	$\dots$	$Paid_{1,k_1,n-1}$	$Out_{1,k_1,n-1}$	$Paid_{1,k_1,n}$	$Out_{1,k_1,n}$
		$ID_{2,1}$	$Paid_{2,1,1}$	$Out_{2,1,1}$	$Paid_{2,1,2}$	$Out_{2,1,2}$	$\dots$	$Paid_{2,1,n-1}$	$Out_{2,1,n-1}$		
2	E_2	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		
		ID_{2,k_2}	$Paid_{2,k_2,1}$	$Out_{2,k_2,1}$	$Paid_{2,k_2,2}$	$Out_{2,k_2,2}$	$\dots$	$Paid_{2,k_2,n-1}$	$Out_{2,k_2,n-1}$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\iddots$				
		$ID_{n,1}$	$Paid_{n,1,1}$	$Out_{n,1,1}$							
n	E_n	$\vdots$	$\vdots$	$\vdots$							
		ID_{n,k_n}	$Paid_{n,k_n,1}$	$Out_{n,k_n,1}$							

Table 2.1.: Claims Development Triangle.

- n accident years from $i = 1,...,n$ and the respective development years from $l = 1,...,n$. The resulting diagonal marks the related calendar year.
- For each i-th accident year, there are k_i known single claims with $j = 1,...,k_i$.
- Each known single claim has an identification number $ID_{i,j}$, $i = 1,...,n$, $j = 1,...,k_i$.
- The payment $Paid_{i,j,l}$ and reserve, usually called the outstanding value, $Out_{i,j,l}$ for $i = 1,...,n$, $j = 1,...,k_i$ of each claim for every development year $l = 0,...,n$.
- Consequently, the incurred values $Inc_{i,j,l} = Paid_{i,j,l} + Out_{i,j,l}$ as sum of the payments and reserves, and the payment ratios $PR_{i,j,l} = \frac{Paid_{i,j,l}}{Inc_{i,j,l}}$, as quotient of the payments and incurred values for each development year $l = 0,...,n$, $i = 1,...,n$, $j = 1,...,k_i$, can be calculated.
- Additionally, the absolute growth and percentage increase of the payments, reserves, incurred values, and payment rations can be calculated. Usually, the percentage increase of the incurred values is used in common reserving methods as loss development factors (LDF).
- The status of a claim derived from the reserve of the loss. If the reserve is set to zero, the claim is assumed to be closed.
- An exposure measure E_i for each accident year. For MTPL treaties, this is usually the annual premium, number of vehicles, or number of policies.

This kind of information is usually available for every MTPL contract $Z \in \{cedant_1, cedant_2, ...\}$ as set of possible cedants. Each company reporting the data also has a cedant specific reporting threshold M_i^Z, which might differ for the accident years $i = 1,...,n$. Usually, all claims exceeding the reporting threshold are reported from ground up, meaning that the full development of the claim sizes from the beginning onwards is known. Nevertheless, there are also cedants where the claims are not reported from the ground up. Furthermore, the payments $Paid_{i,j,l}$ might be shown as incremental or accumulated data. On top of that, it has to be noted that the above shown triangle might be incomplete and that the number of development years is smaller than the number of accident years for some claims. While this is problematic for commonly used aggregated models in particular, as discussed in Section 1.3.2, the performance of SLD models is not affected by this.

2.4. Data Properties and Limitations

As already indicated in the previous section, the data provided is subject to cedant specific influences, also stated as internal factors, like the business mix and volume, underwriting, rating and policy conditions, or claims' handling and definition [75]. Additionally, there are also external factors like inflation and economic factors, legal, political and social factors, or climate and environmental factors [75]. This is also related to idiosyncrasies of primary insurance companies and data limitations in general. Since those effects can influence the accuracy of loss reserving methods and can lead to misestimations and unrealistically forecasts [24, 90], a better understanding of those can provide better fits of the underlying parameters [90]. Hereby it has to be noted that this is not necessarily restricted to MTPL data alone and that those factors are not necessarily independent of each other. The influences, limitations and consequences can be summarized into four groups [75, 145]:

- time dependencies and external factors,
- idiosyncrasies of primary insurance companies,
- number of data points,
- and the homogeneity of the dataset.

One shortcoming that occurs by analysing these effects is that the necessary data for an analysis is not available in most cases [46]. Since the performance of a claims reserving method for a specific environment is defined by the elements of the environment that changes [24], it is important to know possible effects that can influence the data. However, before the internal and external influences and limitations of these four groups can be discussed, it is necessary to show how they impact a claims triangle and what difficulties arise when analysing their impact.

2.4.1. Direction of Influence and Simpson's Paradox

Taking a claims triangle as shown in Figure 2.1, possible effects as mentioned in the previous section are either affecting the accident year, development year, or calendar year [27, 46, 269] as shown in Figure 2.1.

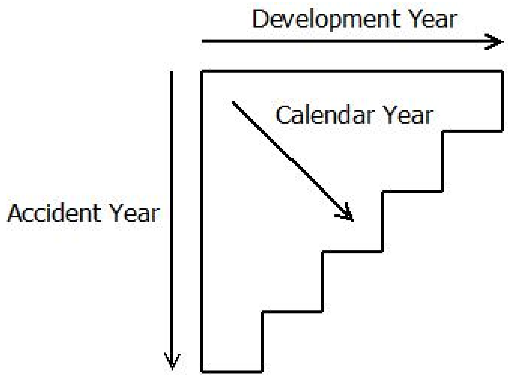


Figure 2.1.: Correlation of Influential Effects for a Claims Triangle.

The two directions of accident year and development year have zero correlation [269] since trends, accident year effects or development year effects, in either direction, are not projected onto the other. However, the calendar year direction can be affected by the accident year as well as the development year. Thus, a trend in the development year respectively the accident year is also projected onto the calendar year [269] resulting in a calender year effect. Overall, all internal and external factors, which can influence the claims triangle, are grouped together there, which makes the detection of possible effects on the data difficult. Thus, it is challenging and in most cases impossible to analyse each impact separately, which is related to the Simpson's Paradox [19, 46]. This phenomenon in probability and statistics refers to situations in which a trend or relationship that is observed within multiple groups reverses when the groups are combined [19]. For the underlying claims triangle, each internal or external factor might show a trend on its own but combined in a single triangle the actuary might see something completely different. Since all influences are overlapping in a claims triangle, the original trends and influences cannot be extracted without prior knowledge. Different effects might cancel out or are even reversed [19, 46]. With this background, the influences and limitations stated in Section 2.4 are further addressed.

2.4.2. Time Dependencies and External Factors

Influences that affect the claims data are commonly measured by trend factors. Those factors measure the changes over time [48] of several time dependent effects, e.g. prices, wages, salaries, care costs, construction cost increases, court decisions, changes in law, as well as political, social, climate and environmental factors [75, 141]. By measuring these effects, it is possible to derive a trend indicator in order to reduce negative effects on the loss inflation of MTPL claims and the disturbance on reserving methods [24]. This is of major importance since an unrecognised inflationary trend consumes capital exponentially [271] and influences the assessment of a claim for its loss value as if it were to be settled immediately or as at the likely future date of settlement [75][11]. If those inflationary effects are recognized, the insurer can apply an indexation in order to modify the claims data and to offset those effects. Hereby, the historical claims data is projected to the current state to make the claims comparable.

In the case of MTPL claims, the repair costs of cars and compensations for bodily injuries are the main drivers of inflation [18, 73]. Claims involving bodily injuries define the long tail character of the MTPL business since it takes time until the final medical condition of the claimant is known and stable [27]. Additionally, the inflation of bodily injury claims is more rapid due to changing legislation and social attitudes [27]. This leads to typical trend effects for the automobile business since it impacts the incurred losses [48], the claim frequency, and might also lead to a classification drift so that the previous risk category of the claim is not valid any more [32, 48, 259]. Climate and environmental factors are more relevant for natural catastrophes and the related motor hull claims and not that much for MTPL claims in particular [75], even though slippery roads might lead to more MTPL claims.

2.4.2.1. Economic Influences

Economic inflation such as changes for prices, wages, salaries, care costs, exchange rates, or construction costs [24, 45, 46, 48, 75, 141, 150, 271] is one of the most known and anticipated trend factors impacting the claims of an insurer. Those changes are usually tracked and published by the statistical institutes for each country in form of the Consumer Price Index (CPI), construction cost index, or the wage index [75] which are expected to be used for an adjustment of the claims data [27]. Nevertheless, the extent to which such an inflation is impacting the claims data together with data cleansing has to be decided by the actuary [27, 75]. Hereby it should be clear that economical inflationary effects operate on the calendar year, effecting the payments and the incurred values [39, 90, 225]. Thus, respective influences can also be seen for the accident and development years as noted in Section 2.4.1. This makes it a challenging task to extract the correct inflation rate itself since indices such as the CPI, Wage, or construction cost index are not available for all insurance markets and it is unrealistic that the seen inflation reflects exactly the economic inflation of the specific business [73].

For MTPL claims data in particular, advancements in safety technology or new materials [259] can also have an impact on the claim sizes. However, the impact of the

[11] Unrecognized inflationary trends have been a principal driver of the collapse of many insurers according to [271].

wage inflation has a major impact on MTPL claims data since those claims consist mainly of income replacements reflecting pre-injury earnings [225] which are used for the estimation of the loss of future earnings.

Focusing on insurance companies, the loss size distribution, which is unknown since the ground-up inflation is difficult to access, is an important source of uncertainty [73]. Consequently, this can make the original estimates between the reserving and settlement dates appear to be inadequate [75]. This is even more challenging for a reinsurance company where the economic inflation affects the layer and reinsurance programms [73]. Since the MTPL business is a long tail business in form of an XL structure, the tail-dependency of inflationary effects are a dramatic problem for this particular business [73] since the reinsurer participates disproportionately in inflation.

For this let $R(D) = \frac{\mathbb{E}(min(Inc,D))}{\mathbb{E}(Inc)}$ be the relief function with priority D and without loss of generality (w.l.o.g.) some claim size Inc. By assuming an inflation of $v > 0$ such that $Inc \rightarrow Inc_v = (1+v)Inc$ than

$$R(D) \rightarrow R_v(D) = \frac{\mathbb{E}\left(min((1+v)Inc, D)\right)}{\mathbb{E}((1+v)Inc)} = \frac{\mathbb{E}\left(min\left((Inc, \frac{D}{1+v}\right)\right)}{\mathbb{E}(Inc)} = R\left(\frac{D}{1+v}\right) < R(D)$$

In order to balance the participation on the claims for the insurer and reinsurer a stabilization is agreed on to adjust the layer depending on a specific index, e.g. the CPI. The respective calculations can be done easily for single losses but are more challenging and related with assumptions for aggregated methods. Thus, the concrete mechanics of stabilization are just discussed briefly for single claims in Section 4.6.2. However, economical or in general, time depending influences are important for the indexation and stabilization of reinsurance claims.

2.4.2.2. Legal, Political, and Societal Influences

Apart from the economical influences, also legal, political and societal effects can drive a trend in the claims data [75, 243]. The legal aspects can be distinguished into trends of court decisions, practices, and legal precedents [259] whereas changes of laws, driven by political decisions, are targeted in legislative aspects [46, 75, 85, 150]. A good example for a change in law and regulations that had a huge impact on MTPL claims experience in the western world was the seat belt legislation, making it compulsory for drivers and passengers to wear safety bells [75] which reduced the number of severe injuries noticably. In the case of court decisions, the level of damages awarded by the courts in compensation cases is an important factor [75]. This is strongly related to a changing attitude in society since the awards will tend to escalate in time, often by far more than the normal amount of economic inflation [75]. Additionally, the public is more ready to take legal action if they are not satisfied with the claims' compensation [75]. Such legal disputes related to a MTPL claim considerably delay the correct estimation of the claim amounts and lead to additional disturbance [27]. Overall, this might impact the claims in form of sudden shifts the actuary should be aware of [75]. It is problematic that award precedents set by court decisions or legislative can affect claims regardless of the claims' occurrence [225]. Even claims that already occurred and are currently in the development phase can change in historic calendar years when the primary insurer is forced to adjust historical payments.

Since the legal aspects are highly related to political and societal influences, they have to be considered as well [75]. A possible societal influence that can have an impact on insurance costs due to a societal change is the claim consciousness in the society leading to more court cases. Additionally, dynamic forces and demographic changes in society have a measurable impact on the insurance experience over time [48]. Those trends can go unrecognized for many years and may lead to inadequate estimates resulting in massive under-reserving and under-pricing [75, 217, 271]. Nevertheless, societal influences can not be measured easily while political and legal influences usually lead to sudden shifts in the claim data.

2.4.2.3. Climate and Environmental Influences

Another factor that may influence the severity and frequency of insurance damages are climate and environmental influences, e.g. severe weather, catastrophes both natural and man-made, and the existence of latent hazards [75]. Therefore, even small changes in the frequency of storms can cause multiple increases in damages [47, 52, 238], which is more related to motor hull claims. Focusing on MTPL claims, severe weather conditions impacting the road conditions like slippery streets due to ice storms or aquaplaning due to heavy rain can influence the claim frequency and severity [75, 182]. Since the size of MTPL claims is dominated by the bodily injury part while the property part is rather small, climate and environmental influences have only a small impact. Moreover, those effects are hard to recognize and not accessed easily which makes a consideration nearly impossible. Nevertheless, an actuary has to be aware that influences from this source might occur and becomes noticeable insidiously.

2.4.2.4. Superimposed Inflation

In addition to the influencial factors mentioned above, the superimposed inflation is another important and challenging problem that insurance companies have to face [49, 175]. However, there are not many references dealing with superimposed inflation due to the prevalence of the Chain Ladder method where no differentiation between normal and superimposed inflation is made [175]. However, the main driver for superimposed inflation is the increase in personal injury claim costs which is far above the economic inflation [49, 175, 225] and can go unrecognized for many years [271]. This includes the development behaviour of bodily injury claims, especially, the increase of medical and nursing costs [29, 259]. Consequently, this is challenging for the actuary in charge with setting reserves and prices because of a bout of undisputed past superimposed inflation [223]. Overall, every time dependent factor that is far above the average economic inflation is a candidate for superimposed inflation.

2.4.3. Idiosyncrasies of Primary Insurance Companies

Apart from the effects discussed in the previous sections, the idiosyncrasies of insurance companies can have an impact on the claim sizes and claim frequencies as well. Especially in the case of MTPL contracts, the trend effects are not only driven by economic inflation [32]. In the case of reinsurance data, the peculiarities are on the side of primary insurance companies since their claims data is reported to the reinsurer. Those idiosyncrasies are related to the settlement rate, reserving behaviour, claims' management, lump sums, management decisions, underwriting, business mix, policy conditions, and the reporting threshold.

2.4.3.1. Settlement Rate and Reopening of Claims

Due to the long tail nature of the MTPL business, the time until a claim is settled can take decades and plays an important role [75]. Every year a claim is still open brings uncertainty for the reinsurer and increases the claim sizes due to inflation [27]. The time until the claim is settled is driven by many different factors such as the agreement on one-time payments, the claims' management in general, and the injury type which is usually unknown to the actuary. If a primary insurance company changes their settlement practice, it influences the development pattern of the business, which can result in a calendar year effect [24, 46, 90, 150]. Since this is not always communicated to the reinsurer such an effect has to be detected and the related data has to be be adjusted [20].

The settlement of a claim is also related to a possible re-opening of already closed claims. This can be driven by political or legal aspects forcing a re-evaluation of single cases or even all cases sharing a specific characteristic [136]. While this is also related to the legislation and jurisdiction of the respective country, it bears another source of uncertainty for the reinsurer.

2.4.3.2. Reserving Behaviour - Case and Bulk Reserves

The adequacy of the reported incurred values as sum of the paid and outstanding values is substantial for the reinsurance pricing [24]. A change of the reserving bahaviour and, thus, on the size of the outstanding values can result in a calendar year effect [32, 150]. The adequacy of the reported reserves is important for a reinsurer in particular since the sensitivity of projections of ultimate losses based on incurred LDFs to changes in the adequacy level of case reserves increases significantly for the long-tail lines [20].

A reinsurer only has a limited possibility to judge the adequacy of the reserves. Firstly, the reported data has to be consistent over time. This refers to the consistency of the data that has been reported initially and might be reported annually due to repricing and contract renewal purposes [75]. Here, also the question arises if consistency is achieved at all over the development periods [75]. Secondly, the purpose of those reserves has to be clarified. It has to be distinguished between reserves reported to bureaus, e.g. individual reserves allocated to specific known cases, and reserves shown in the annual statement that are required to be adequate in the aggregate for all cases since they are not necessarily the same [48]. Thirdly, it is of interest whether the primary insurer sets case of bulk reserves for the portfolio and reported claims. This is also related to the positive or negative run-off of a claim, which is used to express the claims' development compared to the current state. While a positive run-off indicates a decreasing incurred value with a reduced payment for the reinsurer, a negative run-off stands for a worsening of the claim and an increase of the incurred value in the future.

A case reserve is based on the details of the claim and the experience of the company with similar claims. It reflects the overall expected payout for an individual claim and is updated throughout the claims' development process as new information becomes available [21, 207]. Nevertheless, it is extremely unlikely that the provided loss reserve will be the precise amount that is necessary to settle the claim [207]. It has to be mentioned that the adjustment of the reserve regarding the experience is not common since some companies only allow an estimation purely based on the statistical assessment [27]. Furthermore and regarding the consistency of case estimates, the question arises whether an implicit safety margin is already included in the case reserves that are reported to the reinsurer [75]. In comparison to the above, a bulk reserve is designated to specific claims or line of businesses rather than individual claims. This means that each claim fulfilling the bulk reserve requirement is getting the same outstanding value by default. This is usually done for IBNYR claims, for claims where not enough information is available [27], as well as for a potential unanticipated development with case reserves [21]. However, the bulk reserve can be used for numerous reasons. It can also be used to smooth the income, as a tax postponement measure, or to manage financial cash flows [21]. Additionally, an increase in the use of bulk reserves is positively related to reserving errors and manipulation [21]. With focus on long tail lines, the sensitivity of projections of ultimate losses with respect to changes in the adequacy level of case reserves does increase significantly [20], which can be observed for business lines with a lower level of regulation in particular [21].

Overall, given the different characteristics of case and bulk reserves, it is likely that companies use them to achieve their overall reserving goals [21]. However, not only the size of the reserves plays a role but also a possible change between case and bulk reserves [75]. Such a change might lead to an inconsistency of the reserves and to a development year effect. Besides the fact that a reinsurer most likely does not know if a primary insurer is using case or bulk reserves, it is an issue that some cedants set reserves for losses and leave their incurred value untouched over a certain time period. This may occur since the insurer has no further information or uses a bulk reserve for the claims [75]. When those claims are then updated, this might lead to unreasonable development factors and a disturbance in the claims data [21]. With respect to development factors, there is the motivation of setting a higher reserve to have a positive run-off afterwards in practice [75, 83] since an exact and accurate reserve will most likely still develop upwards from a first bureau report [48]. Nevertheless, it cannot be assumed that the LDFs for claims ought to be one or less under the reason of legal requirements to carry reserves adequate to pay all of the outstanding losses [48]. If a significant upward change in the LDFs can be observed, this can indicate an inadequate reserve and it can be assumed that the adequacy of reserves will return to its former level [48]. Overall, a good understanding of the reserving behaviour is necessary. However, in most cases a reinsurer will not get any information about that and has to identify inconsistencies over time to get a hint of possible effects.

2.4.3.3. Claims' Management

Another calender year effect may result from an accelerated, stalled, or changed claim department activity [85, 90]. The claims' handling is one factor over which a primary insurer has the most control [75] and it can be expected that injured people who had their claims managed would have a better outcome in terms of recovery [196]. Due to the cooperation with medical experts, rehabilitation centres [1, 12, 38], and a more efficient usage of medical health services [196], the final payments of a claim can be reduced. This also accounts for a more realistic case reserve, lower overall claim costs, and a shorting of the average claim settlement time [31, 196]. Hence, a good claims' management can have a positive influence on the claims' development by helping the victims and it can help the reinsurer to lower the uncertainty behind the claims' development if a good understanding of the claims department and claims' handling is given [75]. For a possible guideline regarding the claims' handling process, the reader is refered to [171].

Ultimately, the claims' management affects the case reserves and ultimates. Nevertheless, in practice this effect can only be observed by comparing the accounting results for different cedants. Thus, it is not measurable in detail since not enough information is available to the reinsurer.

2.4.3.4. Lump Sums

Road accidents and MTPL claims are one of the largest category for which personal injury claims are made under the tort system [31]. Usually, these tort systems have some drawbacks when it comes to accident cases, such as delayed payment or delayed settlement of the claim [31]. While cases with lesser injuries are settled within a short time period, those with severe injuries suffer a great delay in settlement due to more prolonged medical complications [31]. This is also influenced by the involvement of lawyers which can result in a delayed claim closure as well [43]. Consequently, a prolonged settlement of the claim leads to more uncertainty for the insurance company.

In comparison to that, a no-fault system would close claims faster than it is done under the tort system [43]. Overall, this can be handled by putting a financial value on the run-off of a single claim and do a lump sum payment. This specific amount is then paid to the insured person who waives all claims on the insurance company in return [46]. Nevertheless, this is based on the regularities and compensation systems in the specific country, e.g. in Italy many claims are settled on the basis of lump sums [255]. While lump sum payments immediately settle the claim, this does result in a distortion in the claims' settlement process. However, also the future uncertainty is reduced as long as the claim is not re-opened again.

According to Bryant [31], it is suggested to handle severe injury cases with continuing future losses by structured settlement and to use lump sum payments otherwise to reduce settlement delays and resultant stress [139, 140]. A similar effect can only be observed when benefit payments at retirement take over the compensation [3]. Ultimately, an actuary has to be aware of the occurrence of lump sum payments for the different countries and cedants.

2.4.3.5. Management Decisions

Besides the above mentioned influences, management or business related decisions may lead to calendar year effects as well [243]. Two major aspects have to be considered in case of individual claim reserves. Firstly, a change in the claims management department such as expansion or contraction of staff numbers may lead to calender year effects since the closing rate of claims can change [75, 225]. This might also be the case if another person is handling the claim for a short period of time which can result in discontinuities in the claims' handling process. Secondly, the management can influence the company guidelines and the way of how claims are handled and how reserves are set [75]. Nevertheless, regulatory requirements are limiting the influence. In practice, a reinsurer will not be able to get any information about the guidelines and staff limitations of the primary insurance company and cannot take this into consideration.

2.4.3.6. Underwriting, Business Mix, and Policy Conditions

While the estimation practice of a primary insurer is driven by the class of business itself [75], the respective underwriting of this business also has an impact which can result in a calendar year effect in the claims data [32, 75]. This includes policy conditions, clauses such as the claims notification clause, or the rate adequacy [24, 41, 75, 159]. However, the major importance derives from the business mix and related distributional changes between cars, trucks, motorcycles, fleets, or buses since this leads to a shift in the underlying loss and frequency distribution [24, 46, 75, 92, 150, 259]. This is also related to the exposure and premium level of the related portfolio which impacts the frequency estimation and can lead to a calendar year effect [159, 216, 259]. A change on calendar year trends due to exposure level changes is a special case of a change in business mix on frequency and severity [159].

However, a change of the portfolio and underwriting of the primary insurer for a MTPL portfolio is usually observable for a reinsurer in the data or can be requested by the reinsurer, who can adjust the pricing accordingly. Nevertheless, these influences cannot be measured since an adjustment of the pricing is based on the experience and judgement of the actuary.

2.4.3.7. Reporting Threshold

The underwriting and policy conditions in the previous section addresses the primary insurer's portfolio. However, the clauses in the reinsurance contract and the claims notification clause [41] in particular are also important. While a primary insurer is not under an obligation to report all claims to the reinsurer [27], all claims falling under the notification clause have to be reported. This clause is commonly used for NP business and defines a threshold for which the claims have to be reported to the reinsurer since those could potentially affect the reinsurance layer. While the reporting threshold is market and cedant specific, it is usually set to 75% of the priority or retention of the layer [128, 141]. However, a reporting threshold of 50% is sometimes agreed on [128]. If the incurred loss of a claim exceeds this threshold, its development from ground up, meaning from the first development year, is reported [141]. Hereby, the reinsurer has to trust the information he receives and has to be aware of missing claims where the insurer has not realised that the claim reached the reporting threshold [27]. This is also influenced by the usage of Case of Bulk reserves as stated in Section 2.4.3.2. Thus, an information gap between insurer and reinsurer arises, which impacts the pricing and might require later adjustments. Due to the reporting threshold, it is also common for some markets that cedants report no or just sparse claims data making the pricing of such treaties challenging.

2.4.4. Number of Data Points

The available data for a reinsurance company is subject to many external effects as discussed in Section 2.4.3. and 2.4.2. While most of those influences cannot be accessed and evaluated in detail, the reinsurer faces the problem of missing or incomplete data [46]. These data quality issues combined with a smaller claims data basis due to the reporting threshold and the XL structure in combination with an aggregation of the claims can push currently used pricing methods to the edge of their possibilities [157]. Thus, a pricing based on the data of a cedant might lead to an acceptable result while it can also be inadequate for another cedant with less data [157].

To achieve some kind of consistency for the pricings of different cedants, a reinsurance company is reliant on market models which are based on common features like country, region, or a similar claims' behaviour of another cedant in general. While such a similar cedant might not be available in any case, the reinsurer needs to enlarge the data pool for a possible calibration of a market model in order to avoid the usage of pure assumptions. However, this can only be achieved if the data from different cedants is homogeneous enough.

2.5. Homogeneity of Motor Third Party Liability Data

In order to answer the question of homogeneity for this thesis, MTPL data is further discussed in this respect. First of all, the data that is considered here is from the same line of business and country. A more granular selection is usually not done since this already limits influences from the legal, political and societal side as stated in Section 2.4.2.2. Furthermore, the superimposed inflation, which is usually related to care costs, should be the same for all cedants in a single country. This is similar for the climate and environmental influences as long as the country is not too big, e.g. in the US one might want to do a more granular model based on the different states. The impact of economic influences as stated in Section 2.4.2.1 can also be reduced by applying an indexation to project all claims data to the current level [141]. While this data is not available necessarily for all countries, there is usually the possibility to derive an own index or to do an assumption according to market practice. While this still contains some sort of uncertainty, the time dependencies and external factors can usually be detected and reduced.

With respect to cedant specific influences as stated in Section 2.4.3 this is getting more complicated. Changing the reporting threshold is usually not possible since this is based on market practice to which the insurers, brokers, and reinsurers in a market have caught on over time. Some effects like the usage of lump sums or a change in the business mix can be observed and are usually known to the reinsurer. Even though this information is usually not considered directly for a modelling, an actuary can still push the pricing in one or another direction. However, offsetting this by using some sort of index is not possible here since the impact is usually not measurable for a reinsurer due to the Simpson's Paradox. This also accounts for the influence of management decisions, the usage of case or bulk reserves, knowledge about the settlement rates, or the efficiency of the claims' management of the primary insurer. Here, this is only consistent for each individual cedant and noticeable calendar year effects can be smoothed out in the aggregated data. However, the main driver coming from cedant specific influences is the business mix and the underwriting of the primary insurer. While some insurers have a strict underwriting guideline to focus on good risks and balance of the portfolio, others ignore this and only focus on premium and growth. Combining these cedants will result in a dataset that is overestimating the risk for the insurer with good risks and underestimating the risk for the other insurer.

While combining the data of multiple cedants poses a great opportunity of enlarging the data basis, it does also poses a great risk [54] since all idiosyncrasies of multiple cedants are mixed together. This makes the combined data less homogeneous which results in the idea of subdividing the available data to produce less biased results [13]. While the subdivided data would be more homogeneous [29], this is usually not an option for a reinsurer since the knowledge of cedant specific idiosyncrasies is usually sparse if even existing at all. Moreover, dividing larger and smaller claims, the number of claims per cedant, or the exposure measure would make a transition between those states impossible. Thus, the actuary has to choose between a more inhomogeneous larger dataset, where the underlying idiosyncrasies are not fully known, or a more granular set for which the impact of idiosyncrasies is weakened but the data might be too sparse. Since the data is usually aggregated, the full set of data is commonly used in order to minimize the distorting effects of underlying or procedural changes on the data [20] by assuming that the impact of cedant specific idiosyncrasies is negligible. However, this means that the data used is inhomogeneous to some extent. In practice, a pricing using market data can result in overestimation or underestimation of the portfolio risk depending on the cedant since an average claim severity is priced. Hence, this results in a trade-off between amount of data of a cedant and accuracy of the pricing.

2.6. Methods to Detect and Deal with Influential Effects on Data

In order to make a market dataset more homogeneous, the question arise whether and how trend effects in the data can be detected and dealt with. Since aggregate models are pre-dominantly used, some techniques exist to analyse the aggregated datasets for trend effects. Herefore, the residuals are usually considered by analysing the differences for accident, development, or calendar years for a significant pattern [90]. This is done similarly for the Chain Ladder method [150] after applying a standardisation to the estimated residuals [27] as discussed in Section 2.8.1. Additionally, the separation method [228, 240] can be considered here which estimates the calendar year effect under the assumption that the accident year effect is already known or can be offset [32, 46]. Since this assumption is usually not fulfilled, this approach is not considered further. Another approach focuses on the most recent diagonal of the aggregated triangle. Hereby, the latest diagonal is excluded in order to stabilize the results [27], while other approaches only consider the data from the most recent calendar periods to omit data with different development patterns [46]. Both approaches reduce the number of available data points further which is not an option for a SLD model since the history of each single claim will not be considered fully. Finally, several publications consider regression trend models on aggregated data [16, 45, 269] where the aggregated claims data is transformed. Since this can only be done with aggregated data, this approach is not promising for single claims' adjustment. A good summary of current methods to account for changing patterns for aggregated data like handling calendar year trends, changes in case reserve adequacy, changes in settlement rates, and missing data, can be found in the following publications[12].

While all of these approaches are based on aggregated data and adjust the aggregated to deal with trend effects, this is not suitable for adjusting individual claims. So far, reviewing the scientific literature could deliver only one approach which can be used on individual claims under the assumption that the trend effects have an exponential character [20]. Hereby, an exponential curve is fitted to the case reserves per open claim, paid losses per closed claim, and to calendar year paid losses per closed claim. However, it is questionable if trend effects in practice fulfill this assumption. Apart from that, there is no method to deal with trend effects and influences on an individual claims basis. The main reason for this is the fact that it cannot be decided whether an effect comes from the actual claims' development or due to external effects or idiosyncrasies of primary insurance companies. Since the layer inflation is different from the ground up inflation [73] it is only possible to derive the total layer loss inflation if the loss distributions tail is exponential or Generalized Pareto [73]. However, the tail

[12] [20, 27, 46, 67, 79, 90, 205].

distribution is not known for MTPL business and requires a distribution assumption since it is not unusual that the claim settlement might take more than 50 years. Thus, it is only possible to apply the indexation to the external factors [141] described in Section 2.4.2 while knowing that this can only be considered as an approximation reducing the impact. The decision to apply an index is ultimately up to the actuary resulting in a huge uncertainty and volatility of possible pricing results [23]. An actuary can exclude or adjust unusually large individual losses or select loss severity trends based on the loss data [259] if there is a trend effect in his opinion. Additionally, data from cedants showing a significant different behaviour in respect of trends or structural breaks can be excluded to improve data quality in the end.

2.7. Data Choice, Assumptions, and Shortcomings

Concluding the previous sections, it is clear that a reinsurance company has to choose between a larger market data pool with the risk of inhomogeneity and the cedant data itself that might be too sparse for a sophisticated pricing. This goes hand in hand with the purpose of a SLD model. It has to be decided on whether such a model should only work with the data of a single cedant to be applicable for any market or if it should be applicable to any cedant in a market independent of the amount of data that cedant provides. Due to the practical point of view, the second variant is chosen here in the first place. Thus, the market data is considered further for the modelling by taking the minimum data requirements as stated in Section 2.3. Consequently, such a model cannot be used to estimate a price for a cedant with no additional market data available.

This choice of the dataset automatically leads to an inhomogeneity that can disturb the future pricing [46]. Hereby, it has to be clear that the responsible effects and idiosyncrasies cannot be offset completely since a distinction between overlapping trend and LDFs cannot be achieved [39, 48, 193]. Additionally, common methods to adjust the data for trend effects only work on aggregated data and are not applicable to individual claims data [48]. Consequently, it has to be assumed that external effects as described in Section 2.4.2 can be offset partly by the applied indexation and that a proper index is also available for the respective market, which might not be true in practice [73]. This also accounts for any stable trend in the dataset since unstable trends make the projection of claims less predictable [269]. Additionally, the impact of the idiosyncrasies of primary insurers is assumed to be negligible and that companies with a completely different claims' behaviour than the rest of the market are excluded. While this assumption is indeed questionable, it follows the idea that a healthy insurance market will balance itself in respect of portfolio mixes, loss ratios, and rates. In total, this means that the market dataset is assumed to be approximately homogeneous and that all claims in the market dataset are comparable.

Assumption 2.1
The available market data is assumed to be approximately homogeneous after applying the indexation.

Since this is not true for real data, an actuary has to be aware of possible effects and their occurrence in the dataset. Thus, the Chain Ladder residuals are used later on to clarify the existence of possible trend effects in the data even if this cannot be eliminated.

2.8. Aggregated Pricing Methods

For a later comparison with existing aggregated models as stated in Section 1.3.1, the most commonly used aggregated methods are briefly discussed. This includes the classical Chain Ladder method and its distribution free approach [149, 150] since it is still used predominantly in practice while delivering stable and adequate overall results. Additionally, the Munich Chain Ladder model [183] is assumed since it includes more information in form of an additional paid triangle and its development. The last model that is considered is the Cape Cod method [34, 209] since it uses the exposure data for the predictions which none of the models above consider so far.

Therefore, the incurred value for each claim is cumulated according to the respective accident and development year:

$$C_{i,l} = \sum_{j=1}^{k_i} Inc_{i,j,l} \qquad \text{for } i = 1,\dots,n;\ l = 1,\dots,n$$

This leads to a simplified accumulated claims development triangle as shown in Table 2.2 which is used further.

Accident Year	Development Year				
	1	2	$\dots$	$n-1$	n
1	$C_{1,1}$	$C_{1,2}$	$\dots$	$C_{1,n-1}$	$C_{1,n}$
2	$C_{2,1}$	$C_{2,2}$	$\dots$	$C_{2,n-1}$	
$\vdots$	$\vdots$	$\vdots$	$\iddots$		
n	$C_{n,1}$				

Table 2.2.: Accumulated Claims Development Triangle.

2.8.1. Chain Ladder Model

At the beginning of the 2020s the Chain Ladder method is widely used and the most popular claims reserving method [150]. Originally, it was a purely deterministic procedure for calculating reserves without considering the claims reserving problem in a stochastic framework [249]. The basic idea behind this method is that the ultimate value can be estimated by multiplying the initial claim amount by some development factors $F_{i,l}$:

$$C_{i,n} = C_{i,1} \cdot \prod_{l=2}^{n} F_{i,l}, \qquad \text{with } F_{i,l} = \frac{C_{i,l}}{C_{i,l-1}}.$$

Since the correct development factors $F_{i,l}$ are unknown, they are estimated by considering the overall claims' development for two consecutive development years. Thus, the ultimate value can be estimated for accident years $i > 1$ by:

$$\hat{C}_{i,n} = C_{i,n+1-i} \cdot \prod_{l=n+2-i}^{n} f_l, \qquad \text{with } f_l = \frac{\sum_{i=1}^{n+1-l} C_{i,l}}{\sum_{i=1}^{n+1-l} C_{i,l-1}}, \; l > n+1-i. \tag{2.1}$$

The f_l are the Chain Ladder development factors or age-to-age development factors, which can be smoothed for the development years in order to adjust the overall predicted result. It has to be clear that the number of accumulated claim amounts used to derive the development factors is decreasing with an increasing number of development years. Thus, the predictive power is decreasing over the development periods and the model is more prone for outliers in the data. Additionally, the development of the claims is only considered until the last development year n which might not be enough to cover the development of long tail business lines as MTPL claims. Therefore, a tail factor is usually applied for aggregated models in order to incorporate the possible tail developments. While the tail factor is chosen manually, the ultimate value term in this section refers to the claim amounts derived for the last development period n. Since the Chain Ladder method does not allow for an error estimation, a stochastic framework was established that was able to present estimates for the prediction uncertainty in terms of the Mean Squared Error [149, 150]. This distribution-free approach by Mack [149, 150] sets a meaningful stochastic foundation behind the Chain Ladder method by stating three essential assumptions:

CL1: The cumulative claims' payments $C_{i,l}$ from different accident years $i \in \{1, ..., n\}$ are independent.

For all $i = 1, ..., n$ and $l = 2, ..., n$ it holds:

CL2: $\mathbb{E}\left(\frac{C_{i,l}}{C_{i,l-1}}|C_{i,1},...,C_{i,l-1}\right) = f_l$

CL3: $Var\left(\frac{C_{i,l}}{C_{i,l-1}}|C_{i,1},...,C_{i,l-1}\right) = \sigma_l^2.$

Following the assumption CL1 and CL2, the same predictor as in Formular 2.1 can be obtained:

$$\begin{aligned}
\mathbb{E}(C_{i,n}|C_{i,1},...,C_{i,n+1-i}) &= \mathbb{E}\left(\mathbb{E}\left(C_{i,n}|C_{i,1},...,C_{i,n-1}\right)|C_{i,n+1-i}\right) \\
&= \mathbb{E}\left(C_{i,n-1}\cdot f_n|C_{i,n+1-i}\right) \\
&= ... \\
&= \mathbb{E}\left(C_{i,n+1-i}|C_{i,n+1-i}\right)\cdot \prod_{l=n+2-i}^{n} f_l \\
&= C_{i,n+1-i}\cdot \prod_{l=n+2-i}^{n} f_l, \qquad \text{for } i > 1.
\end{aligned}$$

For this framework, it can then be shown that $f_2,...,f_n$ are unbiased and uncorrelated [149]. Additionally, assumption CL3 allows to derive the mean squared error for the overall reserve estimates [149] by using the unbiased estimator:

$$\hat{\sigma}_l^2 = \frac{1}{n-l}\sum_{i=1}^{n+1-l} C_{i,l-1}\left(F_{i,l} - f_l\right)^2.$$

Since the Chain Ladder model can also be interpreted as a weighted linear regression [16, 152], a regression analysis can be applied. Hereby, a special attention is set to the analysis and plots of the standardised residuals [14, 15, 150] which can be estimated for each $i = 1,...,n-1$ and $l = 2,...,n+1-i$:

$$\hat{r}_{i,l} = \frac{\text{observed deviation}}{\text{standard deviation}} = \frac{C_{i,l} - C_{i,l-1}\cdot f_l}{\hat{\sigma}_l\cdot\sqrt{C_{i,l-1}}} = \sqrt{C_{i,l-1}}\cdot\frac{F_{i,l} - f_l}{\hat{\sigma}_l}.$$

Hereby, the originally occurred claim amount for a respective accident and development year is compared with the estimated value after a standardization. Analysing those residuals allows to detect possible influential effects in the data as discussed in Section 2.4, which is usually done by plotting these residuals:

- Plotting the residuals against the development year can be used to verify whether outliers are contained in the dataset.
- Plotting the residuals against the accident years could show portfolio changes or a different development behaviour in general.

- To check for calendar year effects, a plot against the calendar year is suggested. In such a case, it might be advisable to leave out the data of the irregular calendar years.

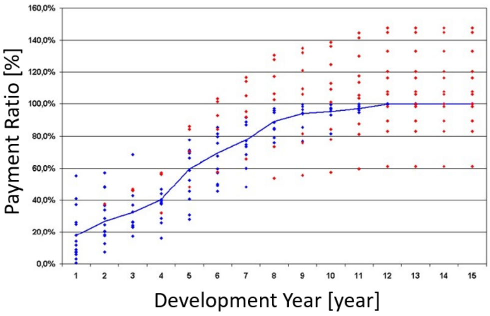
This is also further used to detect possible trend effects in the data. Besides that, some shortcomings and difficulties of the Chain Ladder model have to be considered:

- The Chain Ladder model relies upon the assumption that loss development patterns are stable over time and are not changing from one accident year to another [46, 79].
- The ultimate values prediction depends highly on the latest incurred value on the diagonal and the development factor of the last development year [150]. This makes the method sensitive with respect to outliers and zeros in the data, especially, on the diagonal which is often the case for XL reinsurance.
- Since the Chain Ladder model only includes the accident and development year, all calendar year based dependencies should be removed from the run-offs before the ultimates are calculated [29, 32].
- Among actuaries there is a disagreement whether the Chain Ladder technique should be applied to paid or incurred triangles [243].
- It is not possible to consider expert opinion and knowledge within the Chain Ladder model, which is based only on the settlement process itself. However, an actuary can always adjust the development factors manually which gives some kind of control function.
- Overall, a blind application of the Chain Ladder model can be misleading [32].

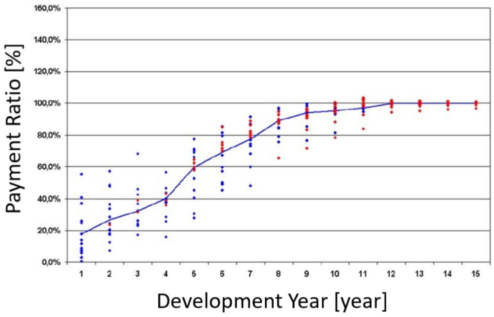
2.8.2. Munich Chain Ladder Model

Among actuaries it is not clear whether methods like the Chain Ladder technique should be applied to paid or incurred triangles, which is also the reason why two Chain Ladder calculations are sometimes performed, one on the paid and one on the incurred triangle. However, considering the payment ratio as defined in Section 2.3 of the extrapolated quadrangles it can be shown that the paid projection is lower for some accident years than the projection on incurred losses but surpasses it in other accident years [183]. This is shown exemplary in Figure 2.2a for the Chain Ladder and Munich Chain Ladder method.

This is due to the fact that the correlation between paid and incurred values is not considered in the Chain Ladder method [183]. Therefore, the Munich Chain Ladder model was developed in order to include the correlation which solves this problem as shown in Figure 2.2b. Therefore, both triangles are used and the payment ratio and



(a) Chain Ladder.



(b) Munich Chain Ladder.

Figure 2.2.: Payment Ratios of the Chain Ladder and Munich Chain Ladder based on Quarg and Mack [183].

its inverse are taken into account in order to smooth the predictors. Since both sets of data, the paid and incurred, are used, the cumulative payments are defined as $C_{i,l}^{Pa}$ and the cumulative incurred values as $C_{i,l}^{Inc}$ for $i = 1,...,n$ and $l = 1,...,n$. This notation is done for clarification only since the cumulated claims' amount stated in Section 2.8.1 is equal to $C_{i,l}^{Inc}$. To state the assumptions of the Munich Chain Ladder method, the notation done by Merz and Wüthrich [265] is considered:

MCL1: The cumulative payments $C_{i,l}^{Pa}$, $l \in \{1,...,n\}$ of different accident years $i \in \{1,...,n\}$ are independent. The cumulative claim amounts $C_{i,l}^{Inc}$, $l \in \{1,...,n\}$ of different accident years $i \in \{1,...,n\}$ are independent.

MCL2: There exist factors f_l^{Pa} and f_l^{Inc} and variance parameters σ_l^{Pa} and σ_l^{Inc} for $l = 1,...,n$ such that following holds for all $i = 1,...,n$ and $l = 2,...,n$:

$$\begin{aligned}
\mathbb{E}\left(C_{i,l}^{Pa}|\mathbb{C}_{l-1}^{Pa}\right) &= f_l^{Pa}\cdot C_{i,l-1}^{Pa} \quad \text{and} \quad Var\left(C_{i,l}^{Pa}|\mathbb{C}_{l-1}^{Pa}\right) = \left(\sigma_l^{Pa}\right)^2 C_{i,l-1}^{Pa}\\
\mathbb{E}\left(C_{i,l}^{Inc}|\mathbb{C}_{l-1}^{Inc}\right) &= f_l^{Inc}\cdot C_{i,l-1}^{Inc} \quad \text{and} \quad Var\left(C_{i,l}^{Inc}|\mathbb{C}_{l-1}^{Inc}\right) = \left(\sigma_l^{Inc}\right)^2 C_{i,l-1}^{Inc}
\end{aligned}$$

with $\mathbb{C}_l^{Pa} = \left\{C_{i,k}^{Pa}, k \le l, i = 1,...,n\right\}$ and $\mathbb{C}_l^{Inc} = \left\{C_{i,k}^{Inc}, k \le l, i = 1,...,n\right\}$.

MCL3: There exist constants λ^{Pa} and λ^{Inc} such that for all $i = 1,...,n$ and $l = 2,...,n$:

$$\mathbb{E}\left(\frac{C_{i,l}^{Pa}}{C_{i,l-1}^{Pa}}\middle|\mathbb{C}_{l-1}^{Pa},\mathbb{C}_{l-1}^{Inc}\right) = f_{l-1}^{Pa} + \lambda^{Pa} Var\left(\frac{C_{i,l}^{Pa}}{C_{i,l-1}^{Pa}}\middle|\mathbb{C}_{l-1}^{Pa}\right)^{\frac{1}{2}} \cdot \frac{\frac{C_{i,l-1}^{Inc}}{C_{i,l-1}^{Pa}} - \mathbb{E}\left(\frac{C_{i,l-1}^{Inc}}{C_{i,l-1}^{Pa}}\middle|\mathbb{C}_{l-1}^{Pa}\right)}{Var\left(\frac{C_{i,l-1}^{Inc}}{C_{i,l-1}^{Pa}}\middle|\mathbb{C}_{l-1}^{Pa}\right)^{\frac{1}{2}}}$$

$$\mathbb{E}\left(\frac{C_{i,l}^{Inc}}{C_{i,l-1}^{Inc}}\middle|\mathbb{C}_{l-1}^{Pa},\mathbb{C}_{l-1}^{Inc}\right) = f_{l-1}^{Inc} + \lambda^{Inc} Var\left(\frac{C_{i,l}^{Inc}}{C_{i,l-1}^{Inc}}\middle|\mathbb{C}_{l-1}^{Inc}\right)^{\frac{1}{2}} \cdot \frac{\frac{C_{i,l-1}^{Pa}}{C_{i,l-1}^{Inc}} - \mathbb{E}\left(\frac{C_{i,l-1}^{Pa}}{C_{i,l-1}^{Inc}}\middle|\mathbb{C}_{l-1}^{Inc}\right)}{Var\left(\frac{C_{i,l-1}^{Pa}}{C_{i,l-1}^{Inc}}\middle|\mathbb{C}_{l-1}^{Inc}\right)^{\frac{1}{2}}}.$$

MCL4: The sets of cumulative payments and claims incurred $\{C_{i,l}^{Pa}, C_{i,l}^{Inc}\}$ for $l = 1,...,n$ are independent for different accident years $i \in \{1,...,n\}$.

The parameter estimation is described in the original paper by Quarg and Mack [183]. The first two assumptions of the Munich Chain Ladder Model MCL1 and MCL2 represent the classical assumptions done for the Chain Ladder model [265]. The third assumption MCL3 adds the conditional correlation coefficients between the incurred and paid values, which is independent of the development years [265]. Assumption MCL4 is needed to derive appropriate estimators for the conditional moments of the different ratios [265]. While it is possible to derive the mean squared error for the Chain Ladder model, only a bootstrap approach done by Liu [142] is known for the Munich Chain Ladder Model. Since the Munich Chain Ladder model is incorporating the correlation between paid and incurred triangles, there are issues when the run-off of the earliest accident years are still unfinished. To overcome these issues, smoothing and extrapolation are required and advisable [183].

2.8.3. Cape Cod

One issue of the Chain Ladder technique is that it depends completely on the last observed values on the diagonal that is projected to ultimate. If those values are not representative, e.g. due to outliers or incomplete data as it is often the case for long-tailed lines of business, the predictions are misleading. Thus, a robustification reducing the impact of possible outliers is done in the Cape Cod method [34, 77, 209, 215, 265, 269]. The Cape Cod method is based on the following assumptions mainly following the notation by Merz and Wüthrich [265]:

CC1: Different accident years are independent.

CC2: There exist parameters $E_i > 0$, $i = 1,...,n$, a constant $\kappa > 0$, and a claims' development pattern β_l, $l = 2,...,n$ with $\beta_n = 1$ such that for all $i = 1,...,n$ the following holds: $\mathbb{E}(C_{i,l}) = \kappa \cdot E_i \cdot \beta_l$.

Hereby, the parameter E_i represents the exposure value for the respective accident year $i = 1,...,n$ in form of the number of vehicles or the premium while κ reflects the average loss ratio of the portfolio. The development pattern β_l could be identified with the Chain Ladder development factors f_l so that:

$$\beta_l = \prod_{k=l}^{n} f_k^{-1}.$$

In order to derive an estimator for the average loss ratio, the Chain Ladder estimates lead to an unbiased estimator for κ that can be robustified [265] for the overall loss ratio to:

$$\hat{\kappa} = \frac{\sum_{i=1}^{n} C_{i,n-i+1}}{\sum_{i=1}^{n} \beta_{n-i+1} \cdot E_i}.$$

The Cape Cod estimator, under usage of a robustified value for $C_{i,n-i}$, is then given by:

$$\hat{C}_{i,n} = C_{i,n-i+1} - \hat{C}_{i,n-i+1} + \prod_{j=n-i+2}^{n} f_j \cdot \hat{C}_{i,n-i+1}$$
$$\text{with } \hat{C}_{i,n-i+1} = \hat{\kappa} \cdot E_i \cdot \beta_{n-i+1}, \text{ for } 2 \leq i \leq n.$$

The Cape Cod method is using the exposure values to estimate a robustified diagonal value $\hat{C}_{i,n-i+1}$ that is less prone to outliers in the diagonal data. The further projection is often done in combination with the Chain Ladder method. Overall, the accident year levels are effectively merged into a single parameter by assuming that the accident years are adjusted to a common level [34, 209]. However, the same final loss ratio is applied to all years, which might not be correct for all portfolios. If the loss ratio is showing a trend in practice, this might lead to deviations in the predictions by only assuming a fixed overall loss ratio.

2.9. Summary

This chapter focused on the available data for a reinsurer in respect of MTPL business connected with an XL reinsurance structure. Hereby, there is a large information gap between the primary insurer and the respective reinsurer. Due to the XL structure and the corresponding clauses in the reinsurance contract, a reinsurer receives only a few claims that might hit the pricing layer. On top of this small data pool, additional influences and effects are distorting the data. This includes external factors like economical inflation, legal and political decisions, societal influences, or superimposed inflation, which might lead to accident, development, or calendar year effects. Additionally, different peculiarities of primary insurers like different claims' handling, the reserving behaviour, or underwriting decisions and business mix changes can affect the data as well. Due to the overlap of all these influences, it is currently not possible to measure the impact of each effect individually. Thus, indexation can only be used on external and obvious effects in the data to reduce the related impact while this cannot be done for the idiosyncrasies of primary insurers. Furthermore, single claims data cannot be adjusted for possible trend effects since it cannot be distinguished between a trend or claims' movement. Hence, it has to be assumed that the data used is approximately homogeneous and that the impact of idiosyncrasies of primary insurers on the data is negligible if the market data is considered. The Chapter concludes with a brief explanation of the three commonly used aggregated pricing methods, the Chain Ladder, Munich Chain Ladder model, and the Cape Cod method. Those methods are later used as benchmark to evaluate the goodness of an individual loss model with respect to its aggregated results. Additionally, the Chain Ladder residuals are later used to detect trend effects in the data even though they cannot be offset for the SLD model.

3. Basic Framework and Clustering

3.1. Required Features in Practice

Before a suitable SLD model and the related model elements can be developed, the required features have to be clarified. It should be clear that the SLD model should be used for reinsurance pricings of XL treaties. From this, specific features and requirements for such a model derive which have to be fulfilled and are driven by market experience:

- Handling sparse claims data: Reinsurance companies deal with larger claims which can result in a small number of known and relevant claims based on the cedants specific reporting threshold. Since the calibration of a model based on, e.g. one or two claims, is not sufficient, the usage of market data itself is a necessary requirement for the practicability of the model.
- New LDFs: In many pricing methods the development factors are often chosen according to already known development factors in the same development year. Due to the small claim number of some cedants, this can lead to biased results, e.g. consider one possible LDF with a value of 1.5 which is used for all claim projections in this development year. Furthermore, a reinsurer wants to predict unknown and unseen developments within the ultimate severity distribution to incorporate the uncertainty of the future claim developments. Thus, the model should use new and unseen development factors for the projection of the claims, which requires the usage of a stochastic modelling approach combined with a triangle free approach.
- Extreme LDFs: Related to the previous point, the possibility of using extreme LDFs during the claims' projection should be possible. Not only for the risk analysis but also to simulate the black swan events, e.g. a very high claim that occurs with a very small probability. These claims have an influence on the tail of a fitted severity distribution, which is of major importance for a reinsurance company in particular.
- Claim settlement: Since all claims are projections on single basis, it is possible to settle claims during this development process. Having a proper settlement process would also allow to develop all claims until they are settled which would make a tail factor superfluous.

- Claim history: It can be seen in real data that after a greater increase of the claims' outstanding value for one development year, the subsequent development year does not show the same behaviour. This implies that the future claim development somehow depends on the historical claims' development which leads to a claims' development memory. This requirement is unusual since current models usually assume the independence between development years resulting in the assumed memorylessness for the claims' development.
- Claim Sizes: The development of larger and smaller claims should differ since this can be observed on real data following the idea by Drieskens et. al. [64].
- Assumptions: Based on the independence assumptions used for the aggregated models, the assumptions stated for the SLD model should be realistic.
- Simplicity: Current SLD models are usually hard to explain, which limits their usage in practice. Due to the communication between actuaries, underwriters, and cedants, the model should be easy to explain to other non-mathematicians resulting in the usage of explainable mathematical models.

These practical motivated requirements are expanding the theoretical requirements from Section 1.3.1. Consequently, such a SLD model would allow to use the stabilisation and layer on individual losses without adding additional assumptions as it has to be done currently. In order to meet the stated requirements, the focus will be set on the implementation of a basic framework in which individual claims can be projected. Therefore, the starting point is set to the market data with the current assumption of having an approximately homogeneous market dataset and new assumptions will be added when they are utilized [5]. All assumptions used are later summarised in Section 4.8.

3.2. Short Description of the Final Model

As described in the introduction, some complications for the development of individual claims arise from the representation as a loss run-off triangle. Thus, a new framework for the development of individual claims has to be set up. This will be explained briefly in order to give the reader a general idea of the model in advance.

This new framework is based on the representation of claims as a point cloud considering the incurred value, the payment ratio, and the LDFs of each single claim and development step. This point cloud is analysed for structural differences using a cluster analysis in Section 3. This serves as a basis to develop individual claims in Section 4. For each cluster identified in the point cloud, a copula is fitted to the LDFs of points lying in the cluster. This represents the basis for the claims' development and is responsible for updating an individual development step. In order to meet the practical requirements, each development step is done under two conditions. Firstly, the drawn development factors are limited with respect to the maximum size depending on the related payment ratio and incurred value of the starting point. Secondly, the current historical claim path is analysed and used to ensure realistic claim paths.

3.3. German MTPL Market Dataset

The SLD model is developed from a practical point of view as stated in Section 1.3.5. Therefore, internal real MTPL claims from the German market provided by a leading reinsurer are used. This dataset contains 5.638 single claims from 1988 to 2015 which represents a market share of more than 50% [36, 131] of the German MTPL market[1] and is used to derive the SLD model in Chapters 3 and 4. The single claims result in $54,803$ data points from ground up and $23,432$ data points after applying the cedant specific reporting thresholds. Moreover, this dataset is used to illustrate the different characteristics of MTPL data as well as the model features.

3.4. Choice of Dataset and Variables

Current models usually consider a claim triangle which is used to set up a pricing model. Therefore, these models stay within the framework of a triangular approach which leads to problems when it comes to single loss development. Additionally, it is disputed among actuaries whether a paid or incurred triangles or even both should be used [243]. Hence, the idea arises to turn away from the representation of a claims triangle and to represent the individual claims and their development in a point cloud, which follows the second idea stated in Section 1.3.5 to consider a triangle-free approach. Based on the data that is available for a reinsurer stated in Section 2.3, the parameters that are of interest and necessary have to be identified first. Hereby, it has to be noted that this is the only reliable information for a reinsurer.

3.4.1. Time Component

When a claim occurs it might take some time until the claim is reported to the insurer. Afterwards, the claims' handling process ticks in and the claim size changs over the development years until the claim is settled. This is the basic lifecycle of a claim

[1] A more detailed market share is left out due to confidentiality conditions by the company providing the data.

without considering a re-opening. Hence, the first component that plays a role here is the time component. This is represented by the accident year, development year, and the calendar year. In commonly used pricing methods, the time component is considered to be the main parameter by choosing the LDFs according to the respective development years [64, 103, 147, 164]. While such an approach is intuitively following the idea of the Chain Ladder model, it also has similar disadvantages.

Since all LDFs per development year are usually considered, it is not given that the chosen development factor is related to a similar claim. The development factors are estimated regardless of the claim size or other claim characteristics. Hence, a small claim could get a development factor related to a large claim and vice versa, which can lead to an unrealistic development of the claim. Even if LDFs are grouped to offset this, e.g. by their incurred values [64], it is not necessarily guaranteed that this grouping is correct. Additionally, the number of available development factors for later development years is reducing which can make a grouping impossible. The sparse number of claims data in later development years is hereby a general challenge when the development year and time component are considered.

Thus, considering the LDFs only with respect to their specific development year alone, does not seem to be a promising approach when considering the projection of single claims. However, it can be seen for real data that the life cycle of a claim can show some patterns based on country level due to the related legislation and jurisdiction. This is related to the feature requirement stated in Section 3.1 of incorporating the claim history in the projection process. Hence, the development year should not be the only parameter to base the future LDFs on but should give some guidance to the claims' development process.

Having a look at the accident years, it cannot be assumed directly that the claims' development of a single claim is independent from the accident years. The reasons here are possible changes in law or court decisions that might lead to an accident year effect and a different claims' handling. Thus, an independence between the claims' development and the accident years is only given if there is no accident year effect. Since this is usually the case for most countries, the independence is further assumed here.

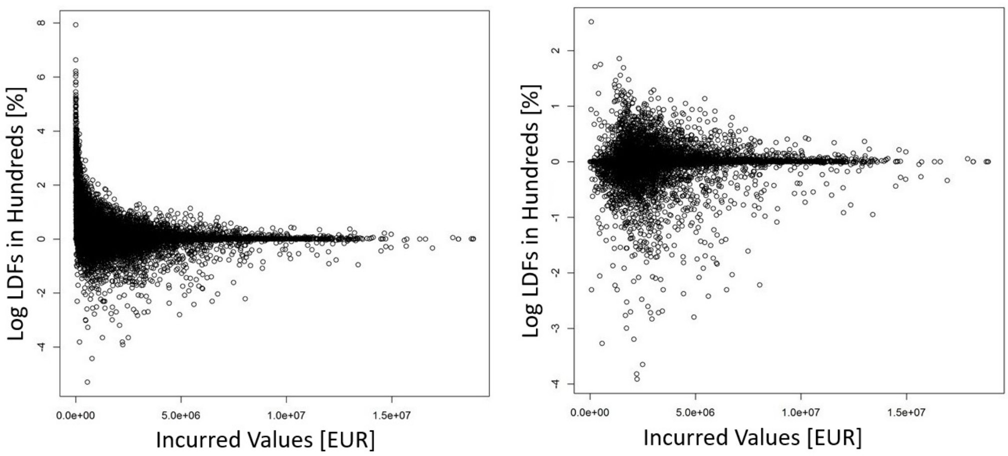
Assumption 3.1
The development of a single claim is independent of the accident year.

Additionally, it is natural to assume that the claims' development for individual claims among themselves is independent. This implies that no relation between MTPL claims exists and that the real claims' development of those claims does not depend on other claims.

Assumption 3.2
The claims' development for individual claims among themselves is independent.

3.4.2. Paid, Outstanding, and Incurred Value

The next parameters to be considered are the paid and outstanding values as well as the incurred value as sum of both. Since these values represent the claim amount, which is then projected to ultimate, they should be considered as an important parameter. Additionally, this is done with the intention to distinguish the development of single claims according to their claim sizes as it is done similarly by Drieskens et al. [64]. This follows the idea that larger claims usually involve severe injuries and disabilities which is related to loss of income and annuity payments. Hence, the size of the LDFs is different in comparison to smaller claims that result from minor injuries or death. This indication is illustrated exemplarily for real German MTPL claims in Figure 3.1. with a distinction for the different reporting thresholds.



(a) Ground-up Losses.

(b) Losses Exceeding Reporting Thresholds.

Figure 3.1.: Loss Development Factors with Respect to the Incurred Values.

The logarithmic LDFs are plotted with respect to the incurred values from ground up in Figure 3.1a and under consideration of the reporting thresholds in Figure 3.1b. Here it can be seen that for both cases the LDFs of claims with a small incurred value have a higher volatility compared to larger claims. The LDFs of larger claims are smaller which leads to a more stable development of those claims. This behaviour is more pronounced for losses below the reporting threshold which is also favoured by the smaller claim amounts. Due to the calculation of the LDFs as quotient of two consecutive incurred values, higher LDFs tend to be related to smaller claim amounts and smaller LDFs to higher claim amounts. Hence it is further assumed that the claims' development depends on the claim size.

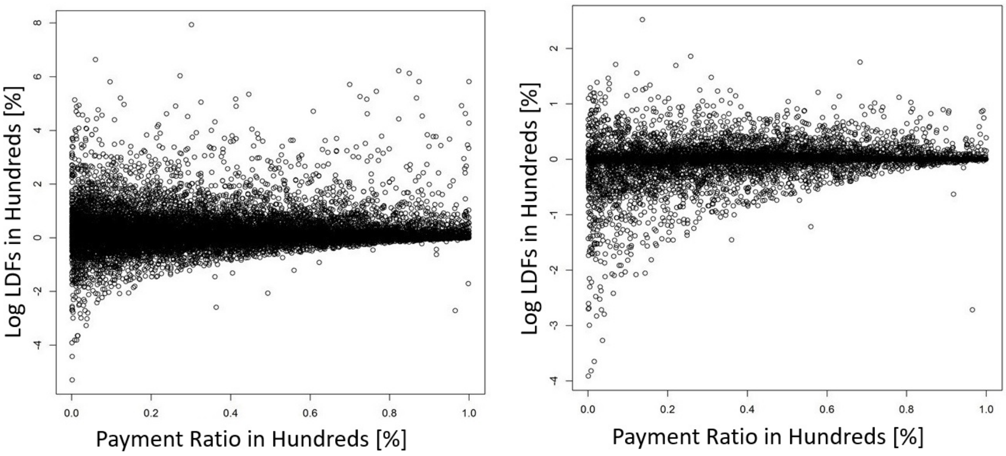
Assumption 3.3
The claims' development depends on the claim size.

However, it is not clear which parameter, the paid, outstanding, or incurred values, should be used to capture this behaviour. The paid value represents the amount that has been paid to the insured so far but is underlying a high volatility, which leads to unstable development factors most of the time [64]. This is related to lump sum payments as discussed in Section 2.4.3.4 and that claims are usually not paid steadily. It is more common to have jumps in the payments over the development period which leads to this high volatility. The outstanding values represent the expected amount the insurer expects to pay until the claim is settled and are highly related to the idiosyncrasies of the insurance company [64] as discussed in Section 2.4.3. Since the development of outstanding values is related to the corresponding paid values, e.g. the outstanding value usually decreases when the insurer pays a larger amount, both the outstanding values as well as the paid values have a similar volatility. Hence, both parameters are not considered to be optimal to base the estimation of LDFs on. In comparison to that, the incurred values are less volatile but involve more modelling complexity [227]. Since this represents the total claim cost for a claim it is more consistent with the claims' behaviour, e.g. if the paid value increases and the outstanding value decreases, the incurred value stays more or less the same. Thus, the incurred value is chosen as parameter to represent the claim size. Hereby, it has to be noted that negative paid values and negative incurred values can occur in practice due to recourses or law decisions. Since this does not represent the majority of claims, it is assumed that the cumulated paid values are monotonously increasing over the development years. However, this assumption is not critical since a later model can be adjusted to deal with negative paid increments and values.

Assumption 3.4
The cumulated paid values are monotonously increasing.

3.4.3. Payment Ratio

The life cycle of a claim can show some pattern that can also influence the further development of a claim as already stated in Section 3.4.1. While this is also related to the claims' development over the different years, claims in their early stage of development should be more volatile than claims in a later stage, where more information about the health condition of a person is known by the insurer. Since a smaller payment ratio is usually related to a large outstanding value for such high claims, this leaves more space for developments as well as miscalculations resulting in a higher volatility. Considering the payment ratio, this is shown exemplarily for real German MTPL claims in Figure 3.2. which is based on the same dataset as Figure 3.1.



(a) Ground-up Losses.

(b) Losses Exceeding Reporting Thresholds.

Figure 3.2.: Loss Development Factors with Respect to the Payment Ratio.

Here, the logarithmic LDFs are plotted with respect to the payment ratio for ground up claims in Figure 3.2a and under consideration of the reporting threshold in Figure 3.2b. It can be seen that the volatility of the LDFs is less distinct if only claims above the reporting threshold are considered. This can be seen by comparing the points lying in the upper right corner in particular[2]. Driven by the reserving behaviour of primary insurance companies, some smaller claims below the reporting threshold but with a high payment ratio show a large LDF due to reserve increases. Moreover, it can be observed that with an increasing payment ratio the volatility of the LDFs decreases.

First of all, this indicates that the development stage or the percentage of the already paid amount of a claim has an influence on the further development. Nevertheless, the ratio between paid and outstanding value could also be considered for this. The reason why this is not done is that this ratio would tend to infinity as soon as the claim is closer to its settlement. In contrast to that, the payment ratio tends towards one for claims near settlement which makes a modelling easier. Additionally, the payment ratio is more similar for different cedants compared to the incurred values since $PR_{i,j,l} \in [0,1]$. Thus, it is further assumed that the claims' development depends on the development stage of a claim, represented by the payment ratio.

Assumption 3.5
The LDF depends on the development stage of a claim.

[2]There are also points in the bottom right corner that indicate a decreasing paid value which is ignored here. Due to the monotonicity assumption of the paid values, those claims will be excluded later.

3.4.4. Loss Development Factor

To develop a claim further, information about the jumps between two development years, the LDFs themselves, have to be known as well. Usually, the LDF is always related to the percentage change of the incurred values. However, there is also the possibility not to consider the percentage but the absolute change with $i = 1, ..., n,\ j = 1, ..., k_i,\ l = 1, ..., n-1$.

$$LDF_{i,j,l}^{Inc,add} = Inc_{i,j,l+1} - Inc_{i,j,l} \qquad LDF_{i,j,l}^{Inc,mul} = \begin{cases} \frac{Inc_{i,j,l+1}}{Inc_{i,j,l}} & , Inc_{i,j,l} \neq 0 \\ Na & , \text{else.} \end{cases}$$

Since this does not influence the overall structure of the dataset in a critical way, this thesis follows the common understanding of a percentage change. Here it has to be noted that the LDF can only be derived from the incurred values and not from the payment ratios. Based on the latter, only the value of the updated payment ratio is known but a separation between the paid and incurred value is not possible.

3.4.5. Exposure Measure

The exposure measures, e.g. number of vehicles, premium income, or number of policies, are not used as parameter for this method. Since this information is based on the underlying accident year, it is used mostly for the calculation of frequencies. Furthermore, there is no logical influence of the exposure value for one accident year and the related claim movements for single claims.

3.4.6. Parameter Set and Basic Framework

In summary, the following parameters will be considered for a SLD model based on a claims triangle with $i = 1, ..., n,\ j = 1, ..., k_i,\ l = 1, ..., n-1$:

- The LDF based on the multiplicative approach $LDF_{i,j,l}^{Inc,mul}$ estimated on the incurred values,
- The incurred value $Inc_{i,j,l}$ representing the claim size,
- The payment ratio $PR_{i,j,l}$ representing the development stage.

Since a choice of the final parameters between the usage of the incurred values or the payment ratio would result in the decision of which parameter is more valuable and trustworthy, this is further investigated in the next section. Hereby, three parameter combinations are considered: the usage of either just the incurred values $(Inc_{i,j,l}, LDF_{i,j,l}^{Inc,mul})$, or the payment ratio $(PR_{i,j,l}, LDF_{i,j,l}^{Inc,mul})$, or the combination of both $(Inc_{i,j,l}, PR_{i,j,l}, LDF_{i,j,l}^{Inc,mul})$. It has to be noted that depending on the constellations a different number of overall points $\tilde{n}$ are given for the related point cloud. Furthermore, the reporting threshold also has an effect on the datasets since it brings in smaller claim values which increases the volatility as shown exemplarily in Figure 3.1. and 3.2. Thus, this distinction is considered in the next section as well.

At this point, it has to be emphasised that the size of the incurred values as well as the size of the payment ratio are related indirectly to the development year. A claim with a high incurred value is most likely related to a later development year than a claim with a small incurred value. Additionally, a claim with a higher payment ratio usually has a longer development period compared to a claim with a smaller payment ratio. Consequently, claims with a longer observed development period and with a usually higher payment ratio are less volatile than claims from earlier years with usually smaller payment ratios.

These findings can be compared to other models where the time component is the main parameter. Here the idea arises that a better prediction can be achieved by considering the payment ratio and incurred values over the development years without a consideration of the related development year. The incurred value and payment ratio already consider the time component indirectly. This is an important point since it is the basis and key idea for the later SLD model.

Assumption 3.6
The incurred value and payment ratio without a consideration of the related development year already consider the time component indirectly.

This directly leads to the conclusion that the LDFs, related to their incurred and payment ratio values, can be represented by a point cloud to set up the basic modelling framework. Having such a point cloud allows a structural analysis in form of clustering to find possible structures, which can be used for a projection of single claims. Thus, a claims' development will be based on similar claims regardless of the accident or development year.

Note here that other parameter constellations also might make sense after setting up the SLD model retrospectively, e.g. paid vs outstanding, payment ratio vs paid, etc. However, considering real observations and a practical point of view, these parameter sets are not intuitive in the first place and are not considered here.

3.5. Clustering - Setting Up a New Model Framework

The classification of similar objects into clusters is an important part of data analysis. Hereby, the clustering as unsupervised learning technique aims at finding structures in a dataset according to the respective cluster parameters. Therefore, it is advisable that the data fulfils some statistical properties, e.g. compact, well-separated, connected, and stable [30], in order to make the clustering work well. This is the basis for realistic and relevant cluster results. Keeping this in mind, it can be seen in Figure 3.1 and 3.2 that not all of these properties are fulfilled for the targeted point cloud. Due to the homogeneity of the MTPL market data, the points of the point cloud are highly connected. This makes the clustering difficult since no strict separation can be done. Consequently, it is questionable if a clustering could be applied here. Nevertheless, following Drieskens et al. [64], the main idea and required feature behind this is that claims with a different development stage and claim size should have a different further development. Therefore, a clustering could find groups of similar claims so that the development of a claim can be based on similar claims from the same cluster. Hence, a clustering with respect to the parameters stated in Section 3.4.6 is still a promising approach. However, the reasonability of the clustering should be analysed and discussed to justify this modelling step.

3.5.1. Cluster Techniques

Generally, cluster algorithms can be divided broadly into hierarchical and partitional clustering techniques. However, sometimes a more detailed seperation into hierarchical, partitional, optimizing, and graph-theoretical cluster techniques is done [94, 118, 126]. Hierarchical cluster methods start either with all points grouped in one partition or with all points separated being a partition on their own and find clusters by dividing or combining different partitions according to their similarity. Compared to that approach, partitional clustering starts with an amount of clusters and tries to rearrange the clusters by applying some kind of exchange function in order to find the most appropriate cluster. Since hierarchical methods tend to produce a variety of possible partitions and the later granularity has to be chosen by the actuary, this makes the usage in a modelling process difficult. Therefore, partitional clustering is considered promising if the number of clusters can be pre-defined. Then, the dataset is divided into non-overlapping subsets of clusters, so that each data object is in exactly one subset without any interference. While this approach is simple and well known to practitioners, a soft clustering approach following a fuzzy clustering methods, where each data point is assigned to all clusters by a different degree, might be a more intuitive way of dealing with a highly connected dataset in a second step. A good and comprehensive overview over cluster analysis can be found in following publications [119, 127].

Focusing on the MTPL point cloud, it has to be evaluated whether a partitional clustering approach, resulting in datasets with strict borders, is advisable. To answer this, several measures to define the reasonability of clustering are discussed in Section 3.5.4. Therefore, some checks on the data with respect to data quality and standardization of the cluster variables have to be done. Afterwards, the cluster analysis itself can be applied. This includes the choice of the optimal number of clusters, which can be determined by applying cluster validity indices as it is done in Section 3.5.4. In the second step the cluster algorithm itself can be performed by using the optimal number of clusters and the related cluster variables. The third and last step is the verification of the goodness of the clustering, which is explained in Section 3.5.7. This can also be done by applying validity indices as well as stability measures. Ultimately, this results in a set of cluster centres that can be used further to allocate the clusters accordingly.

3.5.2. Data Requirements, Preparation, and Relation

After defining the parameters that are used for the clustering in Section 3.4.6, the incurred values, payment ratios, and the LDFs have to be prepared further before a clustering can be applied. This includes a possible data reduction to allow an application in practice, the standardization of the variable, and an analysis of the correlation of the cluster variables.

3.5.2.1. Data Reduction

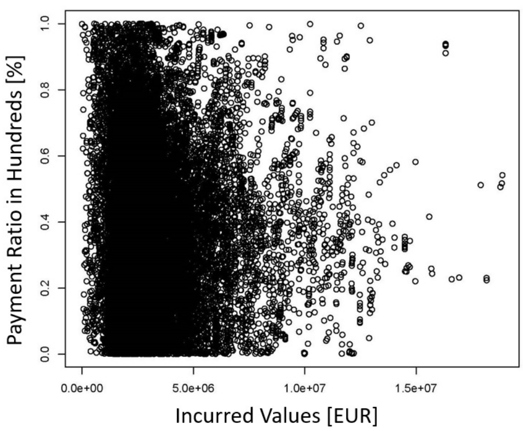
This point targets a later application in practice on usual working machines and is not related to the modelling itself. In the process of clustering data, a distance matrix is estimated, containing the distance between each point in the dataset to each other point related to the several dimensions, which can lead to the point where the available RAM is exceeded. Hence, depending on the size of the market data this can result in memory space and runtime issues which makes an application in practice difficult. In such cases, a reduction of the data points is advisable to fulfil the technical requirements. However, a reduced dataset has to still lead to the same or similar cluster result as done for the whole dataset. One possibility of doing this can be to consider a uniform random sampling of the data points. The clustering is then applied on each sample B_i, $i = 1, ..., k^*$ in order to determine a cluster centre vector of length k^* based on Monte Carlo simulation. Afterwards, the mean of the computed cluster centres from all samples can be used as an approximation of the cluster result for the original dataset and for further analysis. In practice, a sample size of 5.000 data points for the clustering has proven itself to work on usual working machines so that this procedure is applied to all markets where the point cloud consists of more than 5.000 data points.

3.5.2.2. Standardization

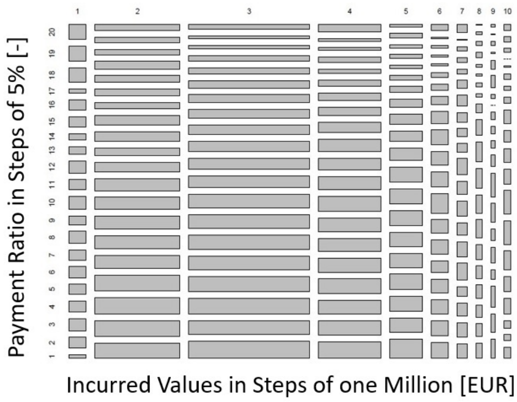
Before a clustering can be performed on the dataset, the related cluster parameters should be comparable. Since the cluster attributes are mixed numerical data, a scaling is applied to standardize the input data. For each dimension, each data point is subtracted by the mean and divided by the standard deviation of the dataset. Afterwards, the data is shifted so that the data is in $\mathbb{R}_{>0}$ for each dimension, which is more intuitive for a later usage.

3.5.2.3. Independence and Correlation of Cluster Variables

In addition to the above, possible relations and their influences between cluster parameters have to be discussed. Since the incurred values as well as the payment ratio are used in order to give an impression of possible LDFs, their relation is of major importance. This is to analyse the *Inc* and $\frac{Paid}{Inc}$ in respect of independence and correlation. The necessity of doing so is shown exemplarily in the German MTPL data and the related contingency table in Figure 3.3.



(a) Incurred Values and Payment Ratio.



(b) Contingency Table.

Figure 3.3.: Relation Between the Payment Ratio and the Incurred Value.

Here, the payment ratios are plotted against the incurred values in Figure 3.3a. together with the related contingency table in Figure 3.3b. In the contingency table, a step size of 5% is chosen for the payment ratio and one million for the incurred values which results in 20 equidistant groups for the payment ratio and 19 for the incurred values. Hereby, the upper 10 groups with respect to the incurred values are grouped so that the 10th group corresponds to all incurred values above EUR 10 million[3]. Subfigure 3.3a shows that for claims below a payment ratio of 50% the distribution for certain payment ratios over the incurred values look similar. Above 50% payment ratio, this changes since only a minority of the claims is already settled. Thus, considering a hypothetical case where all claims in the portfolio are already closed, both variables could be considered to be approximately independent. Since this is not the case in practice and for the data that is used for the clustering, both variables are considered dependent.

This can also be tested by applying the χ^2 test of independence to the dataset, the contingency table, with the hypothesis H_0 : Inc and PR are stochastically independent. The χ^2 related test statistic formula is given by:

$$\chi^2 = \sum_i \sum_j \frac{\left(\tilde{n}_{i,j} - \frac{\tilde{n}_{i,\cdot} \cdot \tilde{n}_{\cdot,j}}{\tilde{n}}\right)^2}{\frac{\tilde{n}_{i,\cdot} \cdot \tilde{n}_{\cdot,j}}{\tilde{n}}},$$

where $\tilde{n}$ is the total number of observations related to the chosen parameter constellation and $\tilde{n}_{i,j}$ corresponds to the two-dimensional frequency distribution of the incurred values and the payment ratio given in a contingency table. Note here that i and j as iterators are related to the number of classes M_i and M_j in the respective contingency table. In order to use a contingency table for this purpose, the fraction $\frac{\tilde{n}_{i,\cdot} \cdot \tilde{n}_{\cdot,j}}{\tilde{n}}$ has to be greater or equal to 5 or, according to the law of Cochran, it should be at least one for all values and 5 for at least 80% [91]. The hypothesis of independence is then rejected if the value of the test static χ^2 is greater than the related $1-\alpha$-quantile for the χ^2 distribution with $(M_i - 1)(M_j - 1)$ degrees of freedom. In order to set up a contingency table for this purpose, it has to be clarified that large claims can occur for all development and accident years and that the payment ratio can have any value between 0 and 1. Due to the long tail character of the business, a lot of claims with a payment ratio below 80% and just a few above as well as just a few large claims compared to many claims with a smaller incurred value are usually available. Thus, several classes have to be grouped together in order to meet the requirements of the test statistic. In the upper case shown in Subfigure 3.3b, the hypothesis of independence is rejected for a significance level of $\alpha = 0.05$.

If both features are independent, then they are also uncorrelated and it does not disturb the clustering in any case. Since this is not the case, it remains to be clarified whether both features are uncorrelated. If both are correlated, it can influence a cluster analysis and distort possible cluster results since they can impact several distance measures. In order to determine a possible correlation, the pearson coefficient r as well as spearmans ρ and kendalls τ could be estimated. While the pearson coefficient is a

[3]This is done in order to fulfil the requirements of a χ^2 test of independence.

measure for linear correlation, spearmans ρ and kendalls τ are rang coefficient based and robust against outliers or do not assume a normal distribution. Besides the correlation coefficient, a test for association between paired samples, using the pearsons product moment correlation coefficient, kendalls tau or spearmans rho can be done as well [105]. Therefore, setting H_0 : there is no correlation, the following test statistic can be used based on pearsons correlation coefficient r:

$$T = \frac{r}{\sqrt{1-r^2}} \cdot \sqrt{\tilde{n}-2}.$$

The test statistic T is approximately t-distributed for $\tilde{n} > 30$ with $\tilde{n}-2$ degrees of freedom. Nevertheless, it has to be mentioned that the sample size has a significant impact on the value of the test statistic. For large samples, it is easy to reject the hypothesis as the value of the test statistic T is increasing, reducing the related p value. Thus, this test statistic cannot be considered here since the results would be misleading and attention has to be paid to the correlation coefficients instead. Hence, the correlation coefficient for the German MTPL dataset in Figure 3.3. can be estimated. The Pearson correlation coefficient has a value of -0.054 showing that no linear correlation between the incurred values and the payment ratio exists. Moreover, spearman's ρ has a value of -0.059 implying that no monotonic relation exists. Moreover, no rank based correlation exists according to kendalls τ with a value of -0.039. Thus, the German MTPL data can be considered to be uncorrelated.

Summing up, it can be stated for the German MTPL data that the observed incurred and payment ratio values are dependent and that both variables are uncorrelated or just correlated weakly with a correlation coefficient around zero. Thus, no impact on the cluster analysis is expected here. This is fulfilled for all test samples used in Chapter 5 that have been analysed in this thesis.

Assumption 3.7
The incurred values and payment ratio are uncorrelated or just correlated weakly.

3.5.3. Cluster Components

As mentioned in Section 3.5.1, the three main steps of the clustering are the determination of the optimal number of clusters by considering cluster validity indices, the clustering itself, and the verification of the cluster quality. Thus, setting up the clustering of the dataset involves several steps that are also related to each other as shown in Figure 3.4.

From a computational point of view, the dataset is clustered for a pre-defined number of clusters by a cluster algorithm, which will be described in Section 3.5.6. The cluster

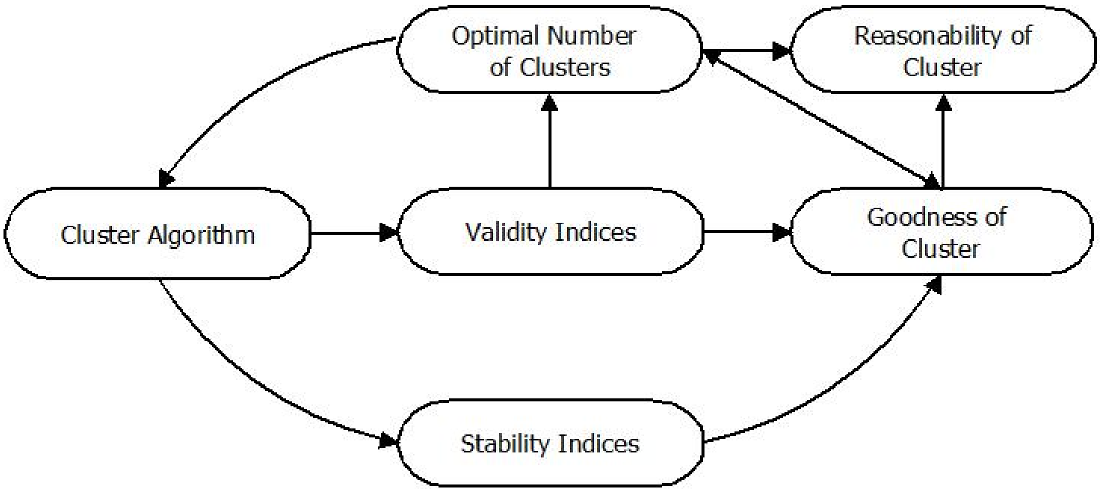


Figure 3.4.: Relation Between the Main Cluster Components.

results can then be used to estimate several validity indices giving feedback about the goodness of the cluster results. By doing this sequentially for a different number of clusters, it automatically results in the optimal number of clusters chosen according to the best cluster fit, see Section 3.5.5. Since this forms a loop depending on the previous results the cluster validity indices will be discussed first, followed by the optimal number of clusters and the cluster algorithm. Additionally, stability indices that are discussed in Section 3.5.7 can be applied, which also gives an indication on the goodness of the clustering. If it is possible to get a good clustering on the dataset and the validity indices state that there is a reasonable cluster, this indicates the reasonability of the clustering. Hence, the reasonability of clusters is defined by the optimal number of clusters and the goodness of the clustering which are both relying on the same validity indices.

3.5.4. Cluster Validity Indices

A main roll in the clustering process is given to the cluster validity indices. Those indices combine different information about the data [44] in order to find the optimal number of clusters and to evaluate the goodness of the cluster results:

- intracluster compactness and intracluster isolation,
- geometric or statistical properties of the data,
- the number of data objects,
- dissimilarity or similarity measurements.

Therefore, cluster validity indices can be separated into three criteria: internal, external, and relative [119]. External indices measure the performance of the clustering by matching the clustering structure to a priori information, e.g. degree of correspondence between cluster numbers and a priori assigned category labels [119]. In contrast to that, internal indices assess the fit between the structure and the data by considering only the input data itself [119]. Relative indices can be used to decide which of two structures is more stable or more appropriate for the dataset [119].

Since the relative indices in form of the stability measuring is used to evaluate the goodness of the clustering in Section 3.5.7 and the external indices require an a-priory input that might be judgemental, internal indices are considered further. Those are analysing the structure of the data itself by assessing the fit between the structure and the data using only the information according to the dataset. Hereby, it has to be recalled that the optimal dataset for clustering, consisting of well separated, connected, and compact points [30], is usually not given for reinsurance data. According to the nature of the data and the chosen cluster parameters, the data is highly connected but not well separated. Thus, indices measuring the smallest distance between data points from different clusters or similar parameters will not deliver meaningful results.

Therefore, some indices as stated by Brock et al. [30] can be considered like the Dunn index [65] or the connectivity [95]. The Dunn index is used to estimate the separation of the clusters by comparing the smallest distance between clusters with the largest intercluster distance. If clusters are well separated and compact, the index is getting larger, which means that it has to be maximized. However, according to the structure of the market data, this index will most likely not deliver meaningful results. Besides that, the compactness of the clusters is assessed by the connectivity which measures the distance of the points from one cluster to all other points in different clusters and has to be minimized. Additionally, three other indices that are commonly used are the silhouette width, which can be used to assess the confidence in the cluster assignement [30, 192], the elbow method [230], and the gap statistic [232] which are both used to determine the optimal number of clusters. These last three internal indices are commonly used in practice and are briefly discussed to provide a short overview of their functionality and performance.

3.5.4.1. Silhouette Coefficient

The silhouette of a cluster can be used to determine the quality of a cluster and belongs to the internal validation indices [192]. The silhouette $s(x^g)$ is determined by calculating the silhouette of each data point x_i^g in the cluster S_g. The average silhouette width $\bar{s}(x^g)$ over all $s(x^g)$ for the different number of clusters S_g with $g \leq G$ and G as the maximal cluster number can be used as an indicator for the optimal number of clusters:

$$g_{opt} = \underset{2 \le g^* \le G}{\arg\max} \underbrace{\frac{1}{g^*} \sum_{g=1}^{g^*} \underbrace{\frac{1}{|S_g|} \sum_{x_i^g \in S_g} \frac{b(x_i^g) - a(x_i^g)}{\max\{a(x_i^g), b(x_i^g)\}}}_{\substack{\text{average silhouette width} \\ s(x^g) \text{ of cluster } g}}}_{\substack{\text{average silhouette width} \\ \bar{s}(x^{g^*}) \text{ over all clusters } g^*}},$$

with $a\left(x_i^g\right)$ as the average dissimilarity of the data point $x_i^g \in S_g$ to all other objects of its cluster S_g and $b\left(x_i^g\right)$ as the average dissimilarity of x_i^g to all objects of its nearest cluster neighbour. The average silhouette width represents the relationship between the within cluster dissimilarity and the smallest between dissimilarity to the nearest cluster. The number of clusters g^* with the highest value should be used for clustering as optimal number of clusters g_{opt}. Moreover, the average silhouette width is an indicator for the goodness of the clustering. A value of $\bar{s}(x^g)$ between 0 and 0.25 represents an unreasonable cluster, a value between 0.25 and 0.5 equals a coincidental cluster, a value between 0.5 and 0.75 depicts a reasonable cluster, and a value between 0.75 and 1 represents a highly probable cluster. Thus, a high average silhouette width above 0.5 most likely represents a cluster number for which clustering is considered to be reasonable. Taking the MTPL data of the German market as an example, the silhouette width is shown exemplarily in Figure 3.5.

The applied cluster algorithm here is the Clustering Large Applications (CLARA) algorithm which will be explained in Section 3.5.6. Hereby, the clustering was done on the ground up claims (upper left figure) and on the data after applying the reporting threshold (lower left figure). The related silhouette width plots of the clusters are shown on the right side. It can be seen that in the first case shown in the upper part of Figure 3.5, the incurred data was separated into 2 clusters. The silhouette width shows a value of 0.79 for the first cluster and 0.67 which indicates that the clustering is reasonable for this case. However, in the second case where the reporting threshold is applied, three clusters have been chosen which leads to a silhouette width of 0.12 for the second cluster. This shows that the clustering is not advised by the silhouette plot.

3.5.4.2. Elbow Method

The elbow method [230] determines the optimal number clusters g_{opt} by calculating the total within-cluster sum of square WSS_{g^*} for several possible clusters $1 \le g^* \le G$. If the euclidean distance is used, its form is shown in the following equation:

$$WSS_{g^*} = \sum_{g=1}^{g^*} \sum_{x_i^g \in S_g} \left(x_i^g - \bar{x}_g\right)^2 \qquad \text{for } g^* = 1, ..., G,$$

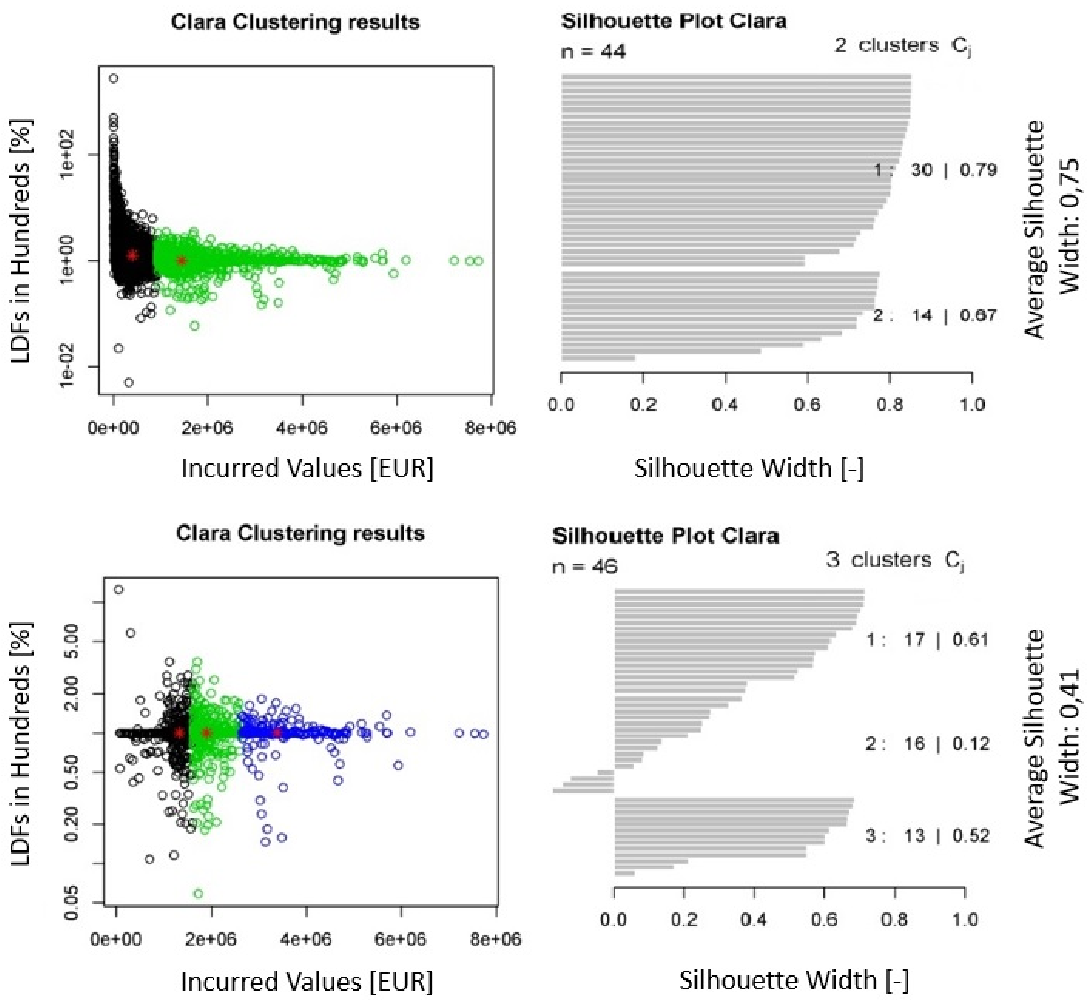


Figure 3.5.: Silhouette Plot.

where $\bar{x}_g$ denotes the mean of points x_i^g lying in cluster S_g. With an increasing number of clusters, WSS_{g^*} tends to zero. Thus, the smallest possible number of clusters with the lowest possible sum of squared error value has to be chosen. This is called the 'elbow' in the plot of WSS_{g^*} against g^*. If the data is not clustered, the within-cluster sum of square curve can be either volatile or smooth and is therefore an insufficient indicator for an appropriate number of clusters in some cases [127]. Since this is a graphical decision method, the plot of WSS_{g^*} against g^* should always be analysed and the actuary has to decide which number of clusters is appropriate.

3.5.4.3. Gap Statistic

The third method to estimate the number of clusters is the gap statistic [232], which is a commonly used approach for deciding the number of clusters [118]. After some studies of the elbow phenomenon [218], the gap statistic provides a statistical procedure to formalize the estimation of the optimal number of clusters using the elbow method described above. The idea of this approach is to compare the change in within-cluster dispersion of a clustering algorithm with the expected dispersion under an appropriate reference null distribution:

$$Gap_{\tilde{n}}(g^*) = \mathbb{E}_{\tilde{n}}\Bigg(\underbrace{log(\sum_{g=1}^{g^*} \frac{1}{2|S_g|} \sum_{x_i^g, x_j^g \in S_g} dist(x_j^g, x_i^g))}_{=W_{g^*}}\Bigg) -$$
$$-log\Bigg(\underbrace{\sum_{g=1}^{g^*} \frac{1}{2|S_g|} \sum_{x_i^g, x_j^g \in S_g} dist(x_j^g, x_i^g)}_{=W_{g^*}}\Bigg).$$

Here $\mathbb{E}_{\tilde{n}}$ denotes the expectation under the reference distribution with sample size $\tilde{n}$ and W_{g^*} denotes the sum of the average intracluster differences. The optimal cluster number according to the gap statistic is the g^* which maximises $Gap_{\tilde{n}}(g^*)$ after taking the null distribution into account. In this framework, a null model of a single component is assumed which can be rejected in favour of a g^*-component model if strong evidence is given [232]. Nevertheless, this method contains a structural bias since the W_{g^*} presupposes spherically distributed clusters which might not be fulfilled in general [137].

3.5.5. Optimal Number of Clusters

Cluster validity indices as described are usually used to determine the optimal number clusters for a dataset. Hereby, it has to be said that the unsupervised choice of the optimal number of clusters is one of the most difficult tasks in cluster analysis [118].

Among the indices described in Section 3.5.4, a total of thirty different validity indices and decision methods are used to determine the optimal number of clusters in this thesis following Charrad et al. [44]. This publication gathers several indices and a complete list of those can be found on page 19 in the respective publication [44] and is also shown in Figure A.2 in the Appendix A.1. The final choice of the optimal number of clusters is then done according to the majority rule. It needs to be mentioned that these validation indices are indicators for different structural characteristics in the dataset. Since the dataset is naturally connected, measures for the smallest distance between cluster points will not work well which leads to the question whether this is ultimately a good approach. Nevertheless, it is not given that the dataset is highly connected for all markets and it has to be kept in mind that these indices are helpful when dealing with extreme values in the dataset. Additionally, this is an easy way to determine the optimal number of clusters automatically. Even if these indices are commonly used in practice, the actuary has to be aware of the fact that the indices are only an indicator for the appropriate number of clusters and can also lead to unreasonable cluster results.

In the case of the three indices described in Section 3.5.4, a possible visualisation for the optimal number of clusters is given in Figure 3.6. based on the same dataset as shown in Figure 3.5. on page 64.

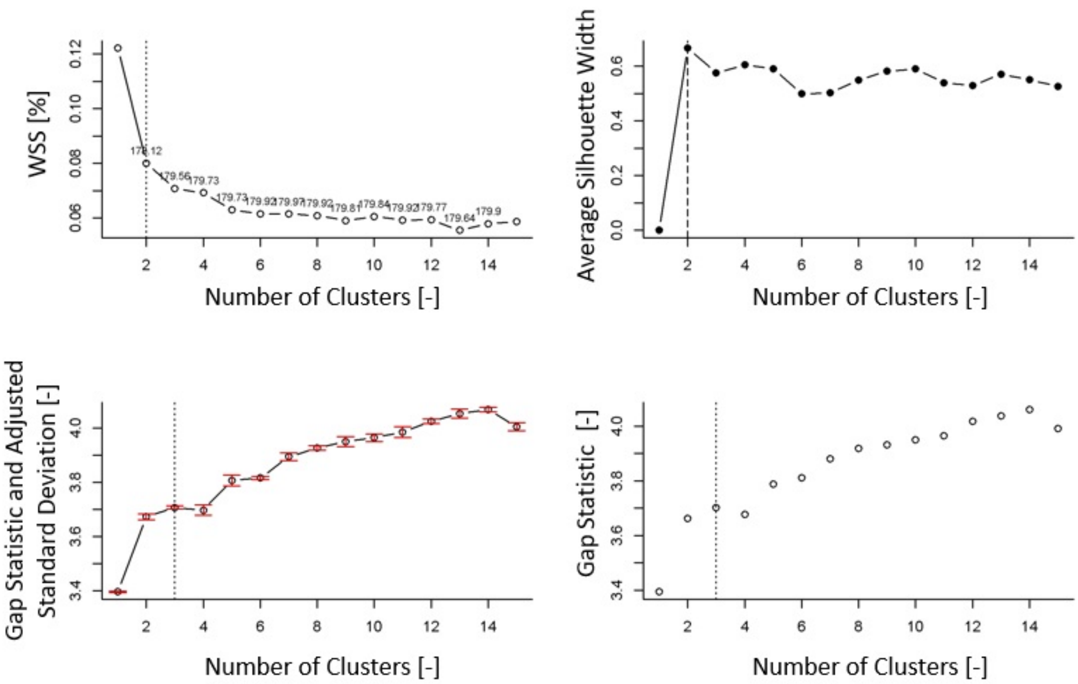


Figure 3.6.: Validity Indices for the Optimal Number of Clusters.

The values of the elbow method from Section 3.5.4.2 are shown in the upper left corner where the ‘elbow’ in the plot is given for 2 clusters. This is also supported by the average silhouette width from Section 3.5.4.1 in the upper right corner. However, this has to be evaluated with caution since it might also be that there is a cluster with a small silhouette coefficient. The information of the gap statistic from Section 3.5.4.3 is stated on the bottom, including the standard deviation in form of the whiskers on the left side and without on the right side. For this dataset, the gap statistic indicates three clusters while the other statistics recommend 2 clusters. Hence, it is not clear which should be the preferred one. Thus, more indices are evaluated to make a reasonable choice for the optimal number of clusters.

3.5.6. Cluster Algorithm

The clustering of data is done using a partitioning cluster algorithm as stated in Section 3.5.1. Therefore, mainly four types of algorithms can be considered here, which are the k-means algorithm, the k-medoid or Partition Around Medoids (PAM) algorithm, the CLARA algorithm, and the Clustering Large Applications Based on Randomized Search (CLARANS) algorithm [220]. While these algorithms have different advantages and disadvantages, one has to be chosen with respect to the required task of clustering the market dataset. Hereby, the data characteristics as well as their complexity, efficiency, and several other parameters [220] are used for a selection and are shown in Table 3.1.

Parameters	**k-means**	**k-medoids (PAM)**	**CLARA**	**CLARANS**
Complexity:	$O(ikn)$	$O\left(ik(n-k)^2\right)$	$O\left(ks^2+k(n-k)\right)$	$O\left(n^2\right)$
Efficiency:	Comparatively more	Comparatively less	Comparatively more	Comparatively more
Implementation:	Easy	Complicated	Complicated	Complicated
Sensitivity to Outliers:	Yes	No	No	No
Predefined No. of clusters k:	Required	Required	Required	Required
Optimized for:	Separated clusters	Separated clusters, small dataset	Separated clusters, large dataset	Separated clusters, large dataset

Table 3.1.: Comparison of Partitioning Cluster Algorithms with a Drawn Sample Size s, Number of Clusters k, Number of Objects n, and Number of Iterations i Based on Pandya and Saket [220].

One of the most known and used partitional cluster method [98, 156] is the k-means algorithm which has been extended in many different ways [109, 118]. Since it counts as an unsupervised learning algorithm [257] this means that no pre-defined target value and reward function exists that guides the output of the cluster algorithm. The algorithm itself includes the following steps:

Initialization: Choose g^* random cluster centres $CC_1^t,...,CC_{g^*}^t$ from the dataset starting with $t = 1$.

Assignment: Each data point x_i, $i = 1,...,\tilde{n}$ is assigned to its nearest cluster centre $CC_1^t,...,CC_{g^*}^t$.

Update: Recalculate the cluster centres $CC_1^{t+1},...,CC_{g^*}^{t+1}$ by taking the mean of all points x_i^g, $g = 1,...,g^*$ allocated to the respective cluster centre CC_g^t.

The steps Assignment and Update are repeated until the algorithm converges, which means that the assignments no longer change. Hereby, it is not guaranteed that the algorithm converges to the optimal solution. The k-means algorithm requires hyper ellipsoids[4], is sensitive with respect to the random starting points, and is highly affected by outliers in the dataset, which is also stated in Table 3.1. The latter is a critical aspect dealing with reinsurance data and large claims that can occur. Therefore, an extension of the k-means, the PAM algorithm [125, 126], is considered. It minimizes the sum of pointwise dissimilarities and is more robust regarding outliers. However, this algorithm is not designed to deal with large datasets in respect of memory requirements and runtime of the cluster algorithm. Therefore, an extension of the PAM algorithm,

[4]An ellipsoid of a higher dimension.

the CLARA algorithm can be applied [126]. The clustering using the CLARA algorithm is divided into two steps. Firstly, a sample is drawn from the set of available data and is clustered using the PAM algorithm. Afterwards, each other object not belonging to the sample is assigned to its nearest representative object. The average distance between each data point and its representative object provides a measure for the cluster quality. This procedure is repeated multiple times and the sample with the lowest average dissimilarity of the clustering is chosen as result. Since the CLARANS algorithm is not as efficient for larger datasets as the CLARA algorithm and has a higher complexity [220], it is not considered for this thesis.

Ultimately, the CLARA algorithm is chosen as best candidate here since it can deal with larger datasets and is not so prone to outliers that might occur in the market data which can also be seen in Table 3.1. Thus, the cluster results based on the CLARA algorithm are considered for the validity indices to estimate the optimal number of clusters in Section 3.5.5. Nevertheless and as mentioned in Section 3.5.1, a softer cluster algorithm might be possible in this case as well.

3.5.7. Stability Indices and Goodness of the Clustering

Besides the validity indices in Section 3.5.4, separate stability indices and extensions of internal cluster validity indices can also be used to determine the goodness of a cluster result. Hereby, a comparison of the cluster results based on the full dataset and a reduced dataset by sequentially removing columns, e.g. cluster parameters, is done. Thus, they measure the stability of the clustering with regard to a reduction of the information in terms of the reduction of dimensions. According to Brock et al. [30], those indices function well when the data is highly correlated, which is not necessarily given for the market data used.

However, some indices that can be used for this are the average proportion of non-overlap (APN) measuring the average proportion of observations not placed in the same cluster. Additionally, the average distance (AD) measuring the average distance between observations placed in the same cluster [30] can be applied. Additionally, the average distance between means (ADM) measuring the distance between the cluster centres and the figure of merit (FOM) measuring the intracluster variance can be calculated [30, 55, 268]. While the validity of these indices is limited due to the already small number of dimensions in the market dataset, another method can also be applied.

Hereby, the clusterwise stability of the cluster results can be measured by calculating the Jaccard similarities. Computational, this can be done by using the 'Clusterboot' method [101], which allows to compute the Jaccard similarities of the original clusters to the most similar clusters in a re-sampled dataset several times. The index of the cluster-wise stability is then given by the mean over these similarities. A value which is close to 1 indicates that the cluster results are stable. This can also be done by adding noise data in order to disturb the cluster algorithm. Although this method provides a reliable index for the cluster-wise stability, it has to be noted that clusters are sometimes only stable because of the inflexibility of the used cluster method [95, 101]. Since the CLARA algorithm samples data multiple times and uses random starting points for the PAM algorithm, the inflexibility is not expected to be problematic for a market dataset.

3.5.8. Reasonability of Clustering

Cluster algorithms find clusters in the data irrespectively of whether or not clustering is advisable [118]. Hence, validity and stability indices as well as logical assessment are the only way to judge the reasonability of a cluster result. Conclusively, the internal validity indices leading to the optimal number of clusters and the stability indices together with the Jaccard similarities can be computed for the clustered market dataset. Hereby, the mean, median, standard deviation, minimum and maximum values are stated as statistical measures to verify the respective results. This is utilized exemplarily in the next section.

If a reduced dataset as stated in Section 3.5.2.1 is used, the mean of the cluster centre vectors over all subsets and simulations is determined and used further for the evaluation.

3.5.9. Impact of Reporting Threshold and Choice of Cluster Variables

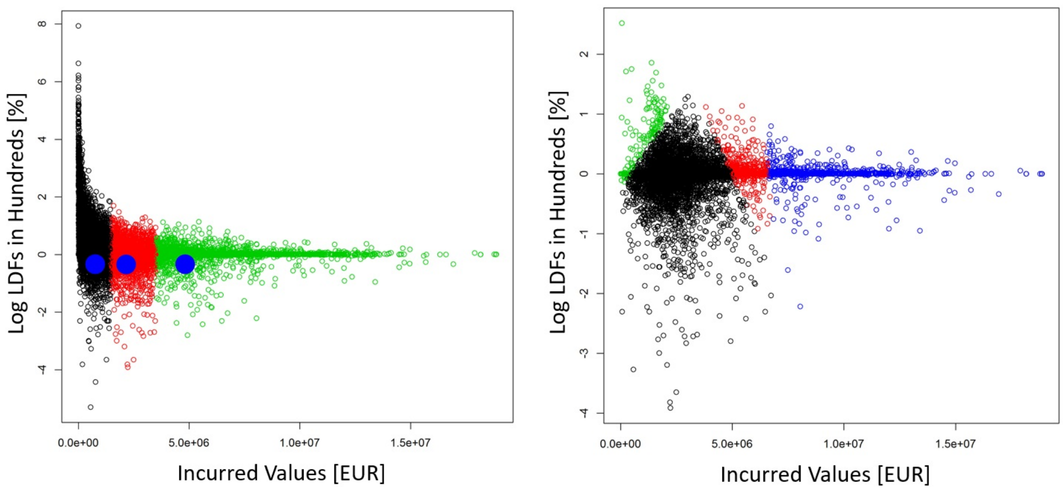
It was stated in Section 3.4.6 that the clustering could be applied on various market datasets due to the choice of the reporting threshold and the set of cluster variables coming from the incurred values and payment ratios in contrast to the LDFs. This is of interest since the reporting threshold of different cedants is setting the threshold for which the claims are reported from ground up and is responsible for the amount of data that is available. In respect of the clustering, this might change the form of the point cloud that is used for the clustering and might lead to different cluster results.

In the case of the three chosen parameters, incurred value, payment ratio and LDF, it can be investigated how the cluster results differ for the resulting point clouds. This is done exemplarily for MTPL claims as described in Section 3.3 of the German market in the following subsections.

Based on the results of Chapter 2, an indexation is applied to the dataset and the reporting threshold. For simplicity reasons, a realistic fixed reporting threshold of EUR 1 million is chosen. In order to test the performance, the data reduction from Section 3.5.2.1 is applied to create 50 reduced datasets and the resulting average cluster centre is considered. The internal cluster validity indices, the stability indices, the optimal number of clusters, as well as the Jaccard similarities for the chosen number of clusters are summarized in tables, shown in the Appendix A.2. Overall, six different cluster results are compared to decide on a new triangle-free framework.

3.5.9.1. Incurred Values and Loss Development Factors

The cluster results based on the incurred values and LDFs as cluster parameters are shown in Table A.1 in the Appendix A.2. The connectivity and silhouette width state a reasonable cluster in both cases while the Dunn index is zero, rejecting the clustering. This was already discussed and is driven by the high connectivity of the point cloud. Following the internal validity indices over the 50 reduced data samples, three clusters are recommended in 25 of them, which is not so pronounced for the datasets where the reporting threshold is applied. Here, it varies between three and four clusters. The resulting clustered market data is shown in Figure 3.7.



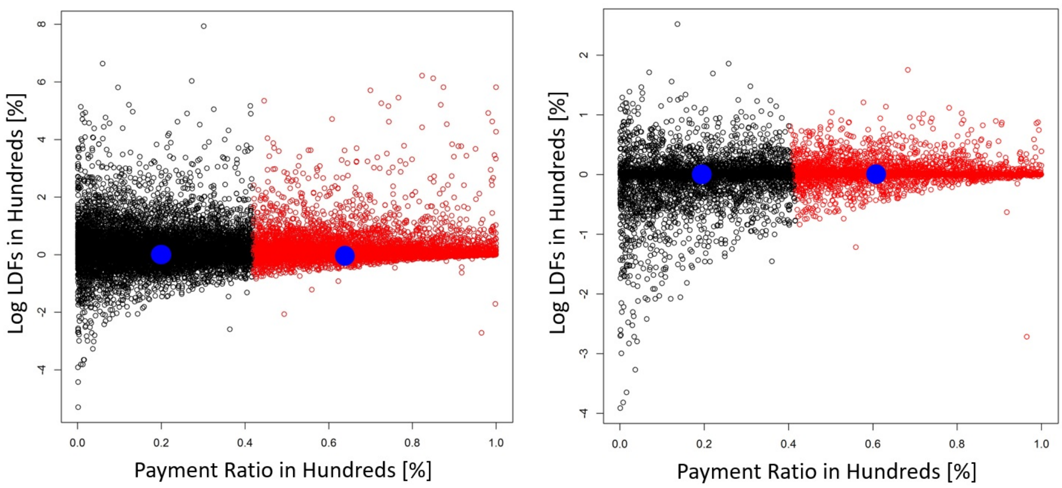
(a) Ground up Losses. (b) Losses Exceeding Reporting Threshold.

Figure 3.7.: Clustered Dataset Using the Incurred Values and Loss Development Factors.

The blue points represent the cluster centres based on the average of the 50 splits and the cluster sequence is 1 black, 2 red, 3 green, and 4 blue. The cluster results for Figure 3.7a are reasonable and divide the dataset according to the different volatilities driven by the LDFs. Furthermore, the Jaccard similarities as well as the stability measures indicate a stable result which is achieved over the simulations. In comparison to that, the cluster centres in Subfigure 3.7b are missing and the cluster result is less reasonable. This is also implied by the stability measures which are not as good as for the clustering considering the reporting threshold. However, as already indicated by the optimal number of clusters, the cluster results for the different simulations vary a lot and depend on the initial cluster means. Consequently, this is an example for an unsupervised clustering failing and leading to insufficient results. Hence, a clustering based on the ground up dataset is preferred in the case of the incurred values.

3.5.9.2. Payment Ratio and Loss Development Factors

The related results considering the payment ratio and LDFs as cluster parameters are shown in Table A.2 in Appendix A.2. The cluster validity indices show a similar behaviour as in the previous analysis. Nevertheless, the connectivity index has a small value of 4.42 for the median compared to 14.95 for the mean and 31.86 for the standard deviation. Due to the random starting points of the cluster centres and the shape of the dataset, some simulations show an unreasonable clustering, which leads to in increased volatility due to points lying not close to 0 on a logarithmic scale. Since those points are usually below the reporting threshold, the validity indices are more stable there. This also explains the Dunn index value, which indicates a highly connected dataset. While the average silhouette width indicates a strong cluster considering the ground up claims, this shifts to coincidental or reasonable clusters if the reporting threshold is considered. Due to the larger volatility in the dataset considering the ground up losses, the number of optimal clusters varies between two and three while this is clearly set to two if the reporting threshold is applied. In Figure 3.8., the complete market dataset is assigned to the determined average cluster centres.



(a) Ground up Losses.

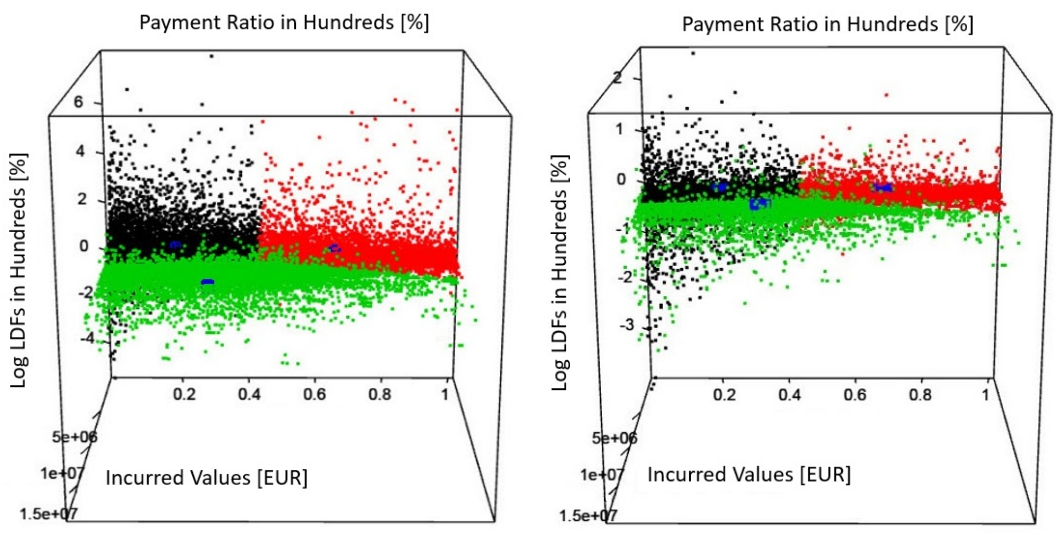
(b) Losses Exceeding Reporting Threshold.

Figure 3.8.: Clustered Dataset Using the Payment Ratio and Loss Development Factors.

The blue points represent the cluster centres and the cluster sequence is 1 black and 2 red. Hereby, the clustering is reasonable for both cases dividing the payment ratio around 0.42 according to the volatility of the LDFs. This behaviour is more pronounced in Figure 3.8b since small claims with a large payment ratio are excluded here. This is also supported by the Jaccard similarities which are close to 1 for the second case. Here it can be seen that the ground up data is prone when noise data is added since the clusters are less clear and pronounced. This lack of stability is also observed comparing the stability indices, which are much smaller for the second case, indicating a more stable and clearer cluster result.

3.5.9.3. Incurred Values, Payment Ratios, and Loss Development Factors

The last parameter set considers the incurred values, payment ratios, and LDFs for the clustering. The related results are stated in Table A.3 in Appendix A.2. The connectivity, silhouette width, and Dunn index show similar results as stated in Section 3.5.9.1 for only using the incurred values and LDFs. While the optimal number of clusters is clearly set to 3 when the reporting threshold is applied, it is less pronounced for the ground up losses. This is driven by the influence of the payment ratio analysed in the previous section, leading to a clearer statement when the reporting threshold is applied while making it less clear for the ground up dataset. The resulting clustered dataset is shown in Figure 3.9.



(a) Ground up Losses. (b) Losses Exceeding Reporting Threshold.

Figure 3.9.: Clustered Dataset Using the Incurred Values, Payment Ratio, and Loss Development Factors.

The cluster centres are stated with a blue point and the cluster sequence is 1 black, 2 red, and 3 green. Comparing Figure 3.9a and Figure 3.9b, the dataset is divided into three clusters with similar shapes. Claims above a specific incurred value are not divided regarding the payment ratio while claims below this threshold are partitioned into two sets according to the payment ratio. The regarded threshold for the payment ratio is around 0.42, which is similar to the results from Section 3.5.9.2. The stability indices indicate that the clustering is more stable compared to the usage of just the incurred values as they are around 20% smaller in the average. This is due to the fact that the used payment ratio stabilises the cluster results for both approaches. According to the Jaccard similarities, the cluster results are stable against the randomness of different cluster starting points. Additionally, it can be seen that the smaller incurred claims have a stabilising affect on the clustered ground up data.

3.5.9.4. Cluster Results and Final Parameter Set

Overall, it is observed that the number of clusters varies when the ground up losses for the incurred values are considered and are more pronounced if the payment ratio is used. This is shown in the choice of the optimal number of clusters as well as the stability indices and Jaccard similarities. Hence, the clustering on the ground up data is recommended if the incurred values are considered and the data exceeding the reporting threshold is recommended if the payment ratio is used. Overall, this comes down to the choice which parameter adds better information. Considering both parameters, the cluster results are less influenced by the reporting threshold. However, the stability indices show a better result for the ground up losses while the internal validity indices are not so clear about the number of clusters. The opposite behaviour can be observed when the reporting threshold is applied. Then the number of clusters is clearly set to 3 while the stability measures are worse. Overall, the cluster result for using all parameters is considered to be more favourable than only applying one of these cluster parameters. Hence, the point cloud based on all parameters, the incurred values, the payment ratios, and the LDFs, forms the triangle-free framework for the SLD model.

Since smaller claims are more prone to the idiosyncrasies of primary insurance companies, the combination of incurred values, payment ratio, and LDFs $\left(Inc_{i,j,l}, PR_{i,j,l}, LDF_{i,j,l}^{Inc,mul}\right)$ for $LDF_{i,j,l}^{Inc,mul} \neq Na,\ i = 1,\ldots,n,\ j = 1,\ldots,k_i,\ l = 1,\ldots,n-1$ with applied reporting threshold is suggested to be used. For simplicity reasons, the point defining the point cloud are further written as $\left(Inc_i^{PC}, PR_i^{PC}, LDF_i^{PC}\right)$ for $i = 1,\ldots,\tilde{n}$. In this case, the clustering is more stable and leads to realistic results while avoiding uncertainty due to idiosyncrasies of primary insurance companies. Furthermore, more information about the claims is considered in this case and the cluster results are also in line with the expectations stated in Section 3.4.

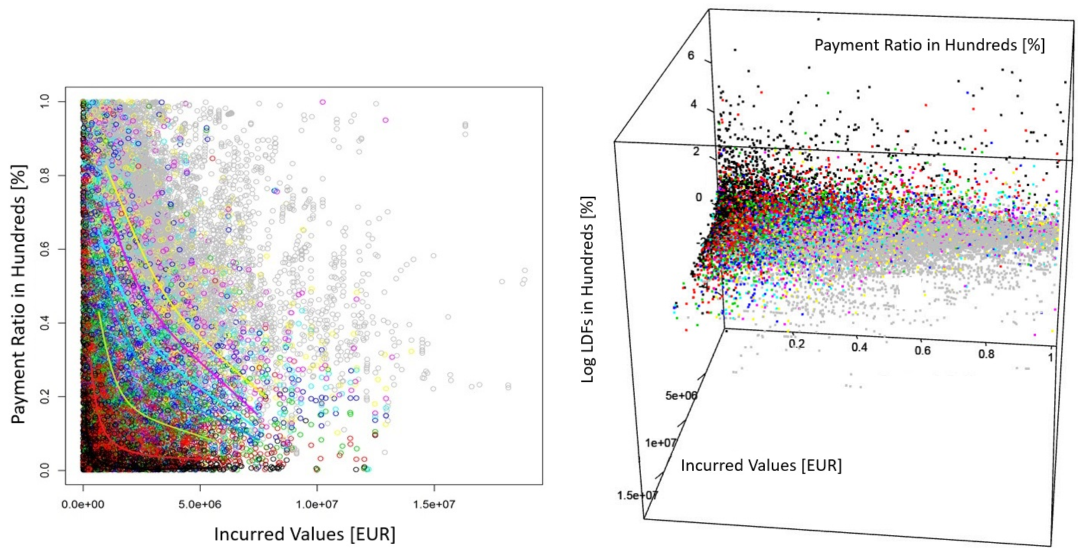
While this cluster result only holds for the German MTPL market so far, it has to be analysed for other MTPL markets as well. Hereby, the cluster results for all other MTPL markets analysed in Section 6 are similar to the cluster results of the German MTPL market since the shape of the point clouds is also similar. Only the number of points differs and a stretching or compressing of the data related to the incurred values or LDFs occur. These similarities for the point clouds can be explained by the great market penetration of MTPL business, a similar occurrence of accidents and injuries across countries, together with a basic standard of compensation given by the motor insurance directive of the EU [71]. Hence, this triangle-free framework following the point cloud is considered to work for other European MTPL markets as well.

3.5.10. Remarks on Cluster Results

Since the payment ratio is considered further, the assumption 3.4 of monotonously increasing paid values results in a lower boundary for the LDFs. This can be obtained by closing a claim and setting the reserve $Out_{i+1} = 0$ to zero as shown in Equation 3.1. Hereby, a short form of the paid values, outstanding values, and payment ratio is used.

$$LDF_i^{min} = \frac{Inc_{i+1}}{Inc_i} \overset{Paid_{i+1}=Paid_i}{\geq} \frac{Paid_i + Out_{i+1}}{Paid_i + Out_i} = PR_i + \frac{Out_{i+1}}{Inc_i} \overset{Out_{i+1}=0}{\geq} PR_i. \qquad (3.1)$$

A second point can be observed when the respective development years are labelled in the clustered dataset which is shown in Figure 3.10.



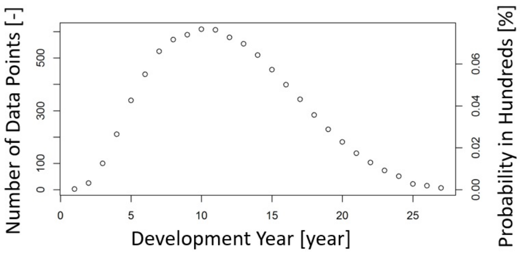
(a) Incurred Values vs. Payment Ratio. Ties of Different Development Years are Marked with a Coloured Line.

(b) 3D Version of Figure 3.10a Rotated by 90 Degrees Including the Logarithmic LDFs.

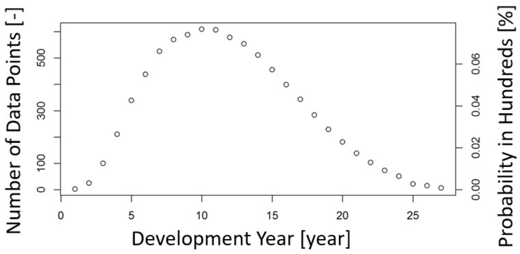
Figure 3.10.: Overlapping Multiple Shell Structure for the Labelled Development Years. Increasing Development Years Starting with the First Year for the Colours: Black, Red, Green, Blue, Cyan, Magenta, Yellow, and Rest is Grey.

The different development years form an overlapping multiple shell structure which can be seen by the coloured lines in Figure 3.10a. Hereby, each shell represents the

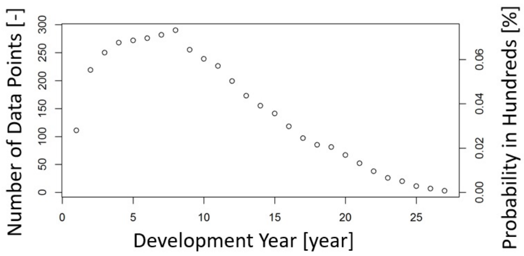
LDFs for a specific development year. Due to the respective volatility of those LDFs, they vary around the mean creating the overlapping multiple shell structure. Additionally, the assumption 3.6 that the time component is indirectly considered using this set of data is supported when analysing the composition of a cluster with respect to the different development years in Figure 3.11.



(a) First Cluster (Black) in Figure 3.9.



(b) Second Cluster (Red) in Figure 3.9.



(c) Third Cluster (Green) in Figure 3.9.

Figure 3.11.: Density of the Development Years for Different Clusters.

Here, the number of data points per development year and their density divided for the three clusters based on the data of Figure 3.9 is shown. The first cluster containing claims with a smaller incurred value and payment ratio highly consists of claims from early development years as shown in Figure 3.11a. The maximum number of points is reached in the third development year and decreases afterwards. This behaviour can be anticipated since claims in their early stages of development are volatile and a future prediction is difficult. Claims with a smaller incurred value and higher payment ratio are located in the second cluster shown in Figure 3.11b. Since the volatility of claims in the second cluster is smaller compared to the first cluster, claims from later development years are expected to be located in this second cluster. The maximum number of data points for this cluster is related to the tenth development year. The shape of the density function is similar to a negative binomial distribution. Hence, the expectation that more claims from later development years are located in that cluster is fulfilled. The third and last cluster gathers claims with higher incurred values irrespectively of the payment ratio. These claims can appear throughout the whole development period, which is related to two main driving factors. Firstly, primary insurers want to set a high reserve for large claims in order to cope with uncertainty in the future development. The second effect is that smaller claims can be located in this cluster due to their development over the time period. Thus, the number of claims decreases almost linearly.

According to the form and position of the shell structure for each tie and the distribution of the development years in the clusters, the commonly used assumption that claims are developed according to other claims in the same development year cannot be supported. Furthermore, the results should be considered as an indicator that there are alternative solutions to the development year as the main parameter for claims' projection.

Analysing the clustered dataset in respect of different cedants, nothing specific can be observed in respect of related idiosyncrasies. This strengthens the assumption 2.1 of an approximately homogeneous dataset across different companies.

The clustering was done considering the percentage change of the incurred values as LDFs. However, the absolute change could also be targeted which does not affect the clustering significantly. However, this thesis follows the classical understanding of the LDFs as stated in Section 3.4.4.

A final remark has to be made in respect of the cluster boarders that are strict for the chosen approach. Considering the market dataset and the high connectivity, a softer approach might be more favourable but automatically leads to a more complex framework on which the further modelling becomes more difficult. Thus, the strict boarders generated by the CLARA algorithm seem to be justified in a first step. However, further analysis of a softening of cluster boarders is advised after enough experience in the application of the developed model has been gathered.

Conclusively, the next step is to choose the LDFs according to the clusters by assuming that claims within the same cluster are more similar to each other than claims in the same development year.

3.6. Summary

The required features for a new SLD model are stated and the available parameters are analysed. This leads to the idea of the basic model framework to consider the claims data as a point cloud based on the incurred values, the payment ratio, and the corresponding LDF. Therefore, the cluster procedure is described briefly and applied on various datasets. Ultimately, the dataset consisting of the incurred values, payment ratios, and LDFs with applied reporting threshold is chosen. This does not include the development years as an own component explicitly, which is usually the main idea behind pricing and reserving models. Instead, the time component is considered implicitly in the cluster parameters and the resulted clusters. Finally, this leads to the point that claims are considered to be similar according to their related cluster which is used further.

4. Copula-Based Single Loss Development Model

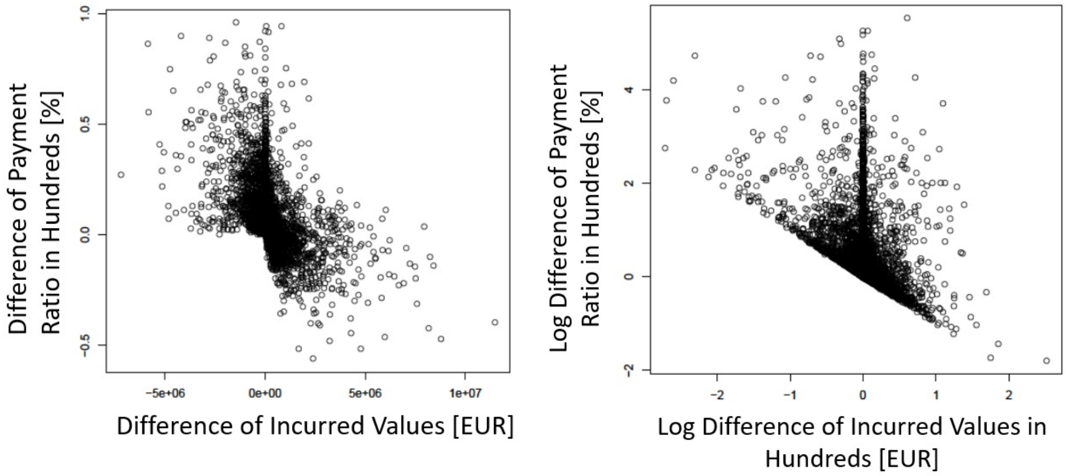
Based on the cluster results from the previous section and the observation that the incurred values and payment ratios are somehow including the time component of the development year, the next step is to develop claims within this clustered point cloud. Here, the main idea is to assume that the development of claims lying in the same cluster is similar.

Assumption 4.1
The development of claims lying in the same cluster is similar.

The main parameters for the development of a claim itself is then described by the incurred value and the related payment ratio. This leads to the idea that not only the incurred value is developed but also the changes of the payment ratio. Both parameters are uncorrelated and dependent as stated in Section 3.5.2.3. However, this must not be true for the changes of both parameters. These changes can be estimated in two different ways. Either by following an additive approach or a multiplicative approach as shown in Section 3.4.4. Hereby, no underlying distribution is assumed. For both approaches with $i = 1, ..., n$, $j = 1, ..., k_i$, and $l = 1, ..., n-1$ it holds:

$$\begin{aligned}
LDF_{i,j,l}^{Inc,add} &= Inc_{i,j,l+1} - Inc_{i,j,l} \qquad & LDF_{i,j,l}^{Inc,mul} &= \begin{cases} \frac{Inc_{i,j,l+1}}{Inc_{i,j,l}}, & Inc_{i,j,l} \neq 0 \\ Na & \text{, else,} \end{cases} \\
LDF_{i,j,l}^{PR,add} &= PR_{i,j,l+1} - PR_{i,j,l} \qquad & LDF_{i,j,l}^{PR,mul} &= \begin{cases} \frac{PR_{i,j,l+1}}{PR_{i,j,l}}, & PR_{i,j,l} \neq 0 \\ Na & \text{, else.} \end{cases}
\end{aligned} \tag{4.1}$$

Comparing the changes of the incurred and payment ratio values, while keeping in mind that the payment ratio is the quotient of the paid and incurred values, it is clear that some sort of dependence has to exist since a change of the incurred value automatically affects the payment ratio. For real claims, primary insurance companies sequentially pay the compensation to the claimants but might increase their case reserves as an update by a large amount which results in a jump of the incurred value in line with a decrease of the payment ratio. Additionally, a settlement of a claim by a lump sum payment will immediately set the payment ratio to 1 while the incurred value may differ from the latest incurred value stated. This related dependence structure for the additive and multiplicative changes are shown exemplarily for MTPL claims of the German market in Figure 4.1 without consideration of the clustering.



(a) Additive Approach. (b) Multiplicative Approach.

Figure 4.1.: Development Steps for the German MTPL Market.

For both approaches, it can be seen that there is some sort of correlation between the changes of the incurred values and the payment ratio. By applying correlation coefficients, this is more pronounced for the additive approach in Figure 4.1a. with a pearsons correlation coefficient of $r = -0.37$, kendalls tau $\tau = -0.26$, and spearmans rho $\rho = -0.36$. This implies a small negative correlation, which is less pronounced for the multiplicative approach in Figure 4.1b. with $r = 0$, $\tau = -0.15$, and $\rho = -0.2$. Furthermore, some pattern can be observed in the changes of the claim development steps which can be seen more easily for the additive approach in Figure 4.1a.:

Case 1: The incurred value increases while the payment ratio value decreases. In those cases, the primary insurer has to adjust the outstanding value for the claim which usually leads to a reduction of the payment ratio and a negative run-off for the claim. This part can be seen in the lower right corner of Figure 4.1a. and is also related to the two examples stated above with an increase in the case reserve or a lump sum payment with an increased incurred value.

Case 2: The incurred value and payment ratio increases. In those cases, the outstanding value of the claim increases while also a larger amount is paid to the insured. This usually leads to a negative run-off as well and is located in the upper right corner of Figure 4.1a. However, the changes are less pronounced in comparison with the other developments.

Case 3: The incurred value decreases while the payment ratio increases. Consequently, the reserve for a claim is reduced. In combination with a stable increased paid amount, this results in a positive run-off for the claim. Usually, lump sum payments to settle claims also fall under this case. Those developments are shown in the upper left corner in Figure 4.1a.

Case 4: The incurred value and payment ratio decreases, which is only possible due to a decreased paid value. Reasons for this can be refunds as a result of a court decision or insurance frauds. Cases like this are not so common for reinsurance claims and due to the assumption 3.4 of monotonously increasing paid values, this case is excluded for the following analysis resulting in the empty space in the bottom left corner of Figure 4.1a.

Case 5: The incurred value is stable while the payment ratio increases. Hereby, the insurer pays an amount while reducing the reserve by the same amount. This is observable as straight vertical line starting at the zero point going upwards in Figure 4.1a.

Assumption 3.4 is also responsible for the straight border towards the bottom left corner for the multiplicative approach seen in Subfigure 4.1b. Since the different cases observable in the changes of the incurred values and payment ratios imply a certain dependence structure, utilizing this dependence structure for the future development steps is promising. Hence, copulas are considered further for this purpose.

4.1. Copula - Development Step

4.1.1. Theoretical and Empirical Copulas

The concept of a copula is based on the idea of splitting a multivariate distribution into multiple parts. Some describing the marginal distributions of each single random variable and one part describing the dependence structure between the marginal distributions [166, 206]. Thus, a copula is a multivariate probability distribution for which the marginals of each variable are distributed uniformly. It provides a relation between a multivariate distribution and their related one-dimensional marginal distributions by describing the dependence structure between those random variables [206]. Moreover, a copula has many useful properties like uniform continuity and the existence of all partial derivatives almost surely (a.s.) [166, 176]. Following Nelson [166], a two-dimensional copula is a function $\mathbf{C}: [0,1]^2 \rightarrow [0,1]$ with the following properties:

- $\mathbf{C}(y_1,y_2)$ is grounded, meaning that $\mathbf{C}(y_1,0) = 0 = \mathbf{C}(0,y_2)$.
- $\mathbf{C}(y_1,y_2)$ has margins, meaning that $\mathbf{C}(y_1,1) = y_1$ and $\mathbf{C}(1,y_2) = y_2$.
- $\mathbf{C}$ is 2-increasing, so that for every y_1,y_2,y_3,y_4 in $[0,1]$ with $y_1 \leq y_3$ and $y_2 \leq y_4$ it holds that: $C(y_3,y_4) - C(y_3,y_2) - C(y_1,y_4) + C(y_1,y_2) \geq 0$.

Overall, this definition applies to a wide group of functions and is equivalent to the following more intuitive and widely used definition [69] that is used in combination with distribution functions:

Definition 4.2
$\boldsymbol{C}(y_1,y_2) = \boldsymbol{C}(\boldsymbol{F}_1(y_1),\boldsymbol{F}_2(y_2)) = \mathbb{P}(\boldsymbol{F}_1 \leq y_1,\boldsymbol{F}_2 \leq y_2)$, *with the copula function* $\boldsymbol{C}()$, *a random vector* $(\boldsymbol{F}_1,\boldsymbol{F}_2)$ *with* $\boldsymbol{F}_1,\boldsymbol{F}_2 \sim U(0,1)$ *and realisations of the random vector* $(y_1,y_2) \in [0,1]^2$.

The fundamental theorem of Sklar [206] states that every multivariate distribution function can be expressed in terms of its marginal distributions. In the two-dimensional case this can be stated as follows according to Nelsen [166]:

Sklar's Theorem
Let $\mathbf{H} : \overline{\mathbb{R}}^2 \to [0,1]$ be a two-dimensional distribution function with one-dimensional marginal distributions $\mathbf{F}_1,\mathbf{F}_2 : \overline{\mathbb{R}} \to [0,1]$ then a two-dimensional copula $\mathbf{C}$ exists so that for all $(y_1,y_2) \in \overline{\mathbb{R}}^2$ the following holds:

$$\mathbf{H}(y_1,y_2) = \mathbb{P}(\mathbf{F}_1 \leq y_1,\mathbf{F}_2 \leq y_2) = \mathbf{C}(\mathbf{F}_1(y_1),\mathbf{F}_2(y_2)).$$

If $\mathbf{F}_1,\mathbf{F}_2$ are continuous, then the copula $\mathbf{C}$ is uniquely defined. Otherwise, $\mathbf{C}$ is uniquely determined on Cartesian product of the ranges of the marginal cumulative distribution functions $Ran(\mathbf{F}_1) \times Ran(\mathbf{F}_2)$. Conversely, if $\mathbf{C}$ is a copula and $\mathbf{F}_1,\mathbf{F}_2$ are marginal distribution functions, then the function $\mathbf{H}$ is a joint distribution function.

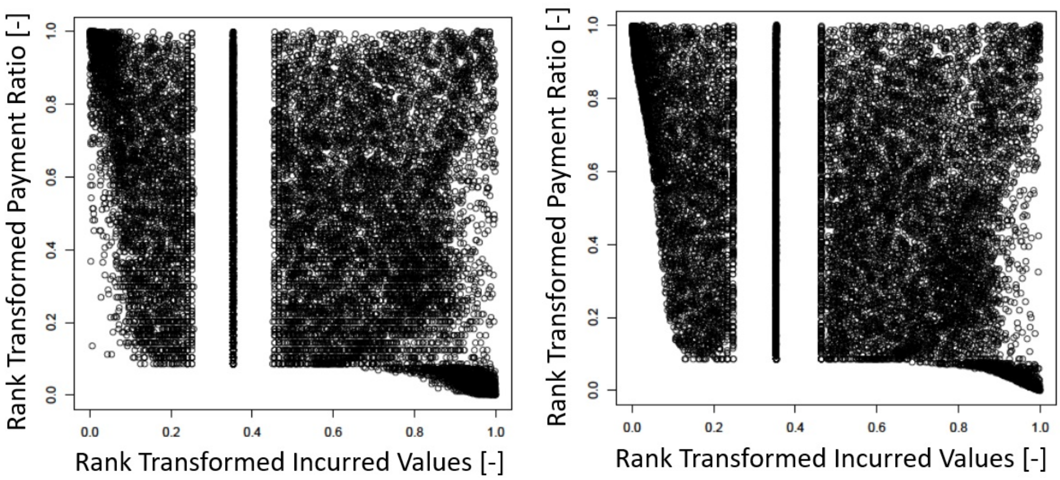
Thus, a copula function $\mathbf{C}$ can be used to model the dependence structure of the incremental changes for the incurred values and payment ratio. Since the dependence structure for this copula function is not clear, the empirical copula build on the available data is analysed. Therefore, the observations of the incremental changes for the incurred values and the payment ratios are rewritten for simplicity reasons to $(LDF^{Inc}_{i,j,l}, LDF^{PR}_{i,j,l}) =: (X^i_1, X^i_2)$ with $i = 1,\dots,N$ with $N = |\{(LDF^{Inc}_{i,j,l},\ LDF^{PR}_{i,j,l}) : LDF^{Inc}_{i,j,l} \neq Na,\ LDF^{PR}_{i,j,l} \neq Na,\ i = 1,\dots,n,\ j = 1,\dots,k_i,\ l = 1,\dots,n-1\}| \in \mathbb{N}$ stating the number of points that are available to build the copula on. The empirical copula is then given by:

$$C^{emp}_N(y_1,y_2) = \frac{1}{N}\sum_{i=1}^{N} \mathbb{1}_{\left(\hat{\mathbf{F}}^i_1 \leq y_1, \hat{\mathbf{F}}^i_2 \leq y_2\right)},$$

with definded pseudo observations based on the rank transformed data,

$$\left(\hat{\mathbf{F}}^i_1, \hat{\mathbf{F}}^i_2\right) = \left(\mathbf{F}^N_1(X^i_1), \mathbf{F}^N_2(X^i_2)\right) = \left(\frac{1}{N}\sum_{j=1}^{N} \mathbb{1}_{\left(X^j_1 \leq X^i_1\right)}, \frac{1}{N}\sum_{j=1}^{N} \mathbb{1}_{\left(X^j_2 \leq X^i_2\right)}\right).$$

Hence, an empirical copula can be seen as the empirical distribution of the rank transformed data. Applying this rank transformation to the additive and multiplicative development factors from Formula 4.1 shown in Figure 4.1 leads to the following dependence structure which is usually referred to as empirical copula[1] shown in Figure 4.2.



(a) Additive Approach. (b) Multiplicative Approach.

Figure 4.2.: Rank Transformed Data for the Empirical Copula.

The scatterplot in Figure 4.2. shows the rank transformed data representing the dependence structure as basis for the bivariate empirical copula for the additive and multiplicative approach. For both, the most unusual observation is the straight line around 36% of the rank transformed incurred values, which is related to a stable incurred and a changing payment ratio value. Since the incurred increments are all 0 for those points, they all get the same rank and are shown as a line in the scatterplot. This is related to Case 5 described in Section 4. One the left of this line all development steps with a decreasing incurred value are located while to the right all points with an increasing incurred value are shown. Thus, it can be seen that for both approaches a negative run-off is more likely, around 52%, compared to a positive run-off with 25% of all developments. Besides that, there are larger groups of points in the upper left and lower right corner related to a positive or negative run-off of claims.

However, such a dependence structure is not common and cannot be modelled by classical archimedian copulas [166] like Gumbel, Clayton, or Frank copulas, nor by parametric elliptical copulas as the Gauss or Student copula. A more flexible copula is required here. Additionally, this excludes parametric copulas since they are too inflexible and not appropriate enough in order to reproduce such a dependence structure, keeping in mind that it must be applicable to other market datasets from other countries as well.

[1] Note here that the empirical copula in this case would be a mapping from $[0,1]^2 \rightarrow [0,1]$ which would be a three-dimensional plot. However, it shows the dependence structure which can be compared to the simulated outputs of known copulas. As a consequence, it is usually denoted as empirical copula.

4.1.2. Bernstein and Grid-type Copula

Since the symmetry, parameter restrictions, and the certain correlation structure of typical copulas are not appropriate, a more flexible copula is required. Therefore, the Bernstein copula and the grid-type copula as non-parametric copulas are chosen as candidates [60, 66, 135, 176, 177, 189, 194]. These copulas are chosen as candidates for several reasons [194]:

- They allow for a flexible, non-parametric and essentially non-symmetric description of dependence structures also in higher dimensions,
- they approximate any given copula arbitrarily well,
- the densities are given in an explicit form and can be easily used for Monte Carlo simulation studies [177],
- and they have lower variances than commonly used non-parametric estimators when used as empirical estimators [194].

Nevertheless, a disadvantage of non-parametric estimators is the bias-variance tradeoff. As the model increases in complexity by adding more parameters, its bias is likely to diminish. However, this increases the variance in return and vice versa. Thus, a balance between underfitting and overfitting has to be found. Additionally, the tail behaviour of the copula cannot be modelled for an asymptotic tail dependence [194]. However, applying this kind of copula is currently the only known solution to take such a dependence structure into account.

The Bernstein copula and the grid-type copula are closely related since the Bernstein copula can be seen as a smoothed variant of a grid-type copula according to Pfeifer et al. [177]. Therefore, the grid-type copula approximates the empirical copula by using indicator functions while the Bernstein copula uses continuous Bernstein polynomials. Hence, the smoothing feature of the Bernstein copula can be useful for sparse datasets since missing data points could be set off by simulating them implicitly. In comparison to that, the grid-type copula is advantageous if many data points are available. In that case, the indicator functions of the grid-type copula can approximate the empirical copula better than the Bernstein copula.

Following the notation of Pfeifer et al. [177], a Bernstein or grid-type copula can be set up by defining a two-dimensional grid with $m_1, m_2 \in \mathbb{N}$ as grid size and N to be the sample size. Then subintervals $I_{p_1,p_2} = \left(\frac{p_1}{m_1}, \frac{p_1+1}{m_1}\right] \times \left(\frac{p_2}{m_2}, \frac{p_2+1}{m_2}\right]$ for all possible choices of $(p_1, p_2) \in T_1 \times T_2$ with $T_i = \{0, 1, ... m_i - 1\}$ describe a two-dimensional grid with $m_1 \cdot m_2$ cells. Let $\mathbb{F} = (\mathbf{F}_1, \mathbf{F}_2)$ be some discrete random vector with uniform margins over T_i. Then the two-dimensional copula density induced by the random vector $\mathbb{F}$ is given by:

$$c^{\phi}(y_1,y_2) = \sum_{p_1=0}^{m_1-1}\sum_{p_2=0}^{m_2-1} \mathbb{P}\left(\bigcap_{i=1}^{2}\{\mathbf{F}_i = p_i\}\right) \cdot \prod_{i=1}^{2} m_i \phi(m_i, p_i, y_i)$$

with $(y_1,y_2) \in [0,1]^2$ and its according copula is given by:

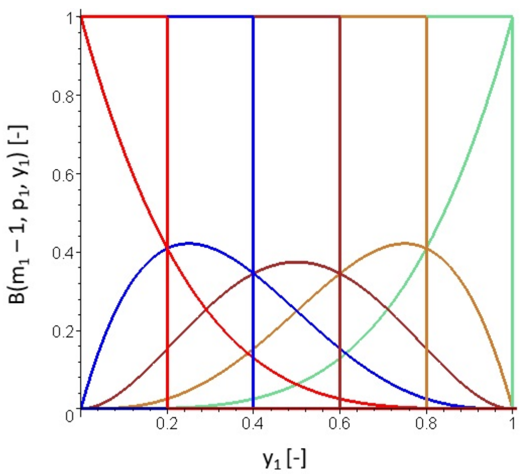
$$C^{\phi}(y_1,y_2) = \sum_{p_1=0}^{m_1-1}\sum_{p_2=0}^{m_2-1} \mathbb{P}\left(\bigcap_{i=1}^{2}\{\mathbf{F}_i \leq p_i\}\right) \cdot \prod_{i=1}^{2} \phi(m_i, p_i, y_i).$$

By applying the Bernstein polynomials or the indicator function respectively for $0 \leq p_i \leq m_i - 1$ for all $i = 1,2$, the equation for the Bernstein or grid-type copula is obtained:

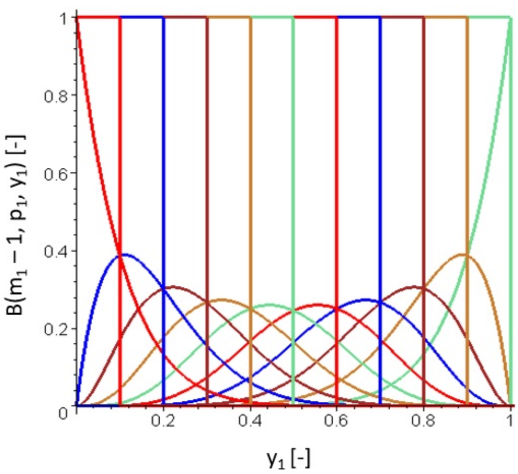
$$\text{Bernstein copula:} \quad \phi(m_i, p_i, y_i) = B(m_i-1, p_i, y_i) = \binom{m_i-1}{p_i} y_i^{p_i}(1-y_i)^{m_i-1-p_i}$$

$$\text{Grid-type copula:} \quad \phi(m_i, p_i, y_i) = \mathbb{1}_{\left(\frac{p_i}{m_i}, \frac{p_i+1}{m_i}\right]}(y_i).$$

Overall, for each grid area the related part of the Bernstein polynomials scaled with the according probability is summed up in order to give the value of the copula itself or the density for (y_1,y_2). While the grid-type copula provides an approximation of the density by applying indicator functions, the Bernstein polynomials allow for a smoother fitting of the density and transitions between the parts of the grid. This smoothing effect is shown in Figure 4.3 as an example for one dimension.



(a) Grid Size Resolution of $m_1 = 5$.



(b) Grid Size Resolution of $m_1 = 10$.

Figure 4.3.: Smoothing Effect of Bernstein Polynomials based on Pfeifer et al. [177].

The approximation improves with an increasing grid size resolution. Nevertheless, this leads to the difficulty of the bias-variance trade-off as mentioned beforehand. Since the calibration of such copulas only requires the probabilities in addition to the grid sizes m_1, m_2, these copulas are flexible and easy to apply.

4.1.3. Fitting the Copula

The procedure of fitting the Bernstein or grid-type copula to the empirical copula is done by fitting the densities which involves two major points. The first one is to choose the grid size, which is the first difficulty since an optimal value in respect to fit and degree of the Bernstein polynomials is still an unsolved problem [177]. Due to comparability, this also applies to the grid-type copula. This is also related to the trade-off between overfitting and underfitting. A grid size that is too small does not approximate the dependence structure sufficiently enough while a grid that is too large does not approximate the overall dependence structure any more. Thus, the grid sizes chosen in Diers et al. [60] with $m_1 = m_2 = 10$ and $m_1 = m_2 = 20$ are considered as sufficient grid sizes and are therefore used. Hereby, the grid size is further denoted as m for simplicity. Secondly, the probabilities for the empirical copula have to be measured. Therefore, this is done using a contingency table with the grid resolution stated above. In order to create this contingency table the relative frequency of the data points are obtained and denoted by $\zeta_{i,j},\ i,j = 1,...,m$. Since the marginal distributions are usually not uniform, this results in an optimization problem which is described and solved by Pfeifer et al. [177]:

$$min! \sum_{i=1}^{m} \sum_{j=1}^{m} (\eta_{i,j} - \zeta_{i,j})^2 \text{ under}$$
$$\sum_{i=1}^{m} \eta_{i,k} = \sum_{j=1}^{m} \eta_{l,j} = \frac{1}{m} \text{ and } \eta_{l,k} \geq 0 \text{ for } k,l = 1,...,m,$$

in order to approximate the contingency table with entries $\zeta_{i,j}$ by a uniform contingency table with entries $\eta_{i,j}$. This optimization problem can be solved by the Karush-Kuhn-Tucker theorem which is in general not straightforward. However, it is also possible to derive a suboptimal solution which leads to a similar result [177]. Hereby, a Lagrange approach by dropping the non-negativity condition is considered.

$$\eta_{i,j} = \zeta_{i,j} - \frac{\zeta_{\cdot,j}}{m} - \frac{\zeta_{i,\cdot}}{m} + \frac{\zeta_{\cdot,\cdot}}{m^2} + \frac{1}{m^2}, \text{ for } i,j = 1,...,m,$$

where the index $\cdot$ means summation. Afterwards, it is possible that negative entries occur. In order to eliminate them while keeping the uniform margins, the following scaling is applied:

$$\bar{y}_{i,j} = \frac{\eta_{i,j} + \gamma}{1 + m^2 \cdot \gamma}, \text{ with } \gamma := -min\{\eta_{i,j} | 1 \leq i,j \leq m\}.$$

Since this suboptimal solution only differs slightly from the optimal solution but is better in respect of programming, this approach is considered further for simplicity. Thus, a contingency table with uniform margins fulfilling the requirements for a copula can be derived. The entries $\bar{y}_{i,j}$ of the contingency table can be used to define the joint distribution of the random vector $\mathbb{F} = (\mathbf{F}_1, \mathbf{F}_2)$ inducing the Bernstein and grid-type copula as described in Section 4.1.2. Overall, this leads to three contingency tables with entries $\zeta_{i,j}$ for the relative frequency, the second one with entries $\eta_{i,j}$ and uniform margins, and the final one $\bar{y}_{i,j}$ with uniform margins and non-negative values for $i, j \in 1, ..., m$.

Applying this to fit the grid-type and Bernstein copula densities to the empirical copula density resulting from the rank transformed data shown in Figure 4.2 by assuming a grid size of $m = 10$ and $m = 20$ leads to the fitted copula densities shown in Figures A.3 and A.5 for the additive approach and Figures A.4 and A.6 for the multiplicative approach in Appendix A.3. In general, both approximations are similar for all approaches and grid sizes. However, in the case of a smaller grid size $m = 10$ the distances between the fit of the Bernstein and grid-type copula densities is sometimes rather large which can be seen in regions where the gradient is huge in particular. This is less pronounced by considering a higher grid size of $m = 20$ where just for some areas the fits show a higher divergent.

4.1.4. Goodness of Fit Test

So far the Bernstein and grid-type copulas can be build following the additive or multiplicative approach as stated in Formula 4.1 for a different grid size. In order to choose the best fit for the related dependence structure, the differences between the fits of the copula densities with respect to the empirical copula can be used to evaluate the goodness of such a copula fit. Therefore, Genest et al. [84] and later Diers et al. [60] propose the Cramer-von-Mises test statistic, which is compared to several other single tests [84]. Since no test is outperforming the others, this test statistic is suggested to be a good choice [78, 84]. Hereby, the difference $\mathbf{C}_N$ between the copula and the empirical copula is measured to decide whether a copula C is coming from some copula class C_0, e.g. $H_0 : C \in C_0$. Therefore, let C_N^{ϕ} be the predicted Bernstein or grid-type copula as estimate of the empirical copula C_N^{emp} under H_0 with N observed data points. Then the test statistic is defined as:

$$S_N^{CvM} = \int_{[0,1]^2} \mathbf{C}_N(\mu, \nu)^2 dC_N^{emp}(\mu, \nu) \tag{4.2}$$

$$= N \cdot \int_{[0,1]^2} \left[C_N^{emp}(\mu, \nu) - C_N^{\phi}(\mu, \nu) \right]^2 dC_N^{emp}(\mu, \nu) =$$

$$= N \cdot \int_{[0,1]^2} \left(\frac{1}{N} \sum_{i=1}^{N} \mathbb{1}_{\left(\mathbf{F}_1^N(X_1^i) \le \mu,\ \mathbf{F}_2^N(X_2^i) \le \nu\right)} - C_N^{\phi}(\mu, \nu) \right)^2 dC_N^{emp}(\mu, \nu) \tag{4.3}$$

which is a Riemann-Stiltjes integral representing a finit sum with $\mathbf{F}_1^N(X_1^i)$ and $\mathbf{F}_2^N(X_2^i)$ according to Section 4.1.1. In order to solve this, the mesh points of the subintervals

I_{p_1,p_2} as stated in Section 4.1.2 are rewritten to m equidistant intervals on $[0,1]$ for both dimensions as $\{0 = \mu_0 < \mu_1 < \ldots < \mu_m = 1\}$ and $\{0 = \nu_0 < \nu_1 < \ldots < \nu_m = 1\}$ in order to solve the Riemann-Stiltjes integral. Therefore, for each subinterval chosen, the squared difference between the empirical copula and the fitted copula is weighted with the probability of the empirical copula on this subinterval as shown exemplarily in Figure 4.4.

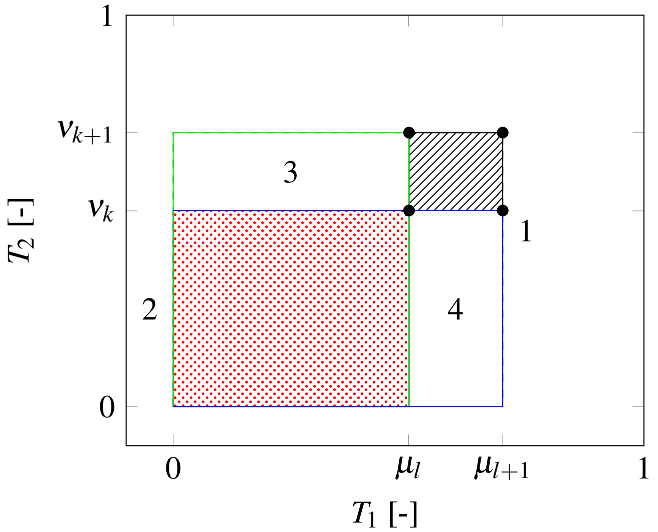


Figure 4.4.: Auxiliary Sketch for the Cramer-von-Mises Test Statistic.

For evaluating the black dashed subinterval shown in Figure 4.4, the whole area 1 is subtracted by the green area 3, by the blue area 4, and the red area 2 is added afterwards. Thus, Equation 4.2 can be formulated as follows using a short and a long form for the estimation shown in Figure 4.4.

$$S_N^{CvM} = N \cdot \sum_{k=0}^{m-1} \sum_{l=0}^{m-1} \Bigg(\underbrace{\frac{1}{N} \sum_{i=1}^{N} \mathbb{1}_{\left(\mathbf{F}_1^N(X_1^i) \in (\mu_l, \mu_{l+1}],\ \mathbf{F}_2^N(X_2^i) \in (\nu_k, \nu_{k+1}]\right)}}_{\text{Short form of the empirical copula}} - C_N^{\phi}(\cdot) \Bigg)^2 \cdot$$

$$\underbrace{\begin{aligned} \cdot \frac{1}{N} \cdot \Bigg[& \sum_{i=1}^{N} \mathbb{1}_{\left(\mathbf{F}_1^N(X_1^i) \le \mu_{l+1},\ \mathbf{F}_2^N(X_2^i) \le \nu_{k+1}\right)} - \sum_{i=1}^{N} \mathbb{1}_{\left(\mathbf{F}_1^N(X_1^i) \le \mu_{l+1},\ \mathbf{F}_2^N(X_2^i) \le \nu_{k}\right)} - \\ & - \sum_{i=1}^{N} \mathbb{1}_{\left(\mathbf{F}_1^N(X_1^i) \le \mu_{l},\ \mathbf{F}_2^N(X_2^i) \le \nu_{k+1}\right)} + \sum_{i=1}^{N} \mathbb{1}_{\left(\mathbf{F}_1^N(X_1^i) \le \mu_{l},\ \mathbf{F}_2^N(X_2^i) \le \nu_{k}\right)} \Bigg] \end{aligned}}_{\text{Long form of the empirical copula}}$$

with $C_N^{\phi}(\cdot) = C^{\phi}(\mu_{l+1}, \nu_{k+1}) - C^{\phi}(\mu_l, \nu_{k+1}) - C^{\phi}(\mu_{l+1}, \nu_k) + C^{\phi}(\mu_l, \nu_k)$ and $\phi()$ according to the Bernstein or grid-type copula. Since the distribution of S_N^{CvM} is unknown, a small value of S_N^{CvM} indicates a good fit [84].

Thus, this allows to evaluate the goodness of the copula fits in respect of the additive or multiplicative approach, the grid size, and the type of the copula. Before doing so, it has to be noted that the test statistic value S_N and its accuracy is influenced by the sample and grid size [60] since the real dependence structure is only approximated by the empirical copula fit C_N^{emp} and the distribution of the assumed copula is only known under H_0 [60, 84]. However, a sample size $N \in [250, 1000]$ is sufficient enough according to Diers et al. [60]. Since the copula will be based on all development steps lying in the same cluster, this is fulfilled for larger markets but not necessarily for smaller ones. Additionally, the unsupervised clustering might lead to a clustering that does not fulfil this requirement. However, this only impacts the assessment of the test statistic while the copula fit itself should still be usable. Consequently, this allows to either set a fixed copula for the development steps or to choose the copula according to the best fit according to the smallest test statistic value S_N^{CvM}.

Based on the cluster results given in Figure 3.9 on page 72 of the German MTPL market, the related copulas are fitted and the values of the Cramer-von-Mises test statistic are shown in Table 4.1 for different grid sizes, the additive and multiplicative approach, and the Bernstein and grid-type copula.

Characteristics		Cluster 1		Cluster 2		Cluster 3	
Copula	Grid size	Add.	Mul.	Add.	Mul.	Add.	Mul.
Bernstein	10	0.425	0.360	1.400	1.510	0.900	0.881
	20	0.130	0.130	0.291	0.293	0.152	0.184
Grid-type	10	0.885	0.896	1.368	1.435	1.017	1.016
	20	0.230	0.223	0.330	0.325	0.180	0.219

Table 4.1.: Cramer-von-Mises Test Statistic Values.

First of all, it can be seen that a higher grid size significantly reduces the value of the test statistic and leads to a better fit of the copula densities since the higher resolution allows for a better approximation of each subinterval. Additionally, the fit of the Bernstein copula density is better compared to the Grid-type copula density except for the second cluster in the case of the additive approach. In the case of the additive and multiplicative approach, both are similar except for the second cluster where the multiplicative approach has a worse performance. Nevertheless, no favourite approach can be chosen here since the performance varies depending on the cluster, the chosen copula, and the grid size. Overall, the test statistic values for the first cluster are the best but get worse for the third cluster and even more for the second. This is a consequence of the time component as shown in Figure 3.10. and how the available developments are distributed over the several clusters. While the information for the first cluster is fully available, it is getting sparse for the second and third cluster. Thus, the empirical copula has subintervals with no or just a few data points. Since these areas are still getting some sort of probability due to the transformations done in Section 4.1.3, this leads to a larger deviation between the fitted and the empirical copula density.

4.1.5. Simulation of Copula Outputs

The related contingency table $\bar{y}_{i,j}$ from Section 4.1.3 can be used to draw possible next development steps from the induced Bernstein or grid-type copula. This can be done efficiently since the Bernstein copula can be represented as mixture of independent beta distributions which leads to an efficient random sampling algorithm [60]. Such a sampling of a two-dimensional vector of random numbers $(\hat{X}_1, \hat{X}_2)$ from the Bernstein copula involves the following two steps:

Step 1: Draw a sample $(q_1, q_2) \in [1, ..., m]^2$ according to the probabilities of the contingency table so that $\mathbb{P}((q_1, q_2) = (i, j)) = \bar{y}_{i,j}$ with $i, j = 1, ..., m$ and $\bar{y}$ as two-dimensional contingency table described in Section 4.1.3.

Step 2: These samples can then be used to evaluate a beta distribution. Thus, a sample $(\hat{X}_1, \hat{X}_2)$ can be achieved by drawing independent $\hat{X}_l \sim Beta(q_l + 1, m - q_l)$ for $l = 1, 2$.

In order to draw a representative of the grid-type copula, a sample $(\hat{X}_1, \hat{X}_2)$ is achieved by drawing independently from a uniform distribution with support on $\left(\frac{q_l}{m}, \frac{q_l+1}{m}\right]$ in the second step. Since a drawn sample represents a two-dimensional point in the multivariate distribution, the development step itself can be estimated by applying the quantile function $\mathbf{Q}_k(\hat{X}_k) = inf\{y_k \in \mathbb{R} : \hat{X}_k \leq \mathbf{F}_k^N(y_k)\}$ for $k = 1, 2$ of the empirical incurred changes $LDF_{new}^{Inc} = \mathbf{Q}_1(\hat{X}_1)$ and of the payment ratio changes $LDF_{new}^{PR} = \mathbf{Q}_2(\hat{X}_2)$. By utilizing this, the respective paid, outstanding, and incurred value as well as the new payment ratio can be estimated. In the case of the additive approach, those values have to be added whereas they have to be multiplied to the latest known incurred and payment ratio values for the multiplicative approach. This can be used further to develop single claims by sequentially drawing development steps from the copula.

4.1.6. Application, Parametrization, and Recommendations

In respect of the less symmetric and unusual dependence structure of the development steps, the choice of a non-parametric copula such as the Bernstein or grid-type copula is advisable. Hereby, the goodness of the copula fits highly depends on the chosen grid size. While the grid-type copula is based on indicator functions and will basically equal the empirical copula for a grid size that is small enough, the Bernstein copula disperses the probability mass over a range of possible outcomes. While this also accounts to a small extent for the Grid-type copula due to the transformation described in Section 4.1.3, this is more pronounced for the Bernstein copula. Thus, the Bernstein copula is preferable for lower grid sizes and a smaller number of data points due to the smoothing effect of the Bernstein polynomial. Consequently, it can be build on a small sample size while the range of simulated development steps also includes a broader range with values that have not been seen so far, which are two of the requirements stated in Section 3.1. Moreover, the Bernstein copula might be a more adequate modelling approach if the dataset is less homogeneous and does not have a strong correlation between the claim developments [60]. Since the copulas are fitted to market data, which is divided into several clusters and only a small correlation is observable, the usage of a Bernstein copula is advised in general.

However, this might lead to cases where the Cramer-von-Mises test statistic values might indicate less appropriate estimates for smaller markets with less data points. Following the goodness of the fits, it is also possible to choose the best copula fit for each cluster here for an automated procedure.

For an application on real data, a few points have to be mentioned. First of all, the usage of such a copula still follows assumption 3.4 since the used quantile functions do not allow for drawing negative values. Secondly, the next development step is always drawn from the copula which is related to the current cluster in which the claim is located in. If the claim development leads into a new cluster, the next development step is drawn from another copula fitted on this new cluster. Hence, a transition between the clusters is possible here. A third point is the application for claims where only an outstanding value is given. In those cases, a multiplicative approach does not work and the first development step has to be done according to an additive approach to get a payment ratio which does not equal zero. The fourth and last point is related to the settlement of single claims. In general, it is possible to develop single claims until they are settled by sequentially drawing enough development steps until the payment ratio equals one. While an increasing payment ratio does limit the number of possibly allowed development steps, it is not possible so far to actually settle a claim. This is due to the fact that the amount of possibly allowed development steps to settle the claim is a null set. Hence, a small workaround is needed for the settlement of a claim by allowing an imaginary payment ratio of two. A possible development step is then drawn without any restriction and a development step which increases the payment ratio above one automatically settles the claim. The incurred value is then determined by evaluating a linear function between the latest and settlement point for a payment ratio equal one.

Furthermore, it has to be kept in mind that the LDFs used are based on historical LDFs which were correct in the past but not necessarily for the future. They can only be counted as an accurate measure for the future development if the present claims have the same degree of reserve adequacy as the past ones [48].

In summary, this allows to develop single claims until they are settled while allowing to have new and unseen development factors. Due to the smoothing feature of the Bernstein copula, it is also possible to deal with sparse claims data. Additionally, the clustering of the dataset allows for a distinction of the developments between claim sizes and the payment ratio. Thus, the four requirements stated in Section 3.1 are already fulfilled.

4.2. Condition I - Loss Development Factor Surface

Current SLD models that serve as a starting point for the proposed model project claims by using other development factors related to the same development year [64, 103]. This might lead to unrealistic development steps when a large development factor is chosen for an already large claim [204]. Thus, the requirement of limiting the development factors to achieve a more realistic claims' development is given.

This leads to two basic questions. Firstly, if this can happen in the currently proposed approach and secondly, if such a behaviour is wanted due to the requirement of simulating extreme LDFs as stated in Section 3.1. To answer this, the LDF structure of a cluster can be taken as example. The development steps are higher for smaller incurred values and diminish with increasing incurred values. Additionally, claims with a smaller payment ratio have a higher LDF compared to claims with a higher payment ratio. Thus, it is possible to have such a development in the current approach. Related to the second question, the main target is to have a realistic development of single claims and to have the possibility of extreme developments. However, such a decision should be up to the actuary since it is also related to the purpose of the model.

Conclusively, a cap of the LDFs has to come in place as long as the cluster result is used to set up the copulas. Due to the model framework such a cap has to scale with the incurred values and payment ratios of the claims and should not depend on the clustering but the whole dataset.

4.2.1. Requirements

In order to restrict the LDFs related to the incurred and payment ratio values, a three-dimensional smooth surface lying on top of the data point cloud shown in Figure 3.9. on page 72 is considered to be a promising approach. Hence, the restriction would scale with the incurred and payment ratio values and would not depend on each cluster but the market data itself.

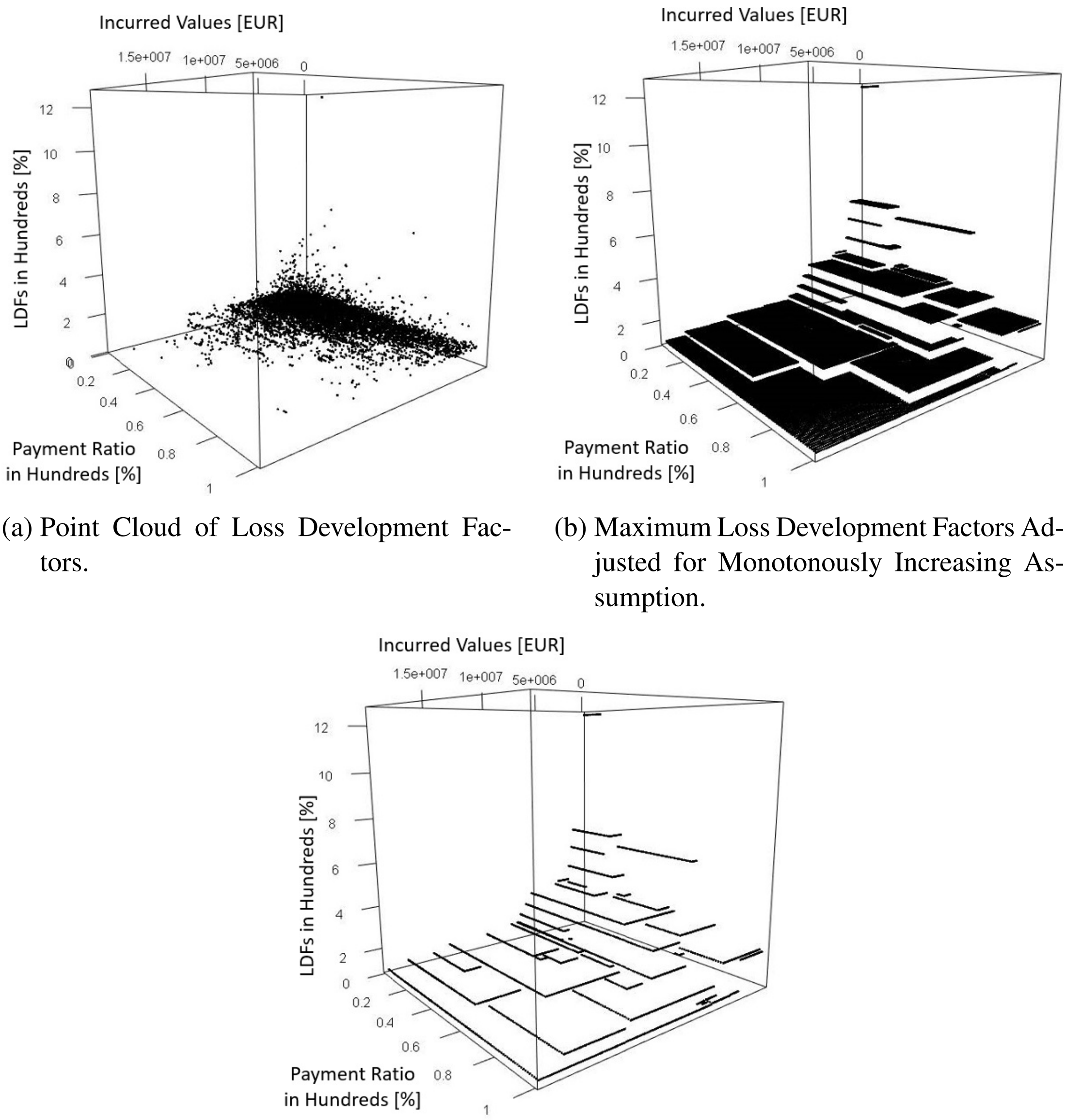
In order to derive such a surface, several aspects have to be kept in mind. First, the surface has to be continuous in order to allow for a maximum LDF for any combination of incurred and payment ratio values. Additionally, not all points in the point cloud can be used. Approximating a surface on the whole point cloud would lead to an overfitting resulting in a spiked surface. Since the required surface should approximate the maximum LDFs at some point where no high LDF has been observed so far, e.g. between two high LDFs there are most likely also smaller development factors located that should not be approximated by the surface, a choice of the possible data points has to be done first. Conclusively, this refers to a smooth surface that is required to lie on the maximal points of the point cloud. Figuratively speaking, a thick blanket has to be thrown onto the point cloud.

To the knowledge of the author, no straight forward procedure exists for this kind of task. Usually, a mesh is already given, which is then getting interpolated. Other methods create a regression plane which is not on top of the point cloud or leads to a perfect fit with many spikes since all points are considered. This results in two main tasks. First, the points in the point cloud which should be considered further for the surface and that represent local maxima for a certain area have to be identified. Second, the space between those points has to be filled following the constraints stated above.

4.2.2. Surface Mesh

Considering the point cloud shown in Subfigure 4.5a with points $\left(Inc_i^{PC}, PR_i^{PC}, LDF_i^{PC}\right)$ for $i = 1, \ldots, \tilde{n}$ as stated in Section 3.5.9.4, it is not intuitively clear how large an area for the local maxima has to be.

The main idea of solving this is that the surface has to be increasing monotonously for diminishing incurred and payment ratio values in order to approximate the maximum values. Therefore, a mesh can be created with m_1^{surf} steps for the incurred and m_2^{surf} steps for the payment ratio values. In order to create equidistant intervals for the grid, the maximal values have to be pre-defined. In the case of the payment ratio, the related interval is $[0,1]$ while the interval for the incurred values is set to $[0, Inc^{surf}]$. This is done for simplicity reasons since the smoothing process will allow LDFs above 1 for all incurred values that are smaller than $Inc^{surf} = \max\limits_{i=1,\ldots,\tilde{n}} \left(Inc_i^{PC}\right) + \varepsilon$ while setting the maximal LDF to 1 for all values above. Hereby this limit is set to the maximal incurred value $\max\limits_{i=1,\ldots,\tilde{n}} \left(Inc_i^{PC}\right)$ in the point cloud increased by some incurred amount ε which should be up to the responsibility of the actuary depending on the respective market. In a later stage, this can be replaced by a function with an asymptotic of 1 for increasing incurred values. Based on the intervals and the step size given, a split can be done into equidistant intervals $\{a_0, a_1, a_2, \ldots, a_{m_1^{surf}}\}$ with $a_i = i \cdot \frac{Inc^{surf}}{m_1^{surf}}$, $i = 0, \ldots, m_1^{surf}$ and $\{b_0, b_1, b_2, \ldots, b_{m_2^{surf}}\}$ with $b_j = j \cdot \frac{1}{m_2^{surf}}$ for $j = 0, \ldots, m_2^{surf}$. Based on each interval $[a_i, a_{i+1}] \times [b_j, b_{j+1}]$ the maximal LDF $LDF_{a_i,b_j}^{surf} = max\{LDF_k^{PC} : (Inc_k^{PC}, PR_k^{PC}) \in (a_i, a_{i+1}] \times (b_j, b_{j+1}],\ k = 1, \ldots, \tilde{n}\}$ is estimated resulting in the points used for the further surface estimation $\left(\frac{a_i + a_{i+1}}{2},\ \frac{b_j + b_{j+1}}{2},\ LDF_{a_i,b_j}^{surf}\right)$. Here it has to be noted that the



(a) Point Cloud of Loss Development Factors.

(b) Maximum Loss Development Factors Adjusted for Monotonously Increasing Assumption.

(c) Maximum Margins for Spline Interpolation.

Figure 4.5.: Preparations for the Maximum Loss Development Surface with a Maximal Incurred Value of EUR 19 Million.

maximal LDF set is related to the middle point of each subinterval which can lead to an underestimaton or overestimation for specific areas. However, due to the smoothing process which is applied later, the surface is going to increase for most areas so that an overestimation is achieved in general, e.g. the surface will be above the point cloud. Since this results in a more conservative approach in the end, it is not seen as a critical point in a pricing procedure which is based on conservative assumptions in general.

Based on this grid and the $m_1^{surf} \cdot m_2^{surf}$ maximal LDF points, the monotonously increasing assumption can be applied. Beginning with the highest payment ratio and incurred value of the lattice $(a_{m_1^{surf}-1}, a_{m_1^{surf}}] \times (b_{m_2^{surf}-1}, b_{m_2^{surf}}]$, the development factors in the mesh are sequentially checked and replaced if the LDF is smaller than the previous one for $i = m_1^{surf} - 1, ..., 0$ and $j = m_2^{surf} - 1, ..., 0$:

$$LDF^{max}_{a_i,b_j} = \begin{cases} LDF^{surf}_{a_i,b_j} & , i = m^{surf}_1 - 1,\ j = m^{surf}_2 - 1 \\ max\{LDF^{surf}_{a_i,b_j}, LDF^{surf}_{a_i,b_{j+1}}\} & , i = m^{surf}_1 - 1,\ j \neq m^{surf}_2 - 1 \\ max\{LDF^{surf}_{a_i,b_j}, LDF^{surf}_{a_{i+1},b_j}\} & , j = m^{surf}_2 - 1,\ i \neq m^{surf}_1 - 1 \\ max\{LDF^{surf}_{a_i,b_j}, LDF^{surf}_{a_{i+1},b_j}, LDF^{surf}_{a_i,b_{j+1}}\} & , \text{else.} \end{cases}$$

Hence, locally small development factors are not further considered and replaced. In total, this leads to a mesh where the LDFs are increasing for diminishing incurred and payment ratio values as shown in Subfigure 4.5b, which fulfils the monotonously increasing assumption.

In addition to that, it can be assumed that for all areas where the development factors are sorted out, a possibly high development factor has not been observed but is possible since it occurs for an even higher incurred or payment ratio value. Thus, only the marginal maximal LDFs are a reliable information as shown in Subfigure 4.5c for $i = m^{surf}_1 - 1, ..., 0$ and $j = m^{surf}_2 - 1, ..., 0$:

$$LDF^{margins}_{a_i,b_j} = \begin{cases} LDF^{max}_{a_i,b_j} & , i = m^{surf}_1 - 1 \\ LDF^{max}_{a_i,b_j} & , j = m^{surf}_2 - 1 \\ Na & , LDF^{max}_{a_i,b_j} \leq min\left(LDF^{max}_{a_{i+1},b_j}, LDF^{max}_{a_i,b_{j+1}}\right) \\ LDF^{max}_{a_i,b_j} & , \text{else.} \end{cases} \tag{4.4}$$

In a next step, the missing points between the maximal margins $LDF^{margins}_{a_i,b_j}$ denoted with Na have to be estimated so that a monotonously increasing smooth surface is created, lying on top of the maximum margins. It should be clear here that the smoothing condition leads to an adjustment and exclusion of some maxima and of the Na's as the space between them.

4.2.3. Qualitatively Constrained Smoothing and B Spline Representation

Since the lattice is given with values of the maximum margins, this can be seen for the respective two-dimensional cases by either fixing sequentially the payment ratio intervals or the incurred intervals in order find a function which fulfils the needed conditions. Here, those related points can be used to sequentially do a spline interpolation with the requirement to get smooth, monotonously increasing functions lying on top of the maximal points. This would create some kind of lattice on which a surface can be interpolated afterwards. Taking w.l.o.g. the first line of the lattice with respect to the incurred values and LDFs, $\left(\frac{a_i+a_{i+1}}{2}, \frac{b_0+b_1}{2}, LDF^{margins}_{a_i,b_0}\right)$ where $LDF^{margins}_{a_i,b_0} \neq Na$

with a fixed $j = 0$ for the payment ratio and $i = 0, ..., m_1^{surf} - 1$. This results in a set

$$\left\{ \left(\frac{a_i + a_{i+1}}{2}, LDF^{margins}_{a_i, b_0} \right) : LDF^{margins}_{a_i, b_0} \neq Na,\, j = 0,\, i = 0, ..., m_1^{surf} - 1 \right\} =: \\ =: \left\{ \left(a_k^\star, LDF_k^\star\right) \right\}_{k=1}^{\hat{n}_{\cdot,0}} \quad (4.5)$$

of points for which the spline interpolation can be applied. For simplicity reasons, this set is rewritten for an easier usage further on. The value $\hat{n}_{\cdot,j}$ denotes the number of points in the set based on the directions, e.g. $j = 0$, of the lattice. The required task can then be performed by considering qualitatively constrained quantile smoothing B-splines. This has been published by Ng and Maechler [167] based on a paper by He and Ng [99]. In this section, only the basic idea is recapitulated briefly by following Ng and Maechler [167] and Ng and He [99].

Therefore, it is assumed that $\left\{ \left(a_k^\star, LDF_k^\star\right) \right\}_{k=1}^{\hat{n}_{\cdot,0}}$ are $\hat{n}_{\cdot,0}$ realizations of a pair of bivariate random variables (X,Y). The task of fitting a smooth function to those data points results in the following minimization problem:

$$\min_h \text{'fidelity'} + \lambda^{Spline} \cdot \text{'}L_p \text{ roughness'}, \quad (4.6)$$

where in this case h is the ψ-th conditional quantile function, $h_\psi(a^\star)$, of Y given $X = a^\star$. Given a scalar $\psi \in [0,1]$ this function has to fulfill the following equation:

$$\mathbb{P}\left(Y \leq h_\psi(a^\star) \,\middle|\, X = a^\star\right) = \psi.$$

The λ^{Spline} is a smoothing parameter that controls the trade-off between 'fidelity' to the data and the 'roughness' to the fit. Hereby, the fidelity to the data is defined as the distance between the data points and the smooth function evaluated at the related values,

$$\text{'fidelity'} = \sum_{k=1}^{\hat{n}_{\cdot,0}} \varepsilon_\psi\left(LDF_k^\star - h_\psi(a_k^\star)\right),$$

under some check function ε_ψ which evaluates the residual distance $d = LDF_k^\star - h_\psi\left(a_k^\star\right)$ by applying a weight function w_ψ:

$$\varepsilon_\psi(d) = w_\psi(d)\,|d| \text{ where } w_\psi(d) := 1 + (2\psi - 1)sgn(d).$$

The roughness can either be defined by the total variation norm or the Maximum norm:

$$\text{'}L_1 \text{ roughness'} = \sum_{k=1}^{\hat{n}_{\cdot,0}-2} \left| h'_\psi\left(a_{k+1}^\star\right) - h'_\psi\left(a_k^\star\right) \right|, \quad (4.7)$$

$$\text{'}L_\infty \text{ roughness'} = \|h''_\psi\|_\infty = \max_{a^\star} |h''(a^\star)|. \quad (4.8)$$

A high fidelity to the data usually implies a small 'L_p roughness' and vice versa explainable as the trade-off between overfitting and underfitting. A perfect fit to the data would result in many spikes with a small fidelity value but a high value for the roughness which leads to an unfavourable fit. Koenker et al. [129] introduced the ψ-th L_p quantile smoothing spline $\hat{h}_{\psi,L_p}(a^\star)$ as a non-parametric estimator for $h_\psi(a^\star)$, which is a solution to the minimization problem stated in Equation 4.6. Furthermore, they have shown that $\hat{h}_{\psi,L_1}(a^\star)$ is a linear smoothing spline of second order for the L_1 roughness in Equation 4.7 while $\hat{h}_{\psi,L_\infty}(a^\star)$ can be approximated by a quadratic smoothing spline of third order for the L_∞ roughness penalty shown in Equation 4.8. Since any q-th order smoothing spline with simple knots at $a^\star_1,...,a^\star_{\hat{n}_{\cdot,0}}$ has an equivalent B-spline representation at the same knot sequence, this allows to find a representation of h_ψ as B-spline. Under the consideration of a general knot mesh $T^{knots} = \{t_i\}_{i=1}^{N+2q}$ with $t_1 = \ldots = t_q < t_{q+1} < \ldots < t_{N+q} < t_{N+q+1} = \ldots = t_{N+2q}$ and N internal knots, the B-spline representation of a smooth function of order q with mesh T is

$$s(a^\star) = \sum_{j=1}^{N+q} e_j B_j(a^\star). \tag{4.9}$$

Hereby, $B_j(a^\star)$ are the normalized B-spline basis functions and e_j are the coefficients for the B-spline basis functions. Hence, the minimization problem stated in Equation 4.6 aims at finding the respective coefficients $\hat{e}_j$ for the B-spline representation in 4.9.

Ng and Maechler [167] and Ng and He [99] show than that Equation 4.6 using a linear B-spline with the L_1 roughness from Equation 4.7 can be written as

$$\min_{\theta\in\mathbb{R}^{N+q}} \sum_{i=1}^{n} \left| LDF_i^\star - \sum_{j=1}^{N+q} e_j B_j(a_i^\star) \right| + \lambda \sum_{i=1}^{N} \left| \sum_{j=1}^{N+q} e_j B'_j(t_{i+q}) - \sum_{j=1}^{N+q} e_j B'_j(t_{i+q-1}) \right| \tag{4.10}$$

and in case of quadratic B-splines using the L_∞ roughness

$$\min_{\theta\in\mathbb{R}^{N+q}} \sum_{i=1}^{n} \left| LDF_i^\star - \sum_{j=1}^{N+q} e_j B_j(a_i^\star) \right| + \lambda \max_{a^\star} |h''(e^\star)| \tag{4.11}$$

where $\theta = (a_1,...,a_{N+q})$. Both minimization problems can be solved by any efficient linear programming algorithm resulting in the optimal set of the coefficients $\hat{e}_j$ and the resulting fitted curve $\hat{m}_{\lambda^{Spline},L_1}$ or $\hat{m}_{\lambda^{Spline},L_\infty}$.

4.2.4. Additional Constraints and Parameter Choice

Since the Equations 4.10 and 4.11 can be solved by a linear program this opens up the possibility to include additional constraints. Those constraints are stated by Ng and Maechler [167] and Ng and He [99] and include monotonicity, convexity, periodicity, and pointwise constraints. Those additional constraints can be included by adding additional equalities or inequalities to the linear program. For the purpose of fitting the surface, one requirement is the monotonicity which has to be fulfilled. Additionally, the surface should lie on top of the local maxima, which can be included by adding pointwise constraints so that the fitted spline must be on or above the marginal points.

The choice of λ^{Spline} as smoothing parameter has a strong effect on the smoothing B-spline. Therefore, the generalized cross-validation criterion is commonly used in least squares based smoothing splines. This criterion is equivalent to the Akaike information criterion and similar to the Schwarz information criterion for a moderate sample size. If the scalar ψ is set to 0.5, the difference is given by the penalty term that is added for the number of parameters, which is replaced by a robust alternative using the mean absolute residuals. This can be seen as a variant of the SIC criterion chosen by Ng and He [99]. Therefore, it is possible to optimize the choice of λ^{Spline} according to the SIC information criterion [99]. Overall, this criterion estimates the relative amount of information lost by the fitted spline. Therefore, it deals with the trade-off between the goodness and the simplicity of the spline fit. Hence, the λ^{Spline} is chosen so that it minimizes the SIC information criterion.

Furthermore, it is possible to delete or add knots used for the fit of the spline [99]. This can be seen as following:

1. The first initial knot set based on the maximum marginal values from Equation 4.4 is checked sequentially for the optimal choice of values using the Akaike information criterion based on the knot mesh T^{knots}, N internal knots and the order of the used spline q:

$$AIC(T^{knots}) = log\left(\frac{1}{\hat{n}_{\cdot,0}}\sum_{k}\varepsilon_{\psi}(LDF_k^{\star} - \hat{m}_{T^{knots}}(a_k^{\star}))\right) + 2\frac{N+q}{\hat{n}_{\cdot,0}}. \tag{4.12}$$

2. In the second step, each knot is removed sequentially to obtain the resulting Akaike information criterion value. The knot giving the largest reduction of the Akaike information criterion is then removed from the knot set. This process is repeated until no more existing knots can be removed.

3. Thirdly, additional knots can be added by taking the mid-points between every adjacent pair of knots in the knot set if the added points lead to a reduction of the Akaike information criterion. If such an improvement is possible, those knots are added.

4.2.5. Application to the Dataset

In order to apply the above described procedure to the dataset, the lattice with the marginal maximum LDFs from Equation 4.4 and 4.5 is considered. The task of fitting the surface is done by sequentially fitting two-dimensional splines. The initial maximal margins are fitted for fixed incurred values in a first step and evaluated at the respective points of the grid. This is used afterwards as an input for another fit by fixing the payment ratios. This recursive procedure is repeated until the resulting surface fit does not change significantly any more with a chosen residual overall distance of $\varepsilon^{fit} = 0.001$. With this choice, the algorithm has proven to deliver an accurate surface while avoiding unnecessarily long runtimes. However, a different value can be chosen here depending on the runtime for the underlying dataset. The derived LDFs of the mesh can then be used for a linear interpolation to obtain the final surface. Hereby, the procedure starts with a fixed incurred value since tests have shown that the surface fit is closer to the original data in that way.

The fitting process is therefore divided into two steps. First, the sparse lattice from the maximum marginal grid is used to estimate the missing values in between. Therefore, the $LDF_k^\star$ of the smallest and largest value $a_k^\star$ as start and endpoints are fixed and the Akaike information criterion from Equation 4.12 is applied. Hence, the missing points in between are approximated while preventing overfitting. Secondly, the smoothing process is started. Therefore, the endpoints are allowed to change and the Akaike information criterion is not considered any more. This is done since experiments[2] have shown that the smoothing process requires more flexibility with respect to each spline based on the associated lattice in order to achieve the smoothness.

Applying the sequential qualitatively constrained quantile smoothing to the maximum margins shown in Figure 4.5c results in the smooth surface which is shown in Figure 4.6 from two different view points.

The obtained surface fulfils the monotonicity condition so that with an increasing incurred or payment ratio value the associated maximum LDF decreases. Due to the not fixed endpoints during the smoothing process, the spline fit approximates slightly higher LDFs than observed so far. However, this is not critical since the differences are small and possibly higher LDFs result in a more conservative projection which is natural for the insurance industry and follows the economical understanding of risk assessment for that segment. However, the LDF surface is only effective, if the smallest possible LDF allowed by the surface for a cluster is smaller than the largest possible LDF which can be drawn from the copula of that cluster.

[2]Based on randomly drawn points of five different markets during the Monte Carlo simulations. Due to the flexibility of the spline approach, this works for all other MTPL dataset and related subsets of the markets considered in Chapter 5. Plots of the surfaces are shown in the Appendix A.4.

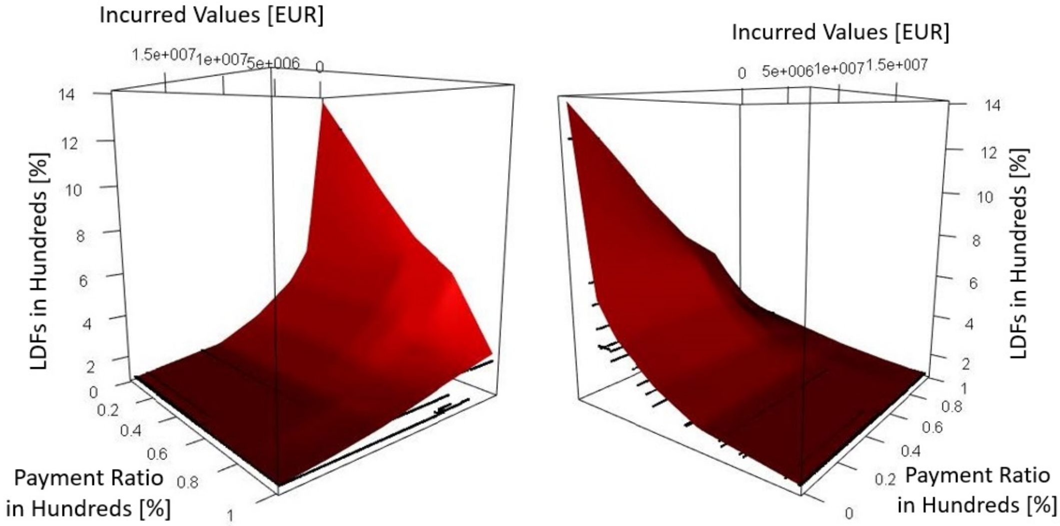
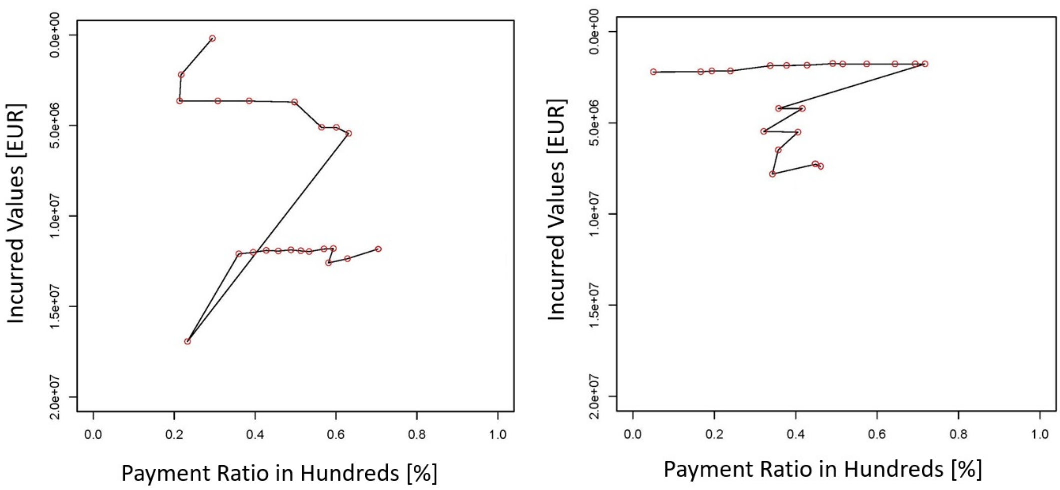


Figure 4.6.: Smooth LDF Surface.

While this is currently only done for negative run-offs in order to have a restriction for the maximum LDFs, a minimal development factor is automatically given due to the restriction that the paid values are increasing monotonously. If this assumption is omitted, a surface could also be fit from the bottom to the dataset in a similar way to have a restriction for the minimal LDFs.

4.3. Condition II - Jump Pattern

When analysing claim paths of real occurred claims some development patterns can be observed. This is shown exemplarily for two real claims of the German MTPL market in Figure 4.7.



(a) Claim 1. (b) Claim 2.

Figure 4.7.: Exemplary Development Paths of Two Single Claims.

Hereby, the first reported value of a claim is the red dot in the upper left corner of each plot while the lines and other red dots show the development path for the claim. The incurred value for the first claim in Figure 4.7a increases slightly until a payout period starts after the second development step. This is interrupted by a larger increase of the reserve after the eighth development step, followed by reserve release and another payout period. In comparison, the second claim in Figure 4.7b gets paid for twelve years after which the reserves increases a few times. Both claims show periods where the claim is paid over time and periods where the claim size in form of the reserve is adjusted. However, after a strong reserve increase there is only a small probability of getting something similar in the next development step. This can be observed for all claims and all market datasets analysed in this thesis. Additionally, the payout periods in Figure 4.7 are related to the annuity payment characteristic of the market and the idiosyncrasies of the insurance companies as discussed in Section 2.4.3. However, the development of claims occurring in a market where mostly lump sums are paid will look differently and will show less development steps with larger jumps.

Hence, it seems to be necessary to catch the related characteristics of the market and to guide the claims' development path in order to project the seen pattern. This can only be done if the memorylessness assumption together with the independence assumption between the development years, which are usually assumed for most models, are discarded. A similar idea was published by Mahon [157], who used the transition between different groups of claims and not of the LDFs related to their payment ratio and incurred values. Nevertheless, the idea of using transition matrix theory for the transition from one state to another with a changing probability, influences the idea that is used further on.

4.3.1. Jump Areas

The intuitive way would be to look for similar claims that have shown a similar development path. However, this results in a few problems. First, it is not clear how a similar development path is defined. Secondly, it is not necessarily given that a claim with a similar development path exists in the market data. Thirdly, the transition probability should not be derived from only a single claim but from a larger group of claims. Overall, the pattern observed in the market data should be projected by considering not the full history of the claim but enough to catch the main characteristics. Otherwise, an overfitting would be done resulting in the same claim paths as observed in the market which is not targeted. Hence, a high volatility in the market data should result in a high volatility of simulated claims and a positive or negative run-off pattern in the market should lead to a positive or negative run-off pattern for the simulations. In summary, the jump pattern should guide the claims' development during the simulations.

In order to assess such a pattern for the claims and to predict the zig-zag curves as seen in Figure 4.7., all one-year development factors $\left(LDF_i^{PC,Inc}, LDF_i^{PC,PR}\right)$ for $i = 1, \ldots, \tilde{n}$ as stated in Section 3.5.9.4 from the point cloud are considered for each single claim $n^{\star}$. The LDF for the payment ratio $LDF_i^{PC,PR}$ is calculated in the same way as it is done for the incurred values $LDF_i^{PC,Inc}$. For simplicity reason, the PC in the numeration is left out further on. The development steps can roughly be divided into the following groups according to the nature of the related development step, which is mathematically stated later in Formula 4.13:

Case 1: The incurred and payment ratio value decreases. This is not possible in the current scenario due to the Assumption 3.4 of monotonously increasing paid value.

Case 2: The incurred and payment ratio changes slightly. The development of the claim is small and nearly negligible in any direction. In order to quantify this, the parameter ω_1 is used to define the radius.

Case 3: This refers to an increased reserve where the incurred value increases but the payment ratio decreases. However, this case is subset into two cases by the ratio of change. For smaller changes below a ratio of ω_2, the claims are assigned to this case. For all other claims above ω_2, they are assigned to case 4.

Case 4: If a claim shows a high increase of the incurred value in addition to a larger decrease of the payment ratio, this is related to larger reserve increases. If the change of such a reserve increase is above ω_2, those development steps are located in this case.

Case 5: This refers to an increasing incurred and payment ratio value. Similar to case 3 and 4, this is divided into a large or smaller increase using a ratio of ω_3. Claims with a smaller ratio than ω_3 are located in this case.

Case 6: If the overall change of the incurred and payment ratio values exceeds ω_3, then it is assigned to this case. Due to the character of this case, development steps in this case are usually rare. However, a distinction is provided here for completeness.

Case 7: If the claim is getting paid with a stable incurred value and an increasing payment ratio, the changes are assigned to this case. This can be observed in Figure 4.2 on page 81 with the straight lines around 38% referring to stable incurred values. However, there is also the possibility to define a tube by allowing a change of ω_4 for the incurred value changes to group claims with nearly no incurred changes into this case as well. The parameter ω_5 is used to distinguish between smaller and larger changes.

Case 8: If the ratio exceeds ω_5, the development steps are assigned to this case. Hence, this includes larger payouts.

Case 9: The last two cases include a positive development of the claim. The incurred value decreases and the payment ratio increases. This is related to decreasing reserves. Similar to the other cases, this is divided according to the ratio ω_6. All positive developments below this threshold are located in this case.

Case 10: If the positive development exceeds ω_6, it is located in this case. Thus, this case includes development steps with a large increase in the payment ratio combined with a decreasing incurred value.

These ten cases are shown exemplarily in Figure 4.8 for the German MTPL market data with differently chosen thresholds $\omega_1 = 0.05$, $\omega_2 = 0.6$, $\omega_3 = 0.55$, $\omega_4 = 0.01$, $\omega_5 = 0.5$, $\omega_6 = 0.6$ to show the different areas. Hereby it has to be kept in mind that the monotonously increasing assumption for the paid values results in a natural border as described in Section 3.5.10, which can be seen towards the lower left corner.

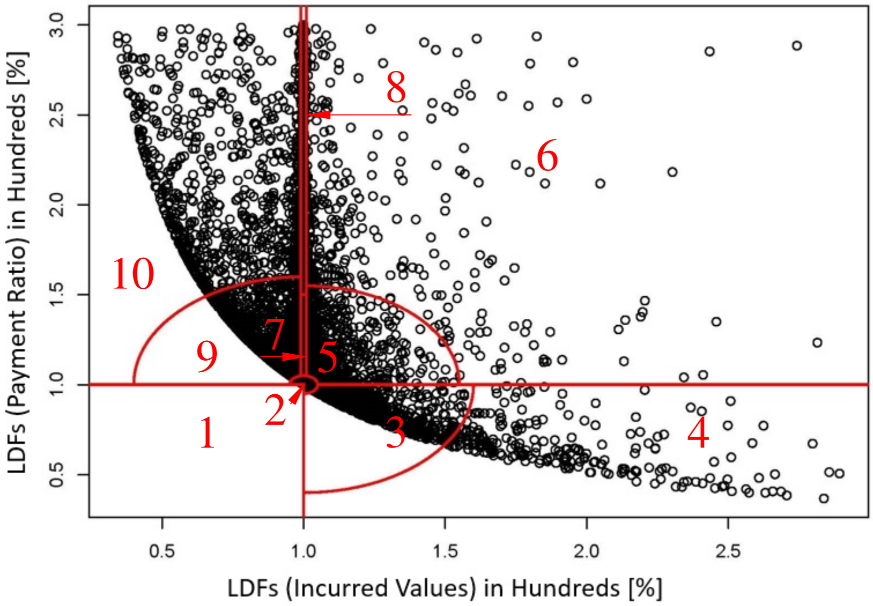


Figure 4.8.: Jump Areas Divided into Ten Cases.

The ratio of change $\xi_i = \sqrt{(LDF_i^{Inc} - 1)^2 + (LDF_i^{PR} - 1)^2}$ is determined by the euclidean norm evaluated for the percentage changes of the one-year development factors for each claim $l = 1, \ldots, n^{\star}$. Those cases will be defined by the function $\theta_i^l(\omega)$ for the l-th claim with the i-th development step based on the related LDFs and the thresholds ω.

$$\theta_i^l(\omega) = \begin{cases} 1 & , \underbrace{LDF_i^{Inc} \leq 1, LDF_i^{PR} \leq 1, \left(LDF_i^{Inc}, LDF_i^{PR}\right) \neq (1,1)}_{:= *} \\ 2 & , \xi_i \leq \omega_1, \neg * \\ 3 & , LDF_i^{Inc} > 1, LDF_i^{PR} \leq 1, \xi_i \in (\omega_1, \omega_2] \\ 4 & , LDF_i^{Inc} > 1, LDF_i^{PR} \leq 1, \xi_i > \omega_2 \\ 5 & , LDF_i^{Inc} > 1 + \omega_4, LDF_i^{PR} > 1, \xi_i \in (\omega_1, \omega_3] \\ 6 & , LDF_i^{Inc} > 1 + \omega_4, LDF_i^{PR} > 1, \xi_i > \omega_3 \\ 7 & , LDF_i^{Inc} \in [1 - \omega_4, 1 + \omega_4], LDF_i^{PR} > 1, \xi_i \in (\omega_1, \omega_5] \\ 8 & , LDF_i^{Inc} \in [1 - \omega_4, 1 + \omega_4], LDF_i^{PR} > 1, \xi_i > \omega_5 \\ 9 & , LDF_i^{Inc} < 1 - \omega_4, LDF_i^{PR} > 1, \xi_i \in (\omega_1, \omega_6] \\ 10 & , LDF_i^{Inc} < 1 - \omega_4, LDF_i^{PR} > 1, \xi_i > \omega_6. \end{cases} \tag{4.13}$$

The used thresholds ω_j, $j = 1, ..., 6$ depend on the observed LDFs for the market and are based on the opinion of the actuary. Since they remain the same for each individual market, they are left out further on for simplicity reasons.

4.3.2. Jump Pattern Recognition and Comparison

The function θ_i^l allows to create a sequence of cases for each individual claim $l = 1, ..., n^\star$. Applied on the single claim shown in Figure 4.7a for all development steps i the sequence would be as follows:

$$(4,3,7,7,5,5,7,5,4,10,7,7,7,7,7,2,7,2,3,9,9). \tag{4.14}$$

Based on these sequences, the main characteristics of the development pattern have to be derived. Therefore, the development from a current case to the next one is analysed by considering the previous development between the cases. Thus, a transition probability depending on the current case is targeted. Hereby, the first intuition is to consider the whole previous development of a case and to compare it with the development of the cases of other claims. However, under the constraint that there should be a sufficient amount of similar comparable claims[3], it can be seen in the data that the history required for this is only based on the most recent changes. Thus, not the full development history is taken into account but the latest case of the claim and how long the claim has already been in this stage. By considering this, longer development periods related to annuity payments as well as the prevention of consecutive larger jumps can be captured.

[3]It was tested on the German MTPL dataset with 20 and 30 similar claims.

The sequence of cases for all claims in the dataset are then used to set up the transition matrices by considering the current case and how long the claims' development has been in this stage. Mathematically, this transition history TH is calculated as follows for all claims $l = 1, ..., n^{\star}$:

$$TH_{\theta_i^l \to \theta_{i+1}^l,\ \sum_{j=1}^{i} \mathbb{1}_{\left(\theta_i^l = \theta_{i-j+1}^l\right)} \cdot \prod_{k=1}^{j} \mathbb{1}_{\left(\theta_i^l = \theta_{i-k+1}^l\right)}}. \tag{4.15}$$

This shows the transition from case θ_i^l to case θ_{i+1}^l and how long the claims' development has been in the stage θ_i^l. The sum is running over all previous cases and is counting if a historical case is equal to the current one. Due to the product, this is only summed up if there has not been a different case in between so far. So the second index number of TH is stating the number of previous development steps in which the development of the claim has been in the same case without any interruption. Applied to the sequence of cases shown in 4.14, this results in the following transition history:

$$\begin{aligned}
&(TH_{4\to3,1}, TH_{3\to7,1}, TH_{7\to7,1}, TH_{7\to5,2}, TH_{5\to5,1}, TH_{5\to7,2}, TH_{7\to5,1}, TH_{5\to4,1},\\
&TH_{4\to10,1}, TH_{10\to7,1}, TH_{7\to7,1}, TH_{7\to7,2}, TH_{7\to7,3}, TH_{7\to7,4}, TH_{7\to2,5}, TH_{2\to7,1},\\
&TH_{7\to2,1}, TH_{2\to3,1}, TH_{3\to9,1}, TH_{9\to9,1}).
\end{aligned}$$

A possible transition matrix can then be derived based on the time a claim has already been in its case. Therefore, the transition is separated into the different times a claim has been in its case defined by the second index of TH. The current case is then defining the column and the consecutive case is defining the row for the transition matrix. Thus, the number of transitions can be counted and the respective probabilities in form of the relative frequency can be estimated based on the time horizon k, the current case i, and the consecutive case j where $\cdot$ means any case:

$$TP_{i,j}^{k} = \frac{\#TH_{i\to j,k}}{\#TH_{i\to\cdot,k}}.$$

The transition probabilities TP are estimated if the sum of the counted values for a column is above a capping threshold $\#TH_{i\to\cdot,k} \geq CT$ to ensure that the probabilities are meaningful and not only based on a few transitions. Since this also depends on the markets and the available number of single claims in the market data, this is also a parameter which has to be set by the responsible actuary. In Figure 4.2. two capped transition matrices are shown as examples for the German MTPL market with a capping threshold of $CT = 30$.

Based on the capping threshold, Figure 4.2a shows the transition probabilities for being in a respective case for one year related to $TH_{\cdot,1}$, while Figure 4.2b shows this for two

		Starting Position									
	Cases	1	2	3	4	5	6	7	8	9	10
Jumps to	1	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
	2	0,00%	43,11%	18,45%	4,87%	6,78%	6,15%	11,71%	6,74%	19,88%	15,86%
	3	0,00%	7,08%	7,77%	5,62%	9,21%	5,05%	5,84%	3,20%	9,55%	4,84%
	4	0,00%	0,82%	0,55%	0,00%	2,13%	2,20%	2,15%	1,94%	1,08%	2,96%
	5	0,00%	4,54%	15,26%	15,36%	28,44%	14,51%	8,19%	4,71%	12,63%	10,35%
	6	0,00%	0,45%	1,11%	3,00%	2,83%	8,79%	1,57%	3,28%	0,77%	2,55%
	7	0,00%	30,67%	44,24%	53,56%	36,23%	34,95%	54,27%	49,49%	36,52%	36,29%
	8	0,00%	2,54%	1,11%	4,87%	2,33%	13,19%	6,52%	18,27%	3,08%	5,65%
	9	0,00%	7,26%	7,63%	5,24%	8,60%	8,13%	4,79%	2,95%	11,86%	9,01%
	10	0,00%	3,54%	3,88%	7,49%	3,44%	7,03%	4,96%	9,43%	4,62%	12,50%
	Total	0%	100%	100%	100%	100%	100%	100%	100%	100%	100%

(a) Transition Probabilities for Being in a Case for One Year.

		Starting Position									
	Cases	1	2	3	4	5	6	7	8	9	10
Jumps to	1	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
	2	0,00%	58,91%	15,22%	0,00%	4,44%	2,63%	9,84%	4,85%	16,88%	0,00%
	3	0,00%	6,03%	8,70%	0,00%	7,66%	2,63%	5,39%	2,43%	9,09%	0,00%
	4	0,00%	1,10%	2,17%	0,00%	0,81%	5,26%	2,49%	4,85%	1,30%	0,00%
	5	0,00%	3,01%	26,09%	0,00%	50,40%	21,05%	4,92%	5,83%	14,29%	0,00%
	6	0,00%	0,27%	0,00%	0,00%	3,23%	0,00%	1,01%	3,40%	0,00%	0,00%
	7	0,00%	22,74%	28,26%	0,00%	22,58%	28,95%	65,50%	54,37%	31,17%	0,00%
	8	0,00%	1,64%	4,35%	0,00%	0,40%	10,53%	4,25%	10,68%	1,30%	0,00%
	9	0,00%	3,84%	15,22%	0,00%	8,06%	7,89%	3,17%	4,37%	22,08%	0,00%
	10	0,00%	2,47%	0,00%	0,00%	2,42%	21,05%	3,44%	9,22%	3,90%	0,00%
	Total	0%	100%	100%	0%	100%	100%	100%	100%	100%	0%

(b) Transition Probabilities for Being in a Case for Two Consecutive Years.

Table 4.2.: Transition Probabilities.

consecutive years related to $TH_{.,2}$. For those transition probabilities, it can be seen that there is a high probability that the next development step will be in the second or seventh case, which is related to the annuity nature of many claims in this market. This is even more pronounced for Figure 4.2b. It can also be seen in Figure 4.2a that the probability of having two consecutive larger outstanding increases is zero referring to case four. While it can already be seen that there are no transition probabilities given for the first, fourth, and tenth case in Subfigure 4.2b, this also appears for the other cases with an increasing number of consecutive years.

Since these transition matrices are based on the developments of the single claims, they cannot be applied if a claim had no development yet. Therefore, also the entry probabilities have to be estimated by considering the first cases θ_1^l of all claims $l = 1, \ldots, n^\star$. Then the entry probabilities are given by the relative frequency of the resulting vector.

4.3.3. Application and Adjustments

The transition and entry probabilities can be used to guide the development of a single claim. Based on the latest transition history TH of a claim, the next case and development area can be drawn. If the claim has no transition history so far, the first case and development area is drawn from the entry probabilities instead. Due to the application of the cutting threshold N, it can also appear that the required transition probabilities are zero since they set to zero in the required transition matrix. In those cases, the previous transition matrices with less consecutive years are checked until the transition probabilities are not zero any more for the required initial case.

Since the transition matrices are covering the market behaviours and guide the claims' development path, this does also give the possibility to incorporate expert opinion from an actuary or other market participants. Hereby, a credibility approach could be chosen by creating an own transition matrix based on the expert opinion and weighting both accordingly. This would allow to, e.g. increase the probability of shifts towards the cases $6, 8$ or 10 to force the closing of claims or towards the cases 9 and 10 to increase the positive run-off of claims. However, a specification and further investigation into this topic is not the focus of this thesis. Nevertheless, it is advised to investigate this topic in further research.

4.4. Development Steps of RBNS Claims

After defining the basic framework in form of the point cloud on which the clustering is applied, the development step can be determined. Therefore, a LDF for the incurred and payment ratio values can be drawn from the copula. Under the restriction of a maximum LDF for the incurred values and depending on the previous development of the claim, a next development step can be estimated. Overall, the following steps are performed for one development step which are also shown in Figure 4.9:

1. The related cluster is determined based on the latest incurred and payment ratio value.
2. The cluster directly leads to the required copula. A further distinction can either be done according to the goodness of fit test for the copulas or by an initial choice of the actuary for the grid sizes and copula type.
3. The maximal possible LDF is determined from the LDF surface resulting in a limit for the maximal incurred value.
4. Based on the latest transition history of the claim, the related transition matrix and the transition probabilities are chosen.
5. The next development step is drawn from the copula [2.] under the restrictions given by the LDF surface [3.] and the development area drawn from the transition matrix [4.].

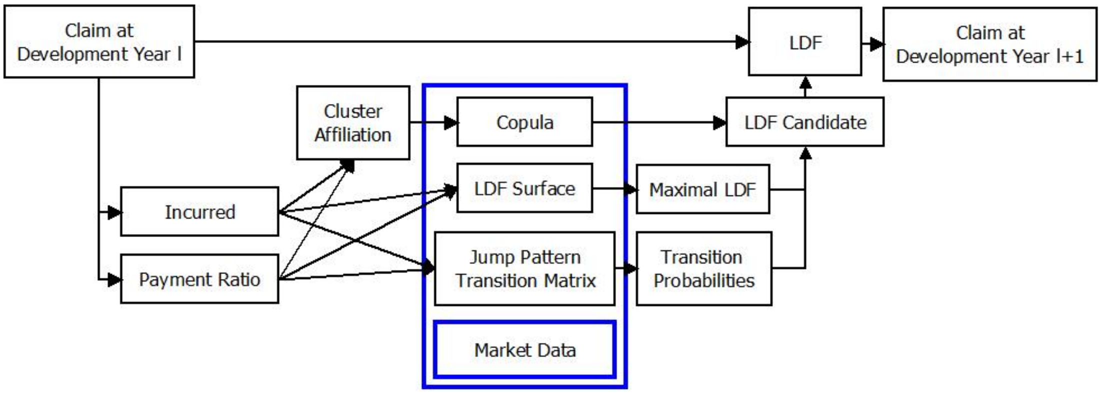


Figure 4.9.: Flowchart of a Development Step.

This can be applied sequentially to a single claim until it is closed to obtain the related ultimate value. This is the case if the payment ratio of the next development step is equal or greater than 1. In those cases, a linear function through both incurred and payment ratio values is used to determine the incurred value for a payment ratio of 1. Due to the stochastic nature of the simulation, the estimation of the ultimate value is done J times following a Monte Carlo simulation to obtain values $\hat{Ult}^{l}_{i,j}$ for $i = 1, ..., n$, $j = 1, ..., k_i$ for $l = 1, ..., J$ simulations. It has to be noted here that the development of a single claim is not necessarily restricted to the end of the claims triangle since there is no restriction with respect to the time component within this framework. The related algorithm behind a single development step is also discussed in Algorithm 1 on page 222 in Appendix C.

The current RBNS development does not consider the usage of extreme LDFs as required in Section 3.1. However, such a feature could be added any time. Therefore, large and unique historical claims can be analysed for extreme motor scenarios. After a projection of the occurred claim sizes to the present state of compensation, frequency and severity distributions could be used to model this feature. Besides that, there are also other ways of modelling this but the focus will be set on the basic model in a first step.

4.5. Modelling of IBNYR Claims

The current modelling of the claims focuses on the development of the RBNS claims. Appart from that, the IBNYR claims have to be modelled as well. The main reason for that is a possible comparison with the aggregated pricing methods stated in Section 2.8 since those methods include the development of IBNYR claims indirectly while a single loss development model is modelling each aspect explicitly.

In the case of a reinsurer, IBNYR claims are usually known to the primary insurer and are below the cedant specific reporting threshold M_i^Z explained in Section 2.3.

During the timelag between the occurrence date and the settlement of those claims, their incurred value could increase so that the total claims' amount is exceeding the reporting threshold or even the priority of the excess of loss contract. Thus, a reinsurer is confronted with those claims many years later when the excess point is overtaken[4]. Hence, those claims stretch far into the future [240].

Since a simulation of the IBNYR claims should be based on the concept of the development of the RBNS claims and the framework presented here is a first approach, a basic IBNYR model is set up. Therefore, the exposure based frequency-severity approach by Schlemmer [198] is chosen and discussed briefly. It is based on the available empirical data and does not use any distributional assumptions about the claims. Therefore, the expected number of IBNYR claims is predicted for each accident year. To estimate the ultimate value for the additional claims, those claims are multiplied by the average claim size derived for the RBNS claims. Hence, the IBNYR claims are not considered for the distribution of the ultimate claims so far.

Therefore, the priority $D_Z \in \mathbb{R}^+$ of the company Z, which should be priced, is taken. All claims where the indexed incurred value exceeds the priority for the first time are counted as an IBNYR claim and are aggregated towards their related accident years $i = 1,...,n$ for each development year $l = 2,...,n$ in order to capture the related reporting delays:

$$IBNYR^{\#}_{i,l} = \sum_{j=1}^{k_j} \mathbb{1}_{(Inc_{i,j,l} \geq D_Z)} \cdot \mathbb{1}_{(Inc_{i,j,l-1} < D_Z)} \cdot \mathbb{1}_{\left(\sum\limits_{l^\star=1}^{l} \mathbb{1}_{(Inc_{i,j,l^\star} \geq D_Z)} \cdot \mathbb{1}_{(Inc_{i,j,l^\star-1} < D_Z)} = 1\right)}.$$

While the first two indicator functions are checking if the claim exceeds the priority for the related development year, the last indicator function is needed to ensure that only the first appearance is counted. Since the number of claims per accident year depends on the underlying exposure value, e.g. a higher number of vehicles indicate a higher number of IBNYR claims compared to a company with a smaller number of vehicles, the exposure value also needs to be taken into account. Hence, the average number of IBNYR claims per exposure over the accident years is estimated per development year $l = 2,...,n$:

$$\overline{IBNYR_l} = \frac{1}{n} \sum_{i=1}^{n} \frac{IBNYR^{\#}_{i,l}}{E_i}.$$

The expected number of IBNYR claims per accident year can then be estimated by multiplying the exposure value per accident year E_i with the related cumulated average number of IBNYR claims per development year $\overline{IBNYR_l}$:

[4] Depending on the markets, those time delays can be 10 or more years for MTPL claims.

$$IBNYR_i^{\#} = E_i \cdot \sum_{l=n-i+2}^{n} \overline{IBNYR_l}.$$

This results in the number of IBNYR claims that are still expected for each accident year $i = 2, ..., n$. Following this approach, there are less IBNYR claims expected for older accident years, which is in line with market observations. Multiplying the expected number of future IBNYR claims with the average claim amounts can be used for the ultimate claims part of the IBNYR claims.

However, backtests done on real data have shown that this procedure overestimates the number of IBNYR claims since not all claims exceeding the priority once are also above the priority when the claim is settled. Thus, the positive run-off of claims is not taken into account so far. This occurs since the number of IBNYR claims is not developed by itself but uses the average claims' ultimate value in the end. Hence, an adjustment for the run-off period has to be applied. Therefore, the latest incurred value of the RBNS claims is compared to their simulated ultimate value to check whether the claim dropped below the priority. Hereby, also older accident years can be taken into account to have a larger database:

$$CAR_i = \frac{\sum_{l=1}^{i} \sum_{j=1}^{k_l} \mathbb{1}_{(\hat{Ult}_{i,j} \leq Prio)} \cdot \mathbb{1}_{(Inc_{i,j,n-i+1} \geq Prio)}}{\sum_{l=1}^{i} \sum_{j=1}^{k_l} \mathbb{1}_{(Inc_{i,j,n-i+1} \geq Prio)}}.$$

Following this, an adjustment rate per accident year can be estimated to reduce the number of IBNYR claims per accident year for the expected run-off of claims in the related market. However, there might also be RBNS claims that are below the priority for their latest incurred value and that can have an ultimate value above the priority. While this development is not covered in the current IBNYR model, checking on market development patterns for run-offs of claims has shown that this can only occur for the minority of claims and that the trend is usually towards a positive run-off. Thus, those cases are omitted here since their impact is considered negligible.

Overall, the number of IBNYR claims is obtained by multiplying the number of IBNYR claims per accident year $IBNYR_i^{\#}$ with the related adjustment rate CAR_i.

4.6. Excess of Loss (XL) Pricing Features

For the pricing on an XL treaty, not only the unstabilised ultimate values are required but the stabilised ultimate loss for the layer. Thus, further aspects are briefly discussed for completeness. These aspects are not necessarily related to the copula-based SLD model presented in this thesis but apply to all pricing methods used for the pricing of XL contracts in general.

4.6.1. Tail Factor and Development

Current aggregated methods use a tail factor which is multiplied to the latest developed column of the projected aggregated triangle to take the tail of the development into account. This is done since not all claims are necessarily settled at the end of the projected triangle. Thus, there is still some uncertainty related to the ultimate value of those claims. The copula-based SLD model allows a projection until all claims are settled but this feature has to be treated carefully. Since the ultimate value is not known for all claims due to the long tail nature of those claims, the current information used is biased, e.g. it is not known for claims that already have 30 years of development, for which ultimate value they will finally settle. Thus, only the settlement for shorter development periods are known.

Hence, plausibility tests should be done according to the lifetime of the claims and the ultimate claim size. Hereby, the lifetime of the claim comes into place since it is unreasonable to simulate a MTPL claim for e.g. over hundred years. Thus, this has to be in line with the experience of the claims department and market observations. However, a distribution of the lifetime of MTPL claims is not available since this highly depends on the peculiarities of the different primary insurers. Additionally, it is hard to judge how adequate such a projection to ultimate is since the current information does not allow for a comparison with real data due to the long run-off periods.

4.6.2. Stabilisation

Since MTPL claims have a long lifetime and are considered to be a long tail branch, the impact of time dependent factors like future inflation have a large impact on XL contracts. Comparing the effect for primary insurers and reinsurers, it is well known that the reinsurer is hit overproportionately by the effect of inflation as shown in Section 2.4.2.1. Therefore, stabilisation clauses are set up that manage the allocation of the inflational impact between the participants. Usually, three different types of stabilisation exist from which one is chosen for each XL contract [185]. Let I_B be the index at the basis year and I_l the index in the l-th development year. The following stabilisation clauses are common in order to derive the adjustment factors f_l^{stab} for each development year [185].

Therefore, the first case 'FIC' is the full index clause where the change of the indices is applied directly without any condition. For the second case 'franchise δ_1', franchise with a margin δ_1, the change of the index is just applied if it exceeds the threshold

FIC	franchise δ_1	SIC δ_2
$f_l^{stab} = \frac{I_B}{I_l}$	$f_l^{stab} = \begin{cases} 1 & \text{,if } \frac{I_l}{I_b} < 1.1 \\ \frac{I_B}{I_l} & \text{,if } \frac{I_l}{I_b} \geq 1.1 \end{cases}$	$f_l^{stab} = \begin{cases} 1 & \text{,if } \frac{I_l}{I_b} < 1.3 \\ \frac{I_B \cdot 1.3}{I_l} & \text{,if } \frac{I_l}{I_b} \geq 1.3 \end{cases}$

Table 4.3.: Forms of Stabilisation Clauses with a Margin of $\delta_1 = 10\%$ and $\delta_2 = 30\%$.

δ_1. The third case is the severe inflation clause with a margin of δ_2, short 'SIC δ_2'. The index is applied if δ_2 is exceeded. In this case, only the additional difference is considered. Thus, the last two cases follow the idea that only in a high-inflation regime the clause becomes operational [185]. Hereby, the choice of the stabilisation clause depends on the respective country and cedant. Those adjustment factors can then be used to stabilise the priority and the limit of a layer. Therefore, stabilisation factors $F^{stab}_{i,j,l}$, $i = 1,...,n$, $j = 1,...,k_i$, $l = 2,...,n$ as quotient of the claim expenses and the inflation adjusted claim expenses can be derived and multiplied by the priority D and limit L in order to derive the stabilised priority $D^{stab}_{i,j,l}$ and limit $L^{stab}_{i,j,l}$:

$$D^{stab}_{i,j,l} = F^{stab}_{i,j,l} \cdot D,\ L^{stab}_{i,j,l} = F^{stab}_{i,j,l} \cdot L, \qquad F^{stab}_{i,j,l} = \frac{Inc_{i,j,l}}{f_1 Paid_{i,j,l} + ... + f_l(Paid_{i,j,l} + Out_{i,j,l})}.$$

It has to be noted here that the priority and limit are stabilised differently for each single claim, which might lead to confusions since the stabilisation will be different each time. Thus, it is also possible to leave the priority and limit untouched and apply the changes to the claims' value instead. However, this is not discussed further here. If the claim is settled, the incurred value used is equal to the ultimate value.

In this thesis, the stabilisation is not applied since it hinders the comparison to commonly used aggregated models. The stabilisation is usually based on single claims' basis while additional assumptions and workarounds have to be done in order to apply this for aggregated claims as well. Since these workarounds are not published and each company might do this differently, it is left out in this thesis.

4.6.3. Layering

The amount a reinsurer has to pay according to an XL structure can be obtained by applying the layering L xs D as stated in Section 1.2 to the ultimate values of each claim $\hat{Ult}^{l,XL}_{i,j}$ for each simulation output $l = 1,...,J$. If the stabilisation is considered, the stabilised priority $D^{stab}_{i,j,l}$ and limit $L^{stab}_{i,j,l}$ have to be applied here:

$$\hat{Ult}^{l,XL}_{i,j} = min\left(L, max\left(\hat{Ult}^{l}_{i,j} - D, 0\right)\right).$$

This gives the part of each single claim which has to be paid by the reinsurer. In the case of currently used aggregated models, the layering is usually applied before the aggregation of the single claims and only the layer damage of the portfolio is then projected to ultimate.

4.6.4. Ultimate Loss Distribution of Single Claims

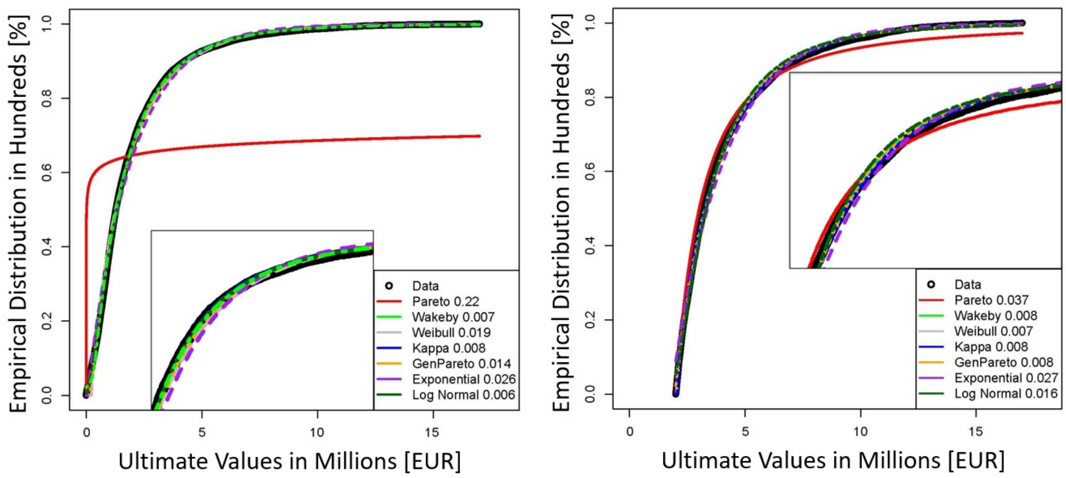
One of the key requirements is the estimation of an ultimate loss distribution for single claims. Therefore, all ultimate claim values $\hat{Ult}^{l}_{i,j}$ over each simulation $l = 1, ..., J$ with or without applied stabilisation are used for the fitting. Here, it is also possible to use the related incurred value at the end of the claims triangle $Inc_{i,j,n}$ to apply a fixed tail factor. Depending on the data and the purpose of the model, e.g. pricing, reserving, or market models, a distribution can be fitted from ground up or only for the layer part. Hereby, the ultimate claim values $\hat{Ult}^{l}_{i,j}$ are realisations of a random variable $\mathscr{X}_{i,j} \sim \mathbf{G}_{\mathscr{X}}$ for the ultimate claim size of a single claims. Such a distribution can then be used further by replacing current distributional assumptions about the claim size of ultimate single claims.

As distributions, commonly used severity distributions as well as more flexible distributions with respect to their shape and location parameters are considered based on best practice:

- Pareto: This distribution is a commonly known severity distribution used for simulation of long tail losses due to its heavy tail.
- Generalized Pareto: This distribution is more flexible compared to the Pareto distribution and is also known as a severity distribution.
- Exponential: This distribution is chosen because it is easy to handle and shows a distribution function typical for severity claims.
- Log-Normal: Is also often used to simulate claims but lags a heavy tail.
- Weibull: Similar to the exponential distribution, it is flexible and also known as a distribution to simulate claim sizes [133].
- Wakeby: This distribution has five parameters with three shape parameters and two location parameters. Thus, it is flexible.
- Kappa: As four parameter distribution with one location parameter, one scale parameter and two shape parameters, it is also flexible and includes many special cases as the generalized logistic, gen Pareto, logistic, gumbel, exponential and uniform distribution [106, 260].

Due to the large amount of parameters for the Wakeby and Kappa distribution, it is expected that those distributions have the best fit to the empirical distribution of the single claim ultimates. Nevertheless, the behaviour and meanings of the several parameters are not explained easily, e.g. for the pareto distribution with one parameter the effect of this can easily be explained. Thus, in practice it is usually required to have more intuitive and widely known distributions. Usually, the pareto and the lognormal distribution are particularly suited to modeling the severity distribution of the ultimate according to Mahon [157]. The goodness of the distributions is estimated by calculating the average residual distance between the empirical and the fitted distribution. The fitting of the parameters is done using the L-moments method [107].

Applying this to the ultimates obtained for the German MTPL market leads to the fits shown in Figure 4.10. For the simulation, 20 splits between training $(3/4)$ and test $(1/4)$ data have been performed and each claim is simulated $J = 50$ times for each split for demonstration purpose. The resulting ultimate values $\hat{Ult}^{l}_{i,j}$ are then taken for the distribution fits for $l = 1, \ldots, J,\ i = 1, \ldots, n,\ j = 1, \ldots, k_i$. Besides that, the average residual distance is stated next to the distributions' name. For more information see Chapter 5.4.3.1.



(a) Simulated Claims from Ground up.

(b) Simulated Claims Exceeding EUR 2 Million.

Figure 4.10.: Distribution Fits for the Ultimate Losses with Residual Distances.

In Figure 4.10a. no threshold is chosen for the distribution fit. It can be seen that a fit of the Pareto distribution is not sufficient in this case. For the other distributions, the fits of the Kappa, Wakeby, LogNormal, and GenPareto are the best, being close to each other with average residual distances between 0.006 and 0.014. The fits of the Weibull and Exponential distributions are a bit worse while the Pareto distribution does not have the correct shape for the data. In the case of a dataset with a threshold of EUR 2 million[5], the fit of the Pareto distribution is better than before. However, the

[5] This is similar to the fits considering a threshold of EUR 1 million and EUR 5 million.

shape of the data is still not fitted correctly. The best distribution fits are obtained by the Wakeby, Kappa, Weibull, and GenPareto distribution due to their high flexibility related to the number of shape parameters with residual distances of 0.007 and 0.008. This is followed by the Exponential and LogNormal distribution. The Exponential and Pareto distributions are the worst fits here due to their inflexibility related to their parameters. According to the distribution fits seen here, the Weibull distribution, which is widely known as claims' severity distribution, as well as the GenPareto distribution are good choices. However, those fits are only trustworthy if the model itself works in a sufficient way which is investigated further in Chapter 5.

4.7. Key Figures

Many key figures such as the claim frequency of ultimate claim values are usually given on an accident year basis. In order to compare the results of the copula-based SLD model with the results of commonly aggregated models, the ultimate values $\hat{Ult}_{i,j}^{l}$ have to be aggregated according to their accident year. Therefore, only the average over all simulations J is used to create the aggregated figures:

$$\hat{Ult}_i^{Agg} = \sum_{j=1}^{k_i} \frac{1}{J} \sum_{l=1}^{J} \hat{Ult}_{i,j}^{l}. \tag{4.16}$$

4.7.1. Ultimate Values and Claim Frequencies

For the pricing of a reinsurance XL treaty, the ultimate values used in Formular 4.16 are usually stabilised and adjusted for the Layer L xs D. Thus, $\hat{Ult}_{i,j}^{l}$ is replaced by $\hat{Ult}_{i,j}^{l,XL}$ for the individual stabilised layer. The resulting layer damage per single claim can then be aggregated for the accident years. Divided by the number of claims per accident year, an average layer damage per year can be estimated. This value is further needed to calculate the layer damage of the IBNYR claims. Therefore, the focus can be set on the claim frequencies.

The total claim frequency is the sum of the claim frequency from the RBNS and IBNYR claims. The first one is obtained by determining how many of the ultimate values exceeded the stabilised priority of the XL contract for each accident year. However, those frequencies are linked to the respective exposure value of the related accident year. Since the next year is of interest for a pricing, this frequency is extrapolated with respect to the expected exposure value. The obtained as-if frequency is then the frequency of accident year multiplied with the quotient of the expected exposure value and the exposure value of accident year i. In case of the IBNYR claims, the frequency per accident year is already given as stated in Section 4.5. Since these frequencies are also related to the exposure value of the related accident year, the same as-if calculation as already done for the RBNS claims has to be done.

Comprehensively, the frequency is determined as the as-if frequency of the RBNS claims lying above the priority of the XL contract plus the frequency derived from the IBNYR model. The total expected layer loss is the sum of the layer losses from the RBNS claims and the IBNYR claims, derived by multiplying the average RBNS layer loss with the expected number of IBNYR claims.

Those figures in addition to the ultimate individual loss distribution can be used further in order to derive the risk premium by assuming a collective model utilizing Wald's equation, the capital requirements, calculate non working capacity, or pricing different layer structures. Let therefore $\mathbb{E}(\mathscr{N})$ be the expected claim frequency and $\mathbb{E}(\mathscr{X}_{i,j})$, $\mathscr{X}_{i,j} > 0$ with $i = 1,...,n$, $j = 1,...,k_i$ the expected claim size of a single claim. Then, it can be assumed that the claim size is following the individual loss distribution derived from the copula-based SLD model $\mathscr{X}_{i,j} \sim \mathbf{G}_{\mathscr{X}}$ with density $\mathbf{g}_{\mathscr{X}}$. Hereby, either the empirical distribution or a fitted distribution as done in Section 4.6.4 can be utilized. The expected layer damage for any layer is then given by:

$$\mathbb{E}(\mathscr{X}) = \int_{D}^{D+L} (x-D)\,\mathbf{g}_{\mathscr{X}}(x)\,dx + \int_{D+L}^{\infty} L\,\mathbf{g}_{\mathscr{X}}(x)\,dx.$$

At this point, an assumption for the respective distribution is usually needed, e.g. the individual losses are pareto distributed. Due to the derived individual loss distribution this is not the case any more, which is the main advantage of a SLD model. Additionally, this can be used to estimate the claim frequency based on different thresholds. The expected claim frequency $\mathbb{E}(\mathscr{N})$ can be calculated from a given threshold ξ^{old} for a different threshold ξ^{new} with w.l.o.g. $\xi^{new} \geq \xi^{old}$, according to:

$$\mathbb{E}(\mathscr{N}(\xi^{new})) = \mathbb{E}(\mathscr{N}(\xi^{old})) \cdot P(\mathscr{X} > \xi^{new}) = \mathbb{E}(\mathscr{N}(\xi^{old})) \cdot (1 - P(\mathscr{X} \leq \xi^{new})).$$

4.7.2. Aggregated Models

Since the ultimate values and the frequencies are derived in a different way for the aggregated models, this is discussed briefly. The claim amounts are obtained due to the aggregated models. However, if the layer structure changes or there is a non-working capacity, a claim size distribution has to be assumed and fitted accordingly. The claim frequency can be estimated by applying the Chain Ladder or Cape Cod method to a frequency triangle followed by an as-if calculation for the expected exposure value.

4.7.3. Mean Squared Error of Prediction

In addition to the other key figures derived above, the respective error terms are also necessary in order to determine the goodness of the model. This derives from the fact that the ultimate amount of claims not yet settled is subject to the outcome of events that have not yet occurred. Additionally, the methods applied correspond to the best practices related to the available data. The error terms have to be determined for the estimated ultimates per accident year $\hat{Ult}_i^{Agg}$. Therefore, the mean squared error of prediction (MSEP), which is also called the prediction error, can be decomposed as follows according to Murphy [162]:

$$\begin{aligned} MSEP(\hat{Ult}_i^{Agg}) &= \mathbb{E}\left(\hat{Ult}_i^{Agg} - Ult_i^{Agg}\right)^2 \\ &= \underbrace{Var\left(Ult_i^{Agg}\right)}_{\text{process risk}} + \underbrace{\underbrace{Var\left(\hat{Ult}_i^{Agg}\right)}_{\text{parameter risk}} + \underbrace{\left(\mathbb{E}\left(\hat{Ult}_i^{Agg}\right) - \mathbb{E}\left(Ult_i^{Agg}\right)\right)^2}_{\text{squared bias of the estimator}}}_{\text{estimation error}}. \end{aligned}$$

Sometimes, the MSEP is also stated as the conditional MSEP based on the specific dataset that was used for the prediction in order to give the average deviation between $\hat{Ult}_i^{Agg}$ and Ult_i^{Agg} due to future randomness only [149]. Overall, the MSEP can be decomposed into the parameter risk, the process risk and the squared bias of the estimator. The parameter risk derives from the fact that the true parameters of the underlying stochastic model are unknown and need to be estimated from the observed data [33, 154, 162]. In combination with the squared bias of the estimator, this is also called the estimation error which equals the parameter risk for unbiased estimators [162]. Additionally, the process error determines the random error of how the ultimate value will deviate from the predicted value.

Since no explicit assumptions are done about the variance or shape of the random variable Ult_i^{Agg}, it is difficult to evaluate the results in terms of an unbiased estimator and variance as well as a significant impact of random noises [46]. The current model of the copula-based SLD model gives no closed form of the estimators which could be used here instead. Thus, the MSEP cannot be stated on the current basis.

However, it is possible to estimate the bias using plug-in estimators and bootstrapping for Monte Carlo simulations [68, 124] which is only one part of the MSEP. Nevertheless, the MSEP can be estimated when a backtest is performed on historically known data, where the ultimate values are already known. However, this will not be done in the later evaluation chapter since the data basis available for this would be too small, which might lead to meaningless results.

4.8. Assumptions and Parameters

One requirement stated in Section 3.1 is to use realistic assumptions that are not too far away from real observations. In order to conclude this, the current assumptions are briefly summarized as follows:

Homogeneity: It is assumed that the used market dataset is approximately homogeneous after applying the indexation. See Assumption 2.1 on page 39.

Accident Year: It is assumed that the claims' development of a single claim is independent of the related accident year. See Assumption 3.1 on page 50.

Individual Claims: It is assumed that the development for individual claims among themselves is independent. See Assumption 3.2 on page 51.

Claim Size: It is assumed that the claims' development depends on the claim size. See Assumption 3.3 on page 52.

Monotonicity: It is assumed that the cumulated paid values are monotonously increasing. See Assumption 3.4 on page 52.

Payment Ratio: It is assumed that the LDF depends on the development stage of the claim. See Assumption 3.5 on page 53.

Time Dependency: It is assumed that the incurred and payment ratio values without the consideration of the development year already consider the time component indirectly. See Assumption 3.6 on page 55.

Uncorrelated: It is assumed that the incurred and payment ratio values are uncorrelated or just correlated weakly. See Assumption 3.7 on page 60.

Cluster Similarity: It is assumed that the development of claims lying in the same cluster is similar. See Assumption 4.1 on page 77.

The most crucial ones are the assumption of a homogeneous market dataset, which is one key element of the used framework. The second one is the time dependency stating that the time component in form of the development year is indirectly considered within the incurred and payment ratio value. The third and last crucial assumption is the cluster similarity saying that the claims' development of claims in the same cluster is similar. This last one is a simplification since it does not necessarily distinguish the claims according to their medical status or between annuities and lump sums.

In terms of the parameters, the clustering does not require any parameter input since the optimal number of clusters can be chosen automatically. Based on the CLARA algorithm the cluster centres are also estimated automatically as part of this unsupervised learning algorithm. This is similar for the estimation of the copulas. Here, the required parameter is the size of the mesh grid for which no optimal solution exists. As it is discussed in Section 4.1.3, it is recommended to leave the choice between 10 and 20. Since the Cramer-von-Mises test statistic allows to choose the best copula fit, this does not require any interference. However, depending on the number of single claims, the actuary might want to choose between the Bernstein and grid-type copula.

Related to the first condition, the LDF surface, the maximal values for the incurred and payment ratios have to be predefined. While the payment ratio has its maximum at 1, this is not easily done for the incurred values since the current model sets a LDF of 1 for all incurred values above the predefined maximum. Additionally, the step size to create the mesh has to be defined as well. In case of the ultimate distribution fit, the residual distance ε^{fit} has to be defined. Here, a value of 0.001 is chosen since it provides a good trade-off between algorithm runtime and accuracy.

The jump pattern as second condition requires to set the classification thresholds ω_i, $i = 1,\ldots,6$ to distinguish the jumps for the different cases. Additionally, the capping threshold CT has to be set up. This is required to ensure that enough jumps are available when determining the relative frequencies. Since this value is related to the number of available claims in the market dataset, no recommendation can be given here.

4.9. Summary

Based on the results of the clustering, the LDFs for each cluster are used to set up a non-parametric copula, which is the basis of the further development of open claims. The drawn development steps for the incurred and payment ratio values are conditioned by two elements. First, the maximum LDF is restricted by the LDF surface according to the incurred value and the payment ratio of the corresponding claims. Therefore, a smooth surface using qualitatively constrained smoothing B-Splines is fitted to the maximum values of the used point cloud. Secondly, the historical development of the claim is taken into account for the next development step. Hereby, the development depends on the current status of the claim and how long the claim has already been in that status. Depending on similar cases observed in the data, the claims' development is guided. Combining this leads to the development of RBNS claims while a basic method is used to incorporate the IBNYR claims. The chapter concludes with further related model elements necessary for the pricing of XL contracts, commonly used key figures, and a summary of the used assumptions and parameters.

In summary, a flowchart showing the main aspects of the copula-based SLD model is shown in Figure C.1 in Appendix C and the following steps are performed for one development step as stated in Section 4.4:

1. Based on the latest incurred and payment ratio value, the related cluster is determined.

2. The cluster directly leads to the required copula. A further distinction can either be done according to the goodness of fit test for the copulas or by an initial choice of the actuary for the grid sizes and copula type.
3. The maximal possible LDF is determined from the LDF surface resulting in a limit for the maximal incurred value.
4. Based on the latest transition history of the claim, the related transition matrix and the transition probabilities are chosen.
5. The next development step is drawn from the copula [2.] under the restrictions given by the LDF surface [3.] and the development area drawn from the transition matrix [4.].

5. General Model Evaluation

The next two chapters focus on the evaluation of the model related to the requirements stated in Section 1.3.1 and Section 3.1. As already stated before, the 'correctness' of a model cannot be proven in principle [26] but several falsification and plausibility checks can be done for validation. Hereby, the focus will be set on the comparison with a-posteriori observations as well as with currently used aggregated models, which is further discussed in Section 5.3. Furthermore, logical test are conducted to compare the model results with the expected ones.

Apart from that, the results in this and the following chapter should be treated with caution since they are subject to influences from different markets and only show some empirical evaluation. Thus, this can only be used to get an intuition about the impact of the several parameters influencing the claims' development.

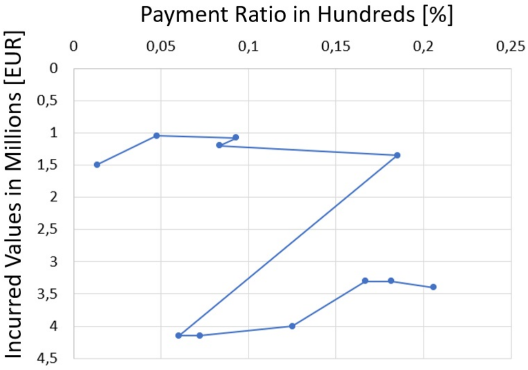
5.1. Impact Analysis of Different Claim Parameters

Since the copula-based SLD model considers several components for the projection like the most recent incurred value and payment ratio in addition to the so far known history of the claim, the impact of those claim input parameters on the ultimate values can be tested. Therefore, the following claim situation examples are evaluated. The German MTPL dataset is chosen as underlying market information, which is further discussed in Section 5.4.1 and 5.4.2. This is done since the following tests are all based on the same model framework and are all subject to the same market idiosyncrasies. Thus, the comparability remains.

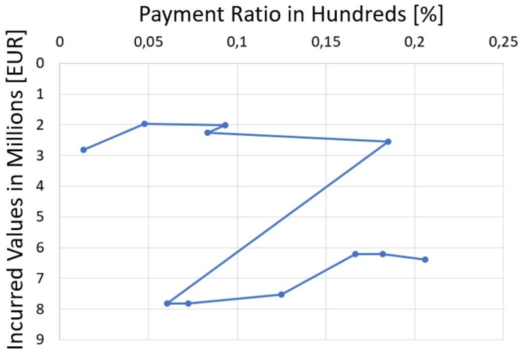
For the test of the claims input parameters, multiple single claims are created based on a claim path seen in the market dataset[1] and are visualised in Figure 5.1. Hereby, every point is a single development step if and only if the incurred or payment ratio value has changed between two development years. This basic claim is then adjusted as follows:

1. The reference claim used for comparison is fictive. Only the course of the claim, shown in Subfigure 5.1a, is typical for a German MTPL claim with a pension share. Five years after its occurrence, it becomes clear whether a rehabilitation is successful or not. In this case, it is not which results in an increase of the

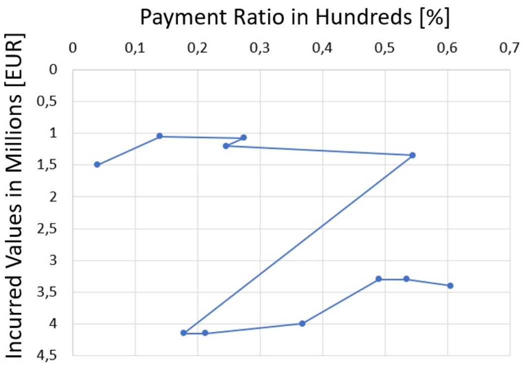
[1] The exact claim sizes are not used for data privacy reasons.



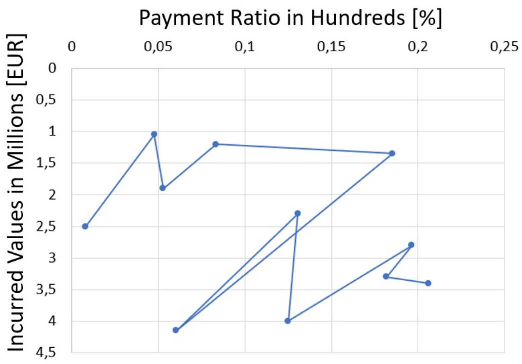
(a) Claim 1.



(b) Claim 2.



(c) Claim 3.



(d) Claim 6.

Figure 5.1.: Several Claim Path Adjustments.

outstanding reserve by the primary insurer. Afterwards, the claim shows a small positive run-off.

2. The same claim as above is chosen but the latest incurred value is increased by EUR 3 million. The split between the latest paid and outstanding values is done so that the payment ratio does not change compared to the reference claim. The history of the claim is then recalculated using the LDFs from the reference claim in order to have the same jump pattern history. The claim's path is shown in Subfigure 5.1b.

3. For the third claim, the latest payment ratio is increased by 40% points. Therefore, the split of paid and outstanding values is recalculated while leaving the incurred value unchanged. The claim's history is again recalculated using the LDFs from the reference claim. The claim's path is shown in Subfigure 5.1c.

4. Starting here, all changes affect the history of the claim and can be considered as extreme scenarios. Therefore, the current course of the claims are changed leaving the latest claim's value, e.g. the latest paid and outstanding value, unchanged. However, the claim paths for claim 4 and 5 are not shown since a visualisation would not add any information. In this case, the claim is known since the first year and remains unchanged until now. Thus, it has a stable development.

5. The second case is that the claim is considered to be an IBNYR claim with a reporting delay of 10 years. Thus, the incurred value is known for the most recent year and is zero all years before.

6. For the last case, the history of the claim is changed so that there are many jumps. These jumps are chosen randomly without any initial intention. The claim is shown in Figure 5.1d.

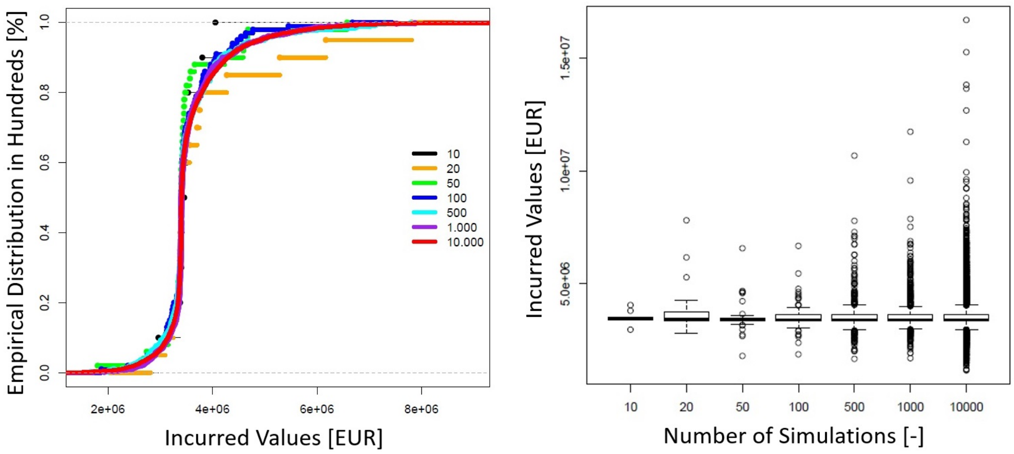
For the comparison of the different effects, the ultimates in form of their distribution and related boxplots are used. Hereby, the boxplot is showing the median, the 25% and 75% quantiles, the whiskers defined as the 1.5-times interquartile distance, and the resulting outliers. Additionally, the densities of the claims' lifetime are compared. Since a smoothing kernel is used for the creation of a smooth density function[2], it appears that probability is given to development years below the 10th one. For those cases, it is referred to the related boxplots and the outliers or whiskers for smaller claim lifetimes. In the case of the second claim with an increased incurred value, the ultimate values have to be subtracted by EUR 3 million in order to make it comparable to the other test cases again. For these comparisons, the simulation with the highest number of simulations is always considered as reference distribution since it approximates the 'true' distribution most accurately based on Monte Carlo simulations.

5.1.1. Change of the Number of Simulations

First, the stability of the simulated claims' ultimate value and lifetime are analysed with respect to the number of simulations. Therefore, the first claim is taken and simulated 10, 20, 50, 100, 500, $1{,}000$ and $10{,}000$ times. Hereby it has to be noted that the number of simulations is equivalent to the number of available points that can be used to fit an ultimate loss distribution. However, this analysis can only be regarded as an indication since different market idiosyncrasies can impact the stability of the ultimate loss distribution. The changes of the different simulated empirical distributions are analysed in Figure 5.2.

Besides the simulated empirical distribution in Subfigure 5.2a, the boxplots of the simulated ultimate claims are shown in Subfigure 5.2b. For the comparison, the simulated empirical distribution with $10{,}000$ simulations is used as reference distribution. In relation to that, the empirical distributions with 10 and 20 simulation runs differ a lot from the reference distribution. While the distribution with 10 simulations is underestimating the tail, the distribution with 20 simulations is overestimating it in the range between 3.5 million and 8 million. Both distributions do not have a similar shape compared to the reference distribution. This improves when the simulation is done with

[2]Here it is referred to the R function 'density' in the R package 'stats' version 4.0.3.

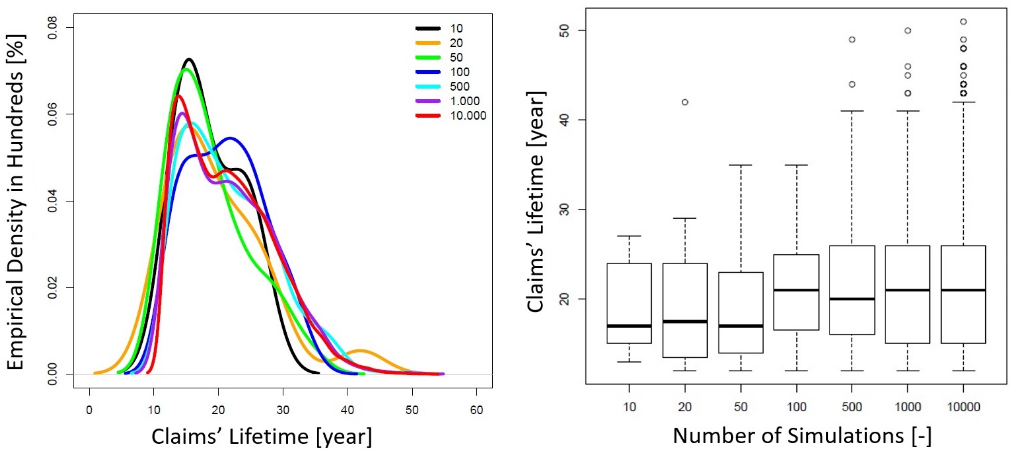


(a) Empirical Distributions. (b) Boxplots.

Figure 5.2.: Ultimate Distribution for Different Number of Simulations.

50 or 100 simulations. While the distribution with 50 simulations still contains breaks, the distribution for 100 simulations looks smoother. However, both distributions still underestimate the tail of the distribution. The distributions for 500 and $1,000$ simulations are more similar to the reference distribution. While there are slight differences, the overall shape does not change significantly by increasing the simulations. These observations are supported by the boxplots as well. The underestimation of the tail can be seen in the smaller volatility for the simulation runs up to 100. The median and interquartile ranges also vary between these simulation runs. Following the observations done for the empirical distribution for the simulation runs above 100, the boxplots for 500 up to $1,000$ simulations look more similar. However, the reference distribution still shows a higher volatility with respect to the outliers for the lower and upper tail.

Besides analysing the changes of the empirical distribution, the simulated lifetimes also vary between the different number of simulations as shown in Figure 5.3.



(a) Empirical Densities. (b) Boxplots.

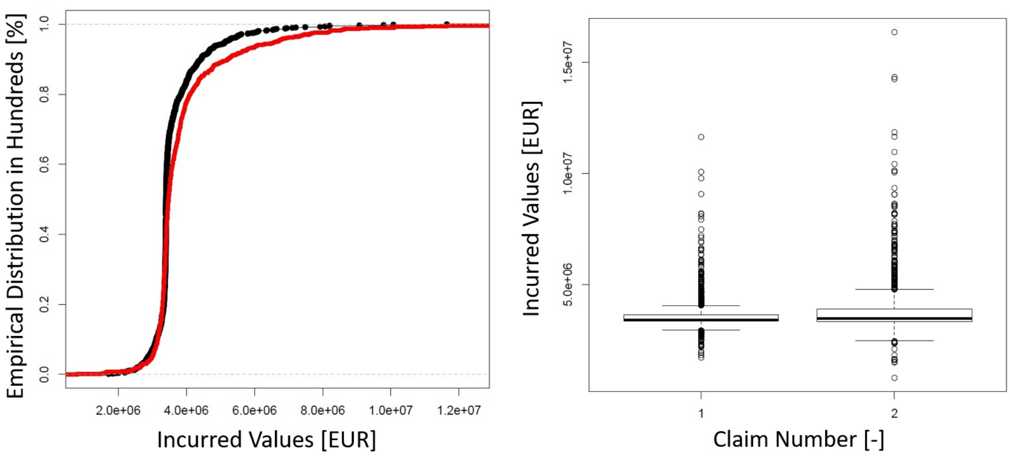
Figure 5.3.: Claims' Lifetime for Different Number of Simulations.

As done for the previous comparison of the ultimate distributions, the density with 10,000 simulations is used as reference here. The densities in Subfigure 5.3a for a smaller number of simulations up to 100 differ a lot and are volatile with respect to the simulation runs. The density for 100 simulations, in particular, follows a different shape. However, with 500 and more simulations the shape is more similar to the reference density. Hereby, the maximum simulated lifetime of the claim also changes significantly across the different number of simulations. This is also shown in the boxplots in Subfigure 5.3b. Hereby, the median is significantly smaller for simulation runs up to 50 compared to the reference distribution. This changes for a higher number of simulation runs starting with 100 and upward. With respect to the whiskers, there are three groups with different volatilities. The first group with 10 and 20 simulations only show a small distance between the whiskers. The second group with 50 and 100 simulations has whiskers going up to 35 years while the last group has whiskers above 40 years.

Consequently, the ultimate loss distribution and claims' lifetime density should be based on at least 500 values in order to get an appropriate fit. With that in mind, the following analysis is done using 1,000 simulations.

5.1.2. Change of Incurred Value

Secondly, the impact of a changed incurred value is analysed. Therefore, the reference claim 1 and the adjusted claim 2 from Section 5.1 are compared in Figure 5.4. For a better comparison, the ultimate values of the second claim are subtracted by EUR 3 million which has no further effect otherwise.

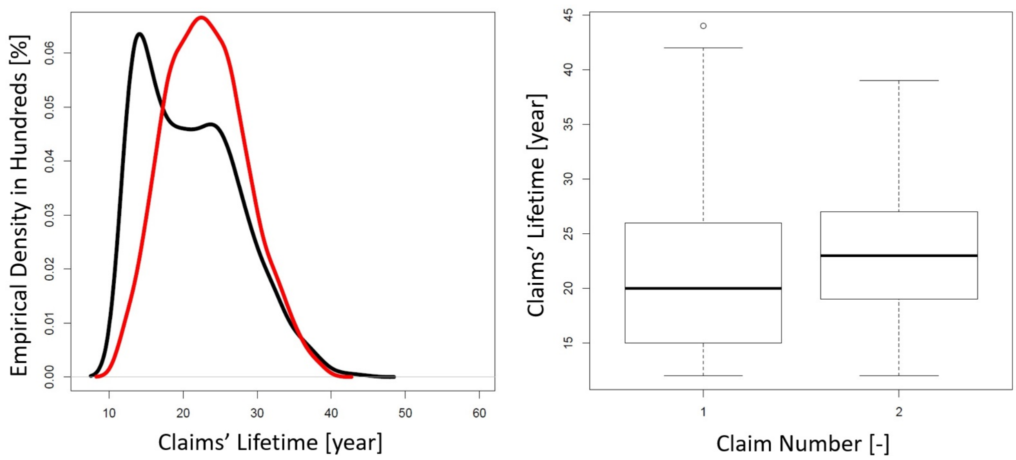


(a) Empirical Distributions.

(b) Boxplots.

Figure 5.4.: Ultimate Distribution for Different Incurred Values Considering the Reference Claim 1 (Black) and the Adjusted Claim 2 (Red).

It can be seen that the tail of the ultimate distribution shown in Figure 5.4a is heavier[3] for the second claim compared to the reference claim. If the incurred value is increased while leaving the payment ratio unchanged, the claim is expected to be a higher risk. During the Monte Carlo simulations, more higher incurred ultimate values are simulated which also follows the initial expectation. This is supported by the boxplots in Figure 5.4b where the interquartile range is larger for the second claim with an increased initial incurred value. While the median is slightly higher, the whiskers of the second claim are also larger. Together with the wider range of the outliers a higher volatility is observed. Since the payment ratio for the second claim is not changed, the higher outstanding value can also result in a larger positive or negative run-off. This also follows market observations where a higher volatility of such claims is expected. Conclusively, a claim with an increased incurred value shows a slightly higher median for the ultimate claim size, more volatility, and a heavier tail for the ultimate distribution which is in line with the expectations. Since the claims can be simulated until settlement, the lifetime of both claims is also compared in Figure 5.5.



(a) Empirical Densities. (b) Boxplots.

Figure 5.5.: Claims' Lifetime for Different Incurred Values Considering the Reference Claim 1 (Black) and the Adjusted Claim 2 (Red).

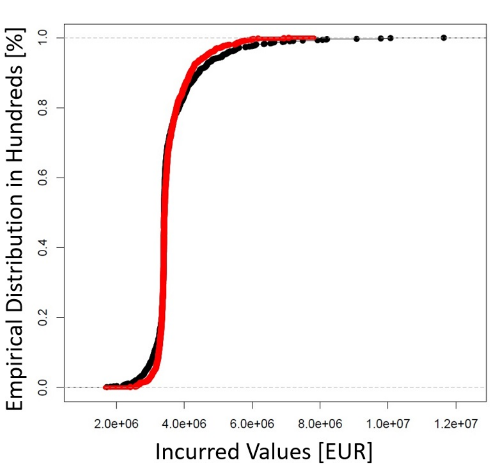
The empirical density of the lifetimes in Figure 5.5a is shifted towards a higher median for the second claim with an increased incurred value. The variance is also smaller for the second claim and more pronounced for the 22nd year. The longest lifetimes are nearly the same while the reference claim is slightly higher. On top of that, the reference claim shows a second local maxima in its density distribution. This occurs because the claim can either have a fast settlement leading to the global maxima of the claim or it can have a longer settlement leading to the second local maxima. In terms of the copula-based SLD model, it is not clear in which direction the reference claim finally develops. However, it is more clear for the second claim since it already has

[3] A heavy tail is characterised by a larger probability that is left for the tail of the distribution.

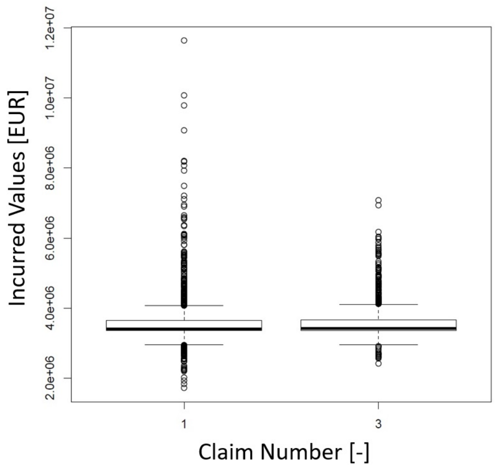
a higher incurred value which implies annuity payments leading to the higher average runtime.

5.1.3. Change of Payment Ratio

The impact of an increased payment ratio is evaluated next. Therefore, the reference claim 1 and the adjusted claim 3 with the increased payment ratio from Section 5.1. are compared with respect to the simulated ultimate values in Figure 5.6. at first.



(a) Empirical Distributions.



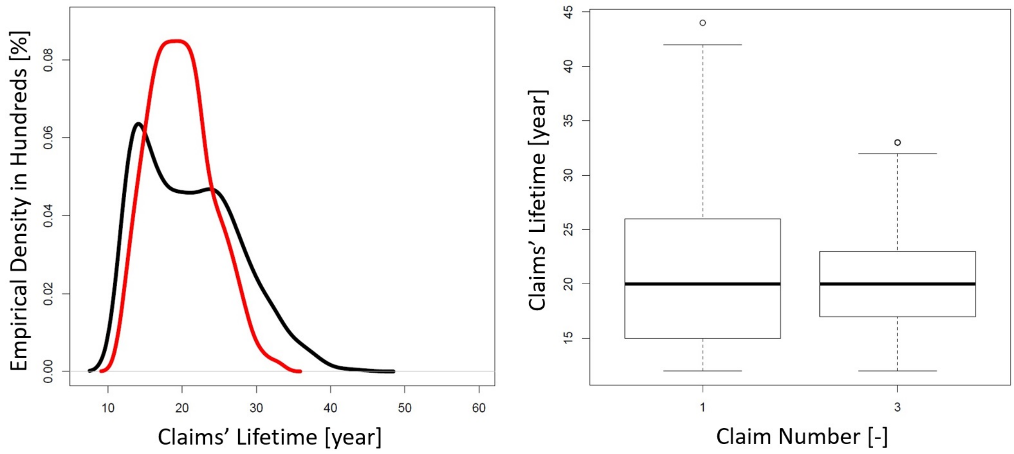
(b) Boxplots.

Figure 5.6.: Ultimate Distribution for Different Payment Ratios Considering the Reference Claim 1 (Black) and the Adjusted Claim 3 (Red).

It can be observed that the distributions show slight deviations compared to an increase of the incurred value. For the ultimate distribution in Figure 5.6a, it can be seen that the upper and lower tails are lighter and that the simulated claim outputs have a smaller variance. This is also supported by the boxplot in Figure 5.6b. Here, the outliers have a shorter range and the volatility is lower while the median and interquartile ranges are similar. Since it is expected that a claim with a higher payment ratio should also have less possibilities for the future development than a claim with a smaller payment ratio, this is in line with the expectation.

With regard to the lifetime of the claim, it is expected that it is settled earlier due to the increased payment ratio analysed in Figure 5.7.

Comparing the densities of the first and third claim, it can be seen that the third claim also has its maximum around the twentieth year, which is similar to the second claim in Figure 5.5. However, the claim is settled slightly faster and shorter lifetimes are simulated in comparison to the first and second claim. This behaviour is expected since a claim with a higher payment ratio is expected to be settled earlier compared to a claim with a small payment ratio. This is also supported by the boxplots in Figure 5.7b. While the median of the third claim is comparable to the median of the first



(a) Empirical Densities. (b) Boxplots.

Figure 5.7.: Claims' Lifetime for Different Payment Ratios Considering the Reference Claim 1 (Black) and the Adjusted Claim 3 (Red).

claim, the whiskers show a smaller interquartile range leading to a smaller volatility. In conclusion, the claim is settled faster if the claim has a higher payment ratio and the probability of simulating longer lifetimes of the claim is reduced.

5.1.4. Change of Historical Development

Finally, the impact of different claim paths and historic developments is evaluated. The reference claim 1 and the adjusted claims 4 with no development, claim 5 as an IBNYR case as well as claim 6 with a high jump frequency from Section 5.1 are compared with respect to the simulated ultimate values in Figure 5.8. The distributions are not shown here since they are too similar and differences are hard to observe.

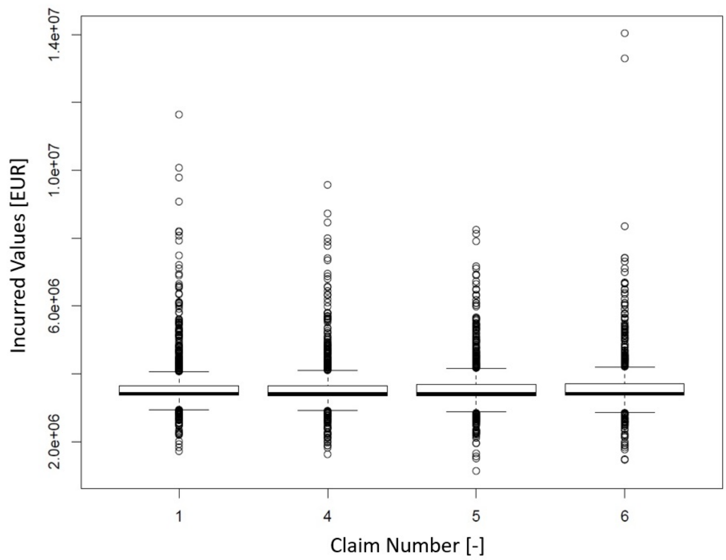
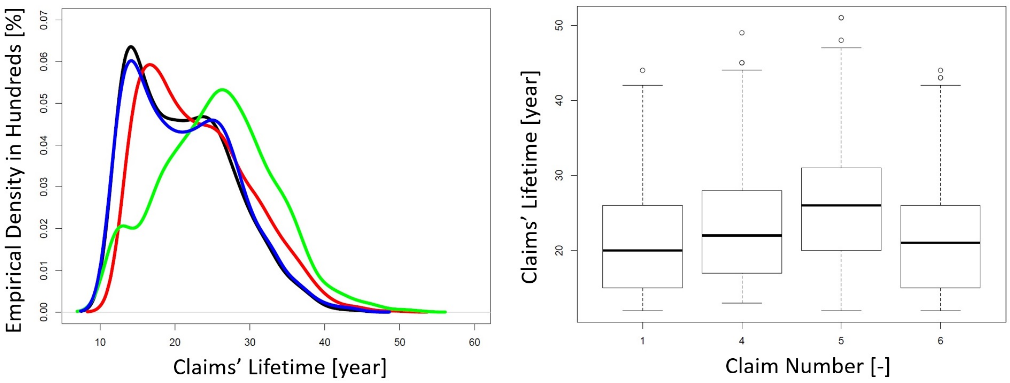


Figure 5.8.: Boxplots for Different Historical Developments.

Compared to the first claim, all other claims show a similar value for the median and a similar range for the whiskers. The only differences occur for the outliers, which show less volatility for the fourth and fifth claim, and for the interquartile distances, which are slightly larger for the fifth and sixth claim. This is expected since the fourth claim has no prior development history which should also result in less development for the further years and, thus, a reduced volatility. The fifth claim is an IBNYR claim, which is developed by using the initial entry probability described in Section 4.3.2. While it shows the smallest volatility with respect to the outliers, it also has a slightly larger interquartile range than the reference claim and the fourth claim. In respect of the claims' development, this IBNYR claim is handled like a claim in the first development year. Due to the used entry probabilities, a higher volatility is expected. However, this is not as significant as expected. The sixth claim with a more volatile claims' history also has a larger interquartile range and shows the largest outlier values. This is expected since the smoothing and stabilising effect of the latest history is missing here. However, the impact of the history is not significant and the differences for the ultimate distributions are minor. This is different when comparing the lifetimes of the claims in Figure 5.9.



(a) Empirical Densities.

(b) Boxplots.

Figure 5.9.: Claims' Lifetime for Different Historical Developments Considering the Reference Claim 1 (Black), and the Claims 4 (Red), 5 (Green), and 6 (Blue) with a Changed Historical Development.

Comparing the densities in Subfigure 5.9a, the first and sixth claim are similar. Both show a second local maxima around the 26th development year and have a similar shape. However, the sixth claim is shifted slightly towards higher claim lifetimes, which can also be observed in the boxplot in Subfigure 5.9b. While the interquartile distance and the whiskers are similar, the median is slightly higher. This is in line with the expectation since only the latest history has an impact on the claims' development. For both claims, this is related to the last development step targeting the fifth case. Thus, the development is similar as well. The fourth claim is similar to the first and sixth insofar that it is shifted. While the others have their first local maxima around the 15th development year, the fourth claim's local maxima is around the 20th development year. This can also be seen in the related boxplot, which is shifted compared to the reference claim. Here, the claim has a constant development and is not expected to close immediately. However, since claims like this have not been observed in the market data so far, only the most recent developments are considered and the development of this claim is delayed. Furthermore, the fourth claim does not show a second local maxima as the reference or the sixth claim. This results from the fact that the jump pattern states the second development case for many years and, thus, a higher probability of having only small developments. Thus, the claim is clearly handled according to annuity payments while this cannot be stated clearly for the reference claim. Consequently, the second maxima does not arise. Apart from that, the tail behaviour of these claims is similar when taking the shift of the fourth claim into account. This includes the maximum simulated lifetimes being around 45 and less than 55 years, which is a realistic range for a long tail market like Germany.

A completely different density is observed for the fifth claim. This claim is considered to be an IBNYR claim that just exceeded the reporting threshold in the most recent year. Since this claim has no development history, it uses the entry probabilities for the first development step. In those cases, many development paths leading to a different density of the lifetime are possible. Considering the boxplot, the median is shifted towards the 26th year while the interquartile ranges are similar to other claims with respect to their length. The upper whisker is also shifted towards longer lifetimes. These shifts are related to the following claim developments. There are two local maxima occurring around the 12th and 28th development year. In the first case, the claim is considered to close soon after opening while the long tail character of the market might lead to an annuity payment in the second case. This would result in a longer lifetime of the claim leading to the second local maxima. Since the jump pattern and history of the claim cannot guide the claim's development, several possible developments are considered here. However, the resulting lifetime of around thirty years is realistic with respect to what has been seen in the data so far and considering that the claim was reported with a delay of ten years.

To summarize these findings, the ultimate values of the claims are similar while the history affects the simulated lifetimes of the claims in particular. This is similar for all claims except for the IBNYR claim, where the largest change can be observed. Due to the missing history, the jump pattern cannot guide the claims' development leading to a wider range of possible lifetimes.

5.2. Simulation Setup and Number of Simulations

The number of Monte Carlo simulations to stabilise the simulated ultimates related to the required runtime is analysed in this section. Since the copula-based SLD model should be applicable and runnable on usual work stations of an employee in a company, the simulations are performed on such a working station with Windows 10, 64 bit System, parallelized on an Intel(R) Core(TM) i7-4765T CPU @ 2.00GHz with 16 GB DDR4 RAM. The reference claim from the previous section was taken for this test.

5.2.1. Accuracy and Runtime Trade Off

In Section 5.1.1, the impact of the number of simulations on the empirical ultimate distribution function of a single claim is analysed together with the impact on the lifetime of such a claim. However, the acquired accuracy of the simulated ultimate distribution has to be seen with respect to the related runtime. Therefore, it has to be kept in mind that only one single claim was developed and tested. The associated runtimes for the different number of simulations are shown in Figure 5.10.

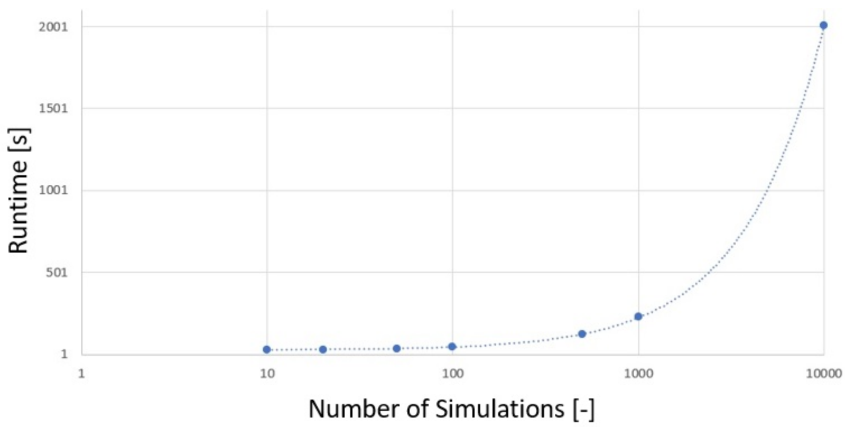


Figure 5.10.: Runtime for the Simulation.

First of all, it can be seen that the runtime follows an exponential function on a logarithmic scale for the number of simulations. This signifies, that the runtime increases linearly with an increasing number of simulations after the simulation process itself is initiated. However, the runtime does not only depend on the number of simulations itself but also on the number of simulated claims in the portfolio, the number of years to be simulated[4], and the lifetime of claims and idiosyncrasies of the market, e.g. claims

[4]The development of a new claim in the first development year takes longer than a claim in the 20th development year.

have to be simulated way longer for a market paying annuities than for a market paying lump sums. Thus, assuming a fixed time frame, the number of possible simulations will differ a lot between different countries and cedants.

Since the main benefit of a SLD model is the resulting ultimate loss distribution based on single claims, at least 500 points should be considered to get a stable ultimate loss distribution as shown in Section 5.1.1. However, this accounts for the ultimate distribution of a single claim as well as for an ultimate loss distribution in general since the fit is based solely on the number of available ultimate values. When simulating a whole portfolio, the overall number of points that can be considered for the ultimate loss distribution fit depends on several other parameters: the number of points in the portfolio, the number of simulation runs for each single claim, and the number of Monte Carlo simulations related to different splits between training and test data[5]. Thus, the overall number of simulations can be changed since a small number of simulations can be sufficient within this constellation in respect of the ultimate loss distribution of a portfolio. However, this does not account for the ultimate loss distribution of each individual claim in this portfolio which can lead to more uncertainty in the ultimate loss distribution fit. With this in mind, the overall runtime for all simulations performed for the market analysis in Section 5.4 and Chapter 6 using 50 simulations for a single claim in each run is around 8 months on the system stated before. Due to the linear runtime increase for a higher number of simulations, this would increase up to 80 month using the minimal required 500 simulations per single claim based on Section 5.1.1. However, it is assumed that the runtime can be improved and optimized for a later application in practice. While this impacts the ultimate loss distribution of each single claim, the main question is whether the fundamental expressiveness of the model is still given if only 50 simulations are performed.

5.2.2. 50 vs. 500 Simulations for the Maltese Market

In order to address this question, the Maltese market is analysed due to the smaller number of single claims in the market and the related runtime. Overall, 318 splits between training $\left(\frac{3}{4}\right)$ and test data $\left(\frac{1}{4}\right)$ are performed for 50 and 500 simulations per single claim and the resulting ultimate values are compared. The splits result in portfolios with around 80 claims, which results in $80 \times 50 \times 318 = 1.272$ million ultimate values in theory, that are considered for an ultimate loss distribution fit. Since the respective accident year is also considered for the split between training and test data, the final number of ultimate value points is lower in practice. In this case, around $300,000$ ultimate values can be used for 50 simulations while around 2.3 million ultimates are available for 500 simulations. While this is a sufficient number of points according

[5] This will be discussed further in Section 5.3.

to Section 5.1.1 in both cases, the individual distribution of each single claim differs here. Moreover, it has to be noted that the following comparison only gives an impression of the impact. Due to the high complexity of the different markets and countries, the results might not be the same for other countries and portfolios. For the market characteristics, it is referred to the later Section 6.1.

First, the ultimate loss distributions as stated in Section 4.6.4 are compared. However, no graphic is included here since the difference between both is not significant and negligible. While the number of simulations has an impact on the shape of the ultimate distribution of each single claim as shown in Section 5.1.1, no significant difference or shape can be observed for the ultimate distribution of the portfolio based on 50 or 500 simulations per single claim. Thus, the average ultimate value per accident year as stated in Section 4.7 and the related boxplots are compared in Figure 5.11.

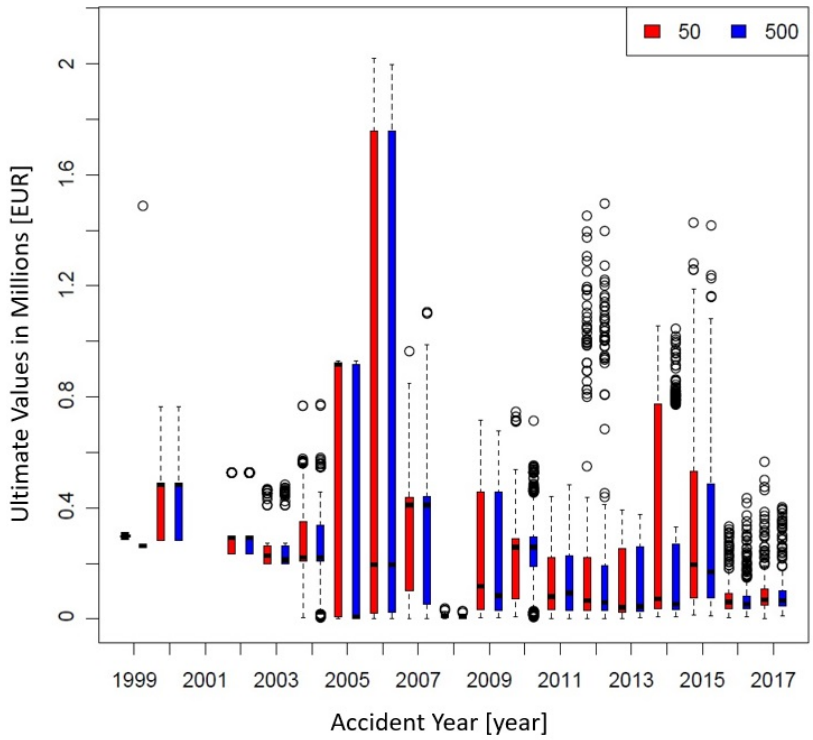


Figure 5.11.: Boxplot of the Average Ultimates for 50 and 500 Simulations.

With respect to the average severities per accident year, it can be seen that the median is similar for both while the whiskers and outliers are slightly different as can be seen in 2004 and 2014. However, the main tendency of the model does not change significantly. Thus, the results with 50 simulation runs per single claim already provide valuable information about the performance of the model so that the main expressiveness is retained. Since the Maltese market has a sparse market data pool, which restricts the splitting between training and test data, it is expected that the performance will be better for other markets with a larger market dataset.

5.3. Evaluation Methods and Data Basis

Four different methods to assess and evaluate the goodness of a model were briefly recapped in Section 1.3.1. While a comparison with experiments and theoretical plausibility checks are not possible, a comparison with a-posteriori observations as well as a comparison with other commonly used models can be done. The first method of using a-posteriori observations is usually called 'backtesting' in the insurance industry and is explained in more detail in the following section. Apart from that, the cross check with other models is also a scientific standard and a good approach in practice [76]. Nevertheless, a comparison will only be done here with commonly used aggregated models since no published and acknowledged SLD model exists that is used for reinsurance pricings as known to the author.

5.3.1. Data Basis and Reinsurance Markets

The copula-based SLD model is tested exemplarily on different markets. The chosen markets are Germany, Malta, Italy, Sweden, and Denmark. These markets have different characteristics in respect of the claim developments and influential effects as discussed in Section 2.4. Furthermore, there is a sufficient market dataset for which the new model is tested. However, this can only indicate the goodness and applicability.

The used MTPL data and claim information is internal real MTPL data from a leading reinsurance company. For the indexation of those countries, the CPI [261] and an additional loading for superimposed inflation is considered for simplicity reasons[6]. The available claims data used is described in Section 2.3 and follows the available information described in Table 2.1 on page 24. Additional information about the used datasets is stated in the related sections.

For the testing, the available market data is split into a training and test dataset for each simulation so that the overall class distribution, related to the distribution of single claims across the accident years, is preserved[7]. In each simulation, all single claims in the training dataset are used for the model calibration, which is applied to the test dataset afterwards where 50 simulations are performed for each single claim and which is used for a later evaluation. Hereby, $\frac{3}{4}$ of the market dataset is considered to be training data while $\frac{1}{4}$ is used for testing[8]. The reason to split the data like this is that the model is tested on data it has never seen before. By repeating this multiple times in a Monte Carlo simulation, the goodness of the model for this specific market dataset

[6] The actual index used is not stated further due to confidentiality conditions by the company providing the data.

[7] For more information see the R package 'caret', version 6.0.80.

[8] See also Section 4.6.4. Note here that a split of $\frac{2}{3}$ to $\frac{1}{3}$ or $\frac{4}{5}$ to $\frac{1}{5}$ are also used commonly in data science.

can be approximated. Hereby, the number of Monte Carlo simulations is related to the number of single claims in the test dataset and the 50 simulations per each single claim and, thus, depends on the related market.

For each of the countries, the related market characteristics are stated briefly using the information of Axco Insurance Information Services [8] as main source together with information provided by local authorities.

5.3.2. Evaluation Method

In scientific literature, the standard is to compare the results of pricing and reserving methods with the results obtained by the Chain Ladder model[9]. This is the case because these models aim to calculate the overall reserve for which the Chain Ladder model is considered as standard. Hereby, a commonly used primary insurance dataset is given by Taylor and Ashe [229] or by Antonio and Plat [4, 179]. There are also other papers using real datasets or purely analytical analysis [273]. However, the kind of data used is usually related to primary insurance data and the Chain Ladder method is usually considered as benchmark. Even if the considered data is reinsurance data, the comparison is based on the projection of the claims [64] into the future. However, considering the Chain Ladder model or a projection as benchmark provides no information about the expressiveness of these methods towards real occurred claims as targeted by a backtest.

Since the SLD model is targeting the ultimate loss distribution in the case of reinsurance pricing, the scientific standard in respect of evaluation method is considered insufficient. Hence, a comparison with commonly used aggregated methods is done in Section 5.3.4 in the pricing procedure but expanded for a backtest to compare the results with actually occurred values in the backtest procedure in Section 5.3.3.

5.3.3. Backtest Procedure

The backtest procedure aims at comparing the projections of claims done by the copula-based SLD model with the claim values that occurred originally. Since the copula-based SLD model utilizes the history of each claim, a sequential addition of this information is of interest as well.

Hereby, the first dataset contains all claims in the test dataset that have a positive incurred value in the first and second development year. For each of the projected development years, a comparison of the claim size distributions for the projected and the

[9]Examples are [28, 57, 149, 190, 203, 229, 253].

originally occurred values is done and the related residual distance is calculated. After this comparison, all claims for which at least the first three development years are known are considered and the same procedure is repeated. This is done sequentially for all $n-1$ development years so that for the last sequence $n-1$ years are known and only the development year n is compared. By doing this, more historical information is added sequentially so that also the impact of the jump pattern can be analysed.

However, it has to be kept in mind that the expressiveness of the backtest procedure worsens for earlier and later development years. In the earlier development years, there are many claims with a short history and in later development years there is a long history but just a few claims. Thus, the comparison lags validity for early development years regarding the claims' history impact and for later development years regarding the number of comparable projected years and the number of claims.

Including the results of commonly used aggregated models here is not possible since this would require a further reduction of the dataset. Hereby, a rectangle has to be constructed within the training data triangle for which a claims triangle could be taken for the comparison. However, this would reduce the data even more leading to a point where the results are not meaningful any more. Furthermore, a comparison with a final ultimate distribution is also not possible since this would require a dataset where all claims are already settled. Due to the long lifetime of annuity claims, no dataset exists that could be used for a comparison. Moreover, using only claims which have already settled would lead to a bias because the ultimate values of longer annuity claims would be missing. Consequently, a comparison with commonly used aggregated models cannot be performed within the framework of a backtesting and requires an extra test procedure, which is stated in the next section.

Overall, this results in a triangle of differences between the average aggregated losses per accident year of the projected and actually occurred claim sizes with respect to the known initial number of development years and the resulting number of development years for the projection. Additionally, a sequence of claim size distributions is obtained for each number of known development years and the related number of projected development years.

5.3.4. Pricing Procedure

In addtion to the backtest procedure described above, another procedure is done to evaluate the goodness of the model. Hereby, the comparison with commonly used aggregated models as described in Section 2.8 is in the focus. Since an application as backtest is not possible here, this procedure predicts the ultimate claims' value in the future, which is also done for pricing purposes. The aggregated methods are applied to the full test dataset while the copula-based SLD model is applied to the single claims instead. After taking the average aggregated values per accident year, both, the results of the aggregated methods and those of the copula-based SLD model can be compared. The related key figures in form of the ultimate claims' amount and the as-if frequency as described in Section 4.7 are can then be compared. Hereby, it has to be kept in mind that currently a simple IBNR model is applied for the copula-based SLD model. The aggregated models that are considered and used as baseline for the comparison are the Chain Ladder model, the Munich Chain Ladder model, as well as the Cape Cod method as discussed in Section 2.8. Hereby, it has to be remembered that the expressiveness of these methods scale with the number of available claims based on the law of large numbers.

Due to the Monte Carlo simulations, these models have to be applied blindly without any manuell adjustment of the Chain Ladder development factors. Thus, the age-to-age development factors and the development of the claims in general are not subject to any smoothing and outlier correction as it is usually done in practice. This has a few consequences for the Chain Ladder and also to its related models, stated by Taylor et.al [227].

- The correct age-to-age development factors fall increasingly short of the historical averages with increasing accident years, and the averages therefore cause a dramatic overestimation of loss reserve for the more recent accident years.
- The incurred Chain Ladder yields negative estimates of loss reserves for the earlier accident years. However, age-to-age development factors in a portfolio tend to be substantially less than unity, and even slight underestimation of them can project the ultimate incurred losses for an old accident year below losses paid to date.

Under these circumstances, the estimates produced by the aggregate models should be treated with caution [227]. This is especially true for the first point mentioned above. Additionally, the Chain Ladder model is applied to the test data triangle without considering the last diagonal. Since the latest diagonal might not be known fully and reported to the reinsurer, this can lead to unnaturally small development factors in the latest diagonal. Thus, the Chain Ladder method is applied twice. Once on the full dataset and the other time on the dataset without the latest diagonal. This is usually not done in scientific literature since the models are usually applied to publicly accessible datasets or to primary insurance data as stated in Section 5.3.2 where this is not an issue. In the case of the copula-based SLD model, the average aggregated ultimate value for each accident year as well as a boxplot of the simulated results for each accident year are shown in the respective evaluation.

Since the main application of the copula-based SLD model would be for the pricing of NP reinsurance contracts, a layering is applied for each country. Firstly, this is done because the claims data for each cedant is only available if claims have exceeded the cedant specific reporting threshold. Doing a pricing with a priority lying below this reporting threshold would lead to an underestimation of the claim frequencies and ultimate values due to the information gap between the primary insurer and the reinsurer. In such a case, it would be assumed that no other claim with a current incurred value between the priority and the reporting threshold exists which is wrong in most cases. When choosing a priority in line with market practice it has to be kept in mind that the aggregated methods are applied without adjusting the underlying LDFs. Thus, an aggregated claims triangle after an application of the priority has to be filled with enough data to ensure that the aggregated model results are stable. Thus, a priority is used for all countries that is slightly above or exactly the reporting threshold. However, it might be the case that the triangle is incomplete and does not contain enough data to allow an application of all aggregated methods. In those cases, this is mentioned in the corresponding analysis and section.

5.4. Example I: German MTPL Market

The first market to be considered is the German MTPL market. This internal real dataset is also used to create the basic framework and to develop the copula-based SLD model in Chapters 3 and 4 as stated in Section 3.3. To recap the market characteristics as stated in Section 3.3, $5,638$ claims in the period from 1988 to 2015 are available, representing a market share of more than 50% of the MTPL market. Before the model can be applied, the data has to be analysed for influential effects due to specific market characteristics targeting the claims' development and behaviour. If not stated differently, the related market information is taken from Axco Insurance Information Services [8].

Additionally, the impact of the different model components is investigated exemplarily for this market. Therefore, the copula-based SLD model is applied in Section 5.4.3. This is followed by applying only the copula in Section 5.4.4, the copula and LDF surface in Section 5.4.5, and the copula and jump pattern in Section 5.4.6. Thus, it is possible to get a grasp of the impact of the different model components.

5.4.1. Market Characteristics

MTPL insurance is one of the compulsory insurance classes in Germany, meaning that every motorised vehicle needs to be insured. There are no tariffs in place and the market is liberalized and in private hands. Overall, the market remains fragmented with 91 insurance companies writing MTPL business in 2019 according to the German Insurance Association [88]. However, the number of insurance companies has decreased from 130 in 1997 to 89 in 2017. The competition for market shares and rates is high [36, 88] and is even more forced due to insurance comparison sites using the increasing price sensitivity [87]. However, this ultimately leads to a good spread of good and bad risks over the different primary insurance companies.

Since Germany is a member of the European Union, there is the need to implement its directives which has been the main driver of development in insurance legislation in recent years. So far, Germany has amended its insurance legislation to introduce all the various and relevant directives [8]. Overall, there is a long tradition of civil litigation and the trust and confidence in the court system is high. Additionally, citizens will use the court system to seek redress for any negligent acts or omissions. However, the costs of going to court are high which may deter claimants from bringing an action to court. This also forces the payments of lump sums as reward in order to avoid court costs and the time consuming trial [8]. It also has to be kept in mind that the claims' development differs for court cases.

The possible compensation for damages is set out in the Civil Code and includes both property damage and personal injury. The compensation for pain and suffering is calculated individually on the severity and duration of pain, the extent of suffering, disfigurement, and mental impairment and is based objectively on medical facts. Even though compensation tables exist, they only provide limited guidance. Additionally, compensation is based on the loss of future income, taking into account re-trainings and the remaining working ability. This type of compensation is usually payed via a pension up to retirement. However, the different parties can agree to settle this due to the payment of a lump sum under respect of a capitalization factor based on the statistical life expectancy and an interest rate. If a case is going through the court system, the majority is settled within four to six years. This might also be a reason why around 90% of the personal injury cases are settled outside of court. Overall, a rough subdivision of the compensation forms results in 60% annuities and 40% lump sums [8]. However, this accounts for all market claims and not for reinsurance claims in particular. Since annuity claims are usually larger, these claims are reported more frequently to the reinsurer, who does not know about the annuity character in most cases.

Over time, severe bodily injury claims have become more expensive due to increasing court awards for pain and suffering and due to rising pensions and health costs. The superimposed inflation in form of increasing costs of advanced medical treatment and compulsory care insurance described in Section 2.4.2 drives the claims' rewards in particular. Additionally, the current low interest rates lead to high present values being subject to the discounting effect. Thus, insurance companies may not be forced to settle these claims by lump sums which leads to a longer run-off. Furthermore, increasing safety measures built into motor cars result in fewer fatal injuries but tend to increase the severe injuries of those that survive accidents [8]. The market developments for the total number of vehicles, accidents, and injuries in Germany according to official statistics since 1991 [131, 213, 214] are shown in Figure 5.12.

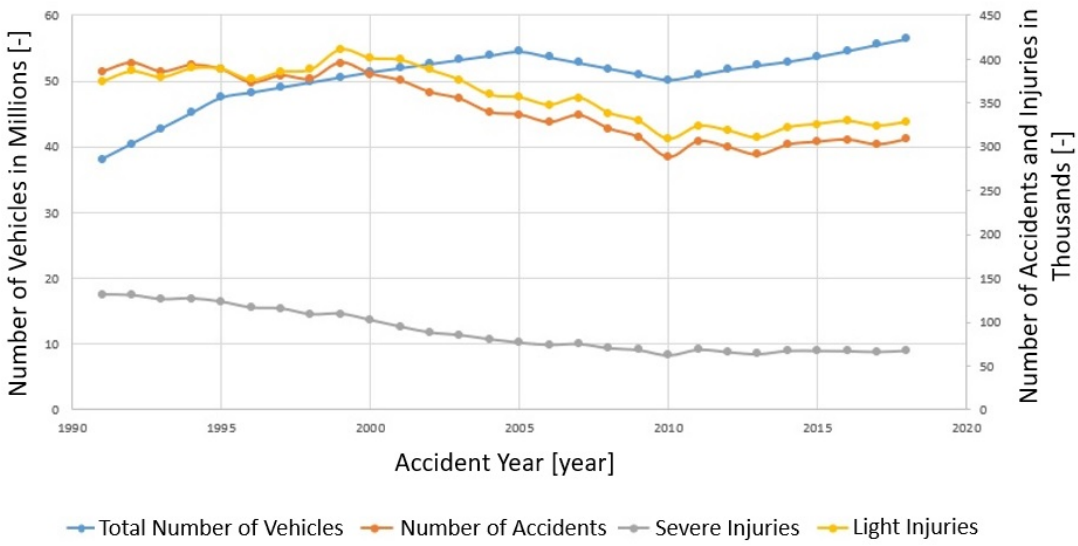


Figure 5.12.: Market Developments for Germany.

The number of light injuries has fallen since 2000 up to 2010 from $401,658$ to $308,550$ and the number of severe injuries has decreased from $131,093$ in 1991 to $70,644$ in 2008. One reason for this might be a law which set the alcohol limit for driving to $0,5‰$ [214]. This has also led to a reduction in loss frequency and, in total, to a reduction of the average size of losses for primary insurers[10] supported by reducing repair costs [8]. However, these reductions have diminished in recent years. For the period between 2005 and 2010, this can be explained partly by a reduction of the number of motorised vehicles, which is a common exposure measure in reinsurance pricing. Since 2010, the number of vehicles is increasing linearly while the number of injured persons, light as well as severe bodily injuries, remains constant. Overall, a reduction of the number of young drivers, a better infrastructure, an overall reduction in annual mileage, more cars per family, and the improved safety of motor vehicles have let to the reductions mentioned above [8].

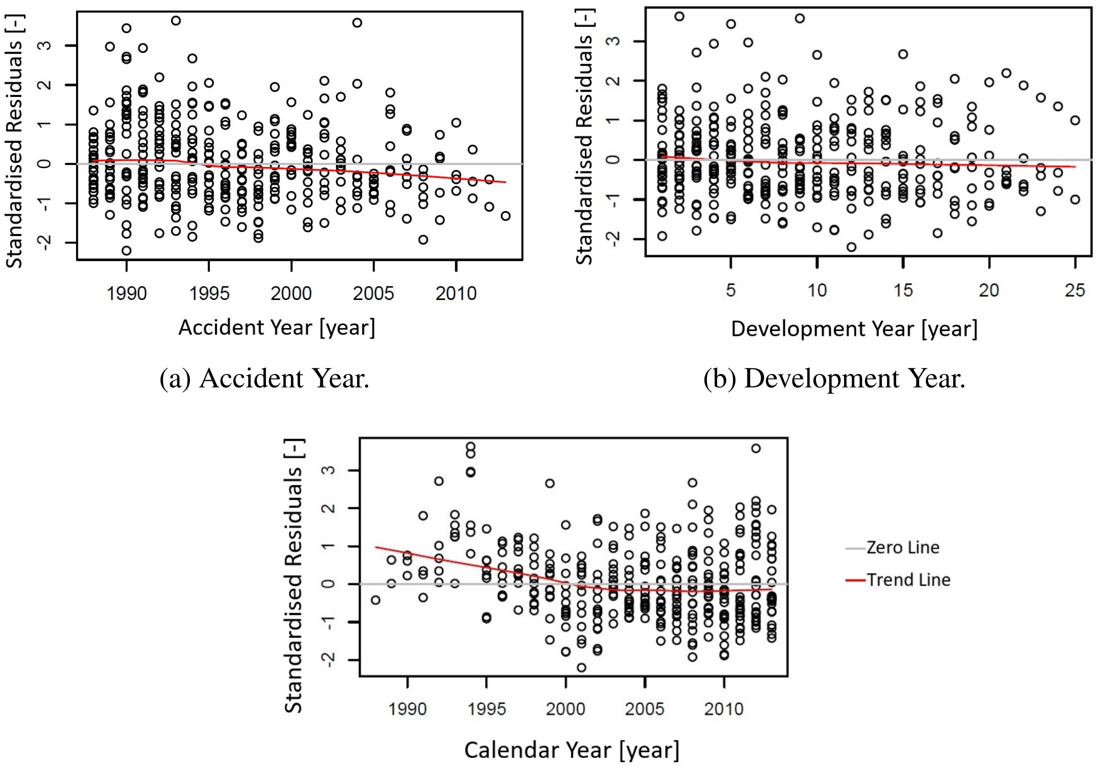
Overall, the market is chosen for the following reasons:

- A lot of data is available and the provided market dataset represents more than 50% of the claims of the total MTPL reinsurance market.
- The MTPL business in Germany with respect to reinsurance MTPL claims is long tail business with mostly annuity payments.
- A long history of claims is available and the overall data quality is consistently good.

[10] Since the XL contracts of reinsurers are only hit by claims above the priority, they are not effected by a reduction of the average size of losses.

5.4.2. Data Analysis, Trends, and Influential Effects

Since influential effects and trends cannot be eliminated from the data on a single loss basis as stated in Section 2.6, these effects should at least be known and explained. Therefore, the standardised residuals of the Chain Ladder method are used as described in Section 2.8.1 and shown in Figure 5.13 for the used MTPL data of the German market. The analysis of the standardised residuals is done with respect to the accident, development and calendar years to identify market changes towards these timelines. However, it has to be kept in mind that this analysis is for the whole market and may not necessarily applicable for each company. It might even happen that a single company shows an opposite trend.



(a) Accident Year.

(b) Development Year.

(c) Calendar Year.

Figure 5.13.: Standardised Residual Plot for the German MTPL Market.

For the accident years in Subfigure 5.13a, the residuals show a slight downwards trend with respect to the more recent years. This is indicated by the red trend line and the shown standardised residuals who have a tendency to be negative for the more recent years. This means that the estimated development factor on a single development step is smaller than the average development factor based on the same development year. Hence, the development factors for more recent accident years are smaller compared to older accident years. It is hard to specify reasons responsible for that. A possible explanation could be a better knowledge of the possible maximum claims' value due to legislative limits and reward practices leading to smaller LDFs for more recent years.

In Figure 5.13b, no tendency or trend can be observed when plotting the standardised residuals against the development years. Thus, this indicates that the overall development of the claims over the different development years is stable. There are still annuities paid and a tendency towards lump sums and an earlier claim settlement cannot be observed for this reinsurance MTPL market dataset.

For the calendar years in Subfigure 5.13c, a decreasing trend until the year 2000 can be observed. Additionally, all residuals in 1994 are positive. This indicates an influential effect for the calendar years. In 1994, the German market was deregulated and there was no supervisory restriction for developing rates and risk factors [231]. Until 1994 the rates were set by the supervisory authority and were increasing steadily, especially from 1991 to 1994. This trend can be seen in the increasing residuals. After the liberalisation, the rates dropped significantly for the next years which can also be seen in the residuals since 1995. Since then, the developments over the calendar years are stable and no clear trend effect can be observed. Thus, for real pricing purpose the years before 1995 should be excluded in order to eliminate this trend effect. However, for the further analysis this effect is kept in mind and the data is used as well. This is done for the following reason. First of all, this calendar year effect is shown by higher individual LDFs for the respective years. Since these are claims in their early development stage, the incurred value as well as the payment ratio tend to be smaller and the development steps higher. Thus, the copula for the first cluster will have more weight on higher developments, the local surface tends to allow higher development factors, and the jump pattern will provide a larger probability for similar jumps in this stage. Overall, the severity of the claims in the first cluster should be slightly higher than without those years. However, the overall impact should be minor since the above described effects are reduced by other claims located in the first cluster. Secondly, by excluding those years, later development stages representing the long tail character of the reinsurance MTPL market as well would not be available for a comparison.

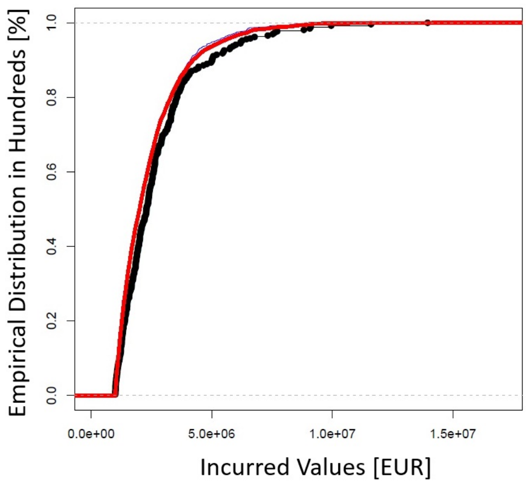
5.4.3. Application of the Full Model

First of all, the full copula-based SLD model with all model components is applied to the dataset. Due to the long runtime of the necessary simulations only 20 splits between training and test data are performed. Each single claim is simulated 50 times and a layer EUR 2m xs EUR 1m is considered for the pricing procedure representing a usual working layer[11] in practice.

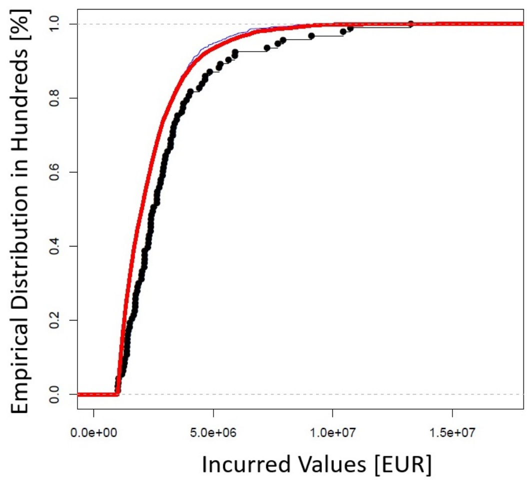
[11] A layer is called a 'working layer' if it is hit by claims frequently.

5.4.3.1. Backtest Procedure Results

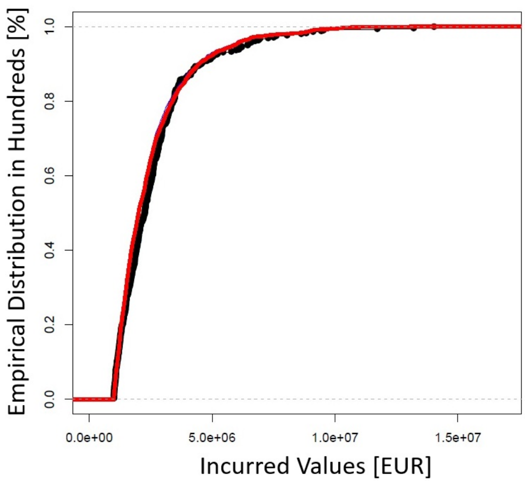
For the backtest procedure explained in Section 5.3.3, the actually occurred latest ultimate distribution (black) and the newly simulated distribution for the ultimates (red) are analysed. Furthermore, the starting distribution of the respective development year is shown as a thin blue line to give an indication about the developments[12]. In Figure 5.14 four of those plots with a different number of known and predicted development years are shown exemplarily for one simulation run in order to give an impression of the observed effects. Hereby, the development year is abbreviated as 'DevYear'. Moreover, the calculated average squared distance between the actual and the simulated ultimate distribution is based on the average distance for all simulations and is shown in Appendix A.11.



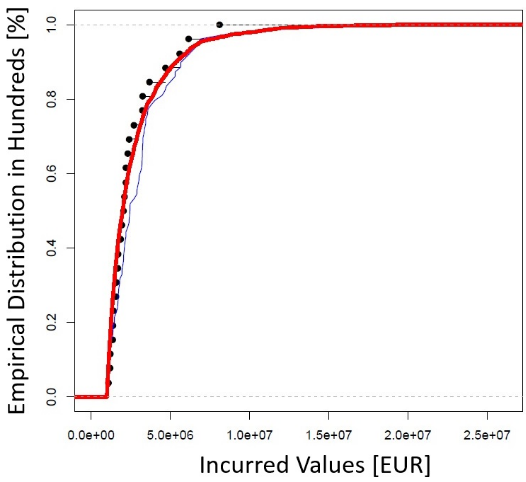
(a) DevYears Known: 2, Predicted: 17.



(b) DevYears Known: 2, Predicted: 22.



(c) DevYears Known: 8, Predicted: 15.



(d) DevYears Known: 23, Predicted: 26.

Figure 5.14.: Severity Distribution Comparison Considering the Full SLD Model.

Focusing on the residual distances shown in Appendix A.11, it can be seen that the simulated distribution fits are close to the actual distribution if at least three known

[12]It has to be noted that the red line is on top of the blue line if the simulation has not changed the loss distribution.

development years are considered and no more than 20 development years are simulated. For less known development years and more simulated development years, the fits are getting worse. The former is due to the effect that the known claims have a short history leading to a worse fit due to the jump pattern selection. For later simulated development years, the number of data is starting to reduce so that the predictive power is decreasing naturally. Additionally, the negative run-off of the long tail MTPL business is coming in place, leading to a larger spread between the compared distributions.

Since the residual distances cannot be used to analyse the shape of the distributions, the exemplary distribution fits in Figure 5.14 can be considered. In Subfigure 5.14a, it can be observed that the simulated ultimate distribution based on two known development years stays close to the other distributions until the 17th development year. Then the actual distribution is getting more severe for claims in excess of EUR 3 million while the simulated distribution stays with the starting distribution. For all following years, the increasing severity of the actual distribution is not modelled correctly as shown in Subfigure 5.14b. The simulated distribution underestimates the actual severity. With more known development years, the deviation between the distributions is getting smaller as shown in the residual matrix. However, the distributions do not show a strong movement and are relatively constant over the development years. This can be seen in Subfigure 5.14c. Here, the 15th development year was predicted with eight already known development years. In these seven years of development, the claims' distribution only changed slightly. Furthermore, the positive run-off that can be observed in Subfigure 5.14d is not simulated correctly for these three further years of development. Besides that, the number of available actually occurred claims is getting sparse so that the black distribution should be considered carefully. Nevertheless, the simulated distribution is less severe compared to the starting distribution.

Overall, it can be summarized that the simulated distribution tends to stay close to the starting distribution. The actual distribution only changes slightly until the 17th to 19th development year. Then, the distribution becomes more severe and the deviation to the simulated distribution increased. This is also supported by the smaller number of available claims for the actual distribution fit, which is good if three development years are already known. However, the negative run-off is not modelled correctly, especially, for later development years.

5.4.3.2. Pricing Procedure Results

Besides the Backtest procedure, a comparison with traditionally used aggregated models is performed as described in Section 5.3.4. The main focus is set on the ultimate claim values per accident year and the related frequency. The comparison of the ultimate aggregated claim values per accident year are shown in Figure 5.15.

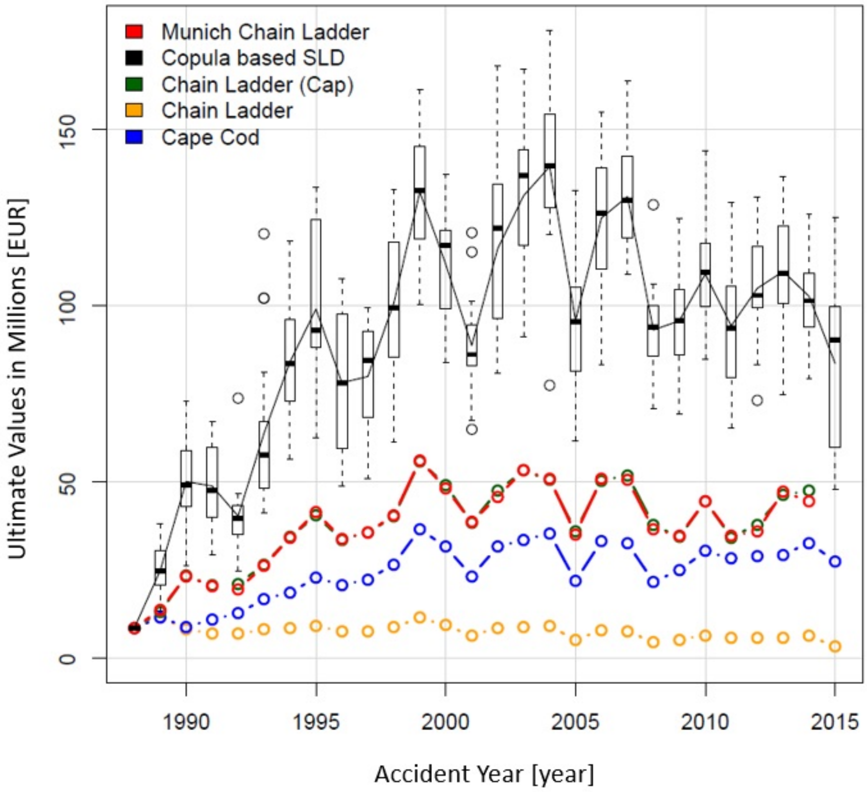


Figure 5.15.: Ultimates of Different Methods Considering the Full Model for the German MTPL Market.

The ultimate values per accident year are shown for the aggregated methods and the copula-based SLD method considering all model components. The latter includes the average shown as line and the related boxplots per accident year. Hereby, multiple effects can be observed. First of all, it is notable that the ultimate values of the Chain Ladder method based on the full triangle are significantly smaller compared to the ultimate values of the Chain Ladder method without the last diagonal which stay on a similar level. For this dataset, the latest diagonal of the market data is not known completely which results in small development factors on the latest diagonal and the seen effect. Thus, the Munich Chain Ladder method is also applied on the triangle without the latest diagonal since the values on the latest diagonal, especially for older years, can result in issues for the Munich Chain Ladder method as well.

Secondly, the ultimate values of the Chain Ladder without the last diagonal and of the Munich Chain Ladder method are close together and tend to differ in more recent years. Hereby, those values are taken as benchmark for the other methods following the scientific standard as stated in Section 5.3.2. The results of the Chain Ladder method and Munich Chain Ladder method are driven by the law of large numbers which stabilizes the LDFs for this dataset. Thirdly, the Cape Cod method and the copula-based SLD model are both applied to the full triangle including the latest diagonal. While the results of the Cape Cod method are around a third smaller than the Chain Ladder values, the ultimate values of the copula-based SLD model are around three times higher.

Additionally, the aggregated ultimates for the respective accident years are plotted as boxplots to see how the results vary over the simulations. It can be seen that the median and the mean, which should be used later on, differ slightly in some cases. The distribution of the ultimate values is skewed for some accident years, e.g. 1995, which leads to this effect. Additionally, the interquartile distance for some years can be up to 30 million. The whiskers and the outliers show that the results are volatile considering that a single claim can only have a maximum of EUR 2 million due to the layering.

Taking a look at the as-if frequencies in Figure 5.16, it can be seen that the as-if frequencies at EUR 1 million are close to each other, except for the copula-based SLD model.

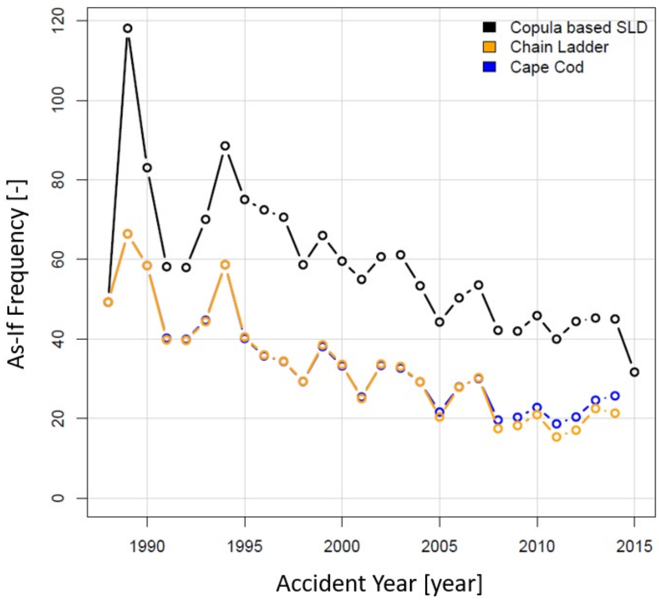


Figure 5.16.: As-If Frequency at EUR 1 Million Considering the Full Model.

Since the number of claims is large for the German market, this leads to stable development factors resulting in a similar estimation of the as-if frequency by the Cape Cod and the Chain Ladder method. After 2007, the Cape Cod model predicts slightly more claims above EUR 1 million than the Chain Ladder. In comparison to that, the copula-based SLD model predicts around 20 to 30 claims more per accident year. It is notable, that this occurs for all accident years.

In conclusion, nearly all simulated ultimates of single claims are hitting the full layer for the copula-based SLD model and are exceeding the priority while this is not predicted as such by the aggregated methods. Thus, the higher frequency as well as the higher ultimate value per accident year are a result of the lag of simulating positive run-off of the claims over the simulations. However, finding a way to better model the positive run-off for the single claims would result in a smaller as-if frequency and simultaneously a smaller ultimate value per accident year which might be more accurate.

5.4.4. Application of the Copula Only

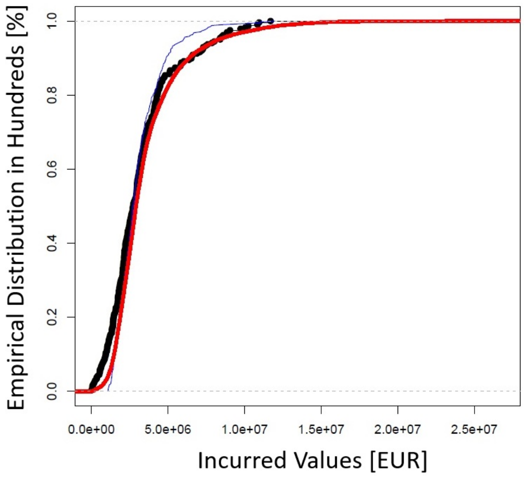
In addition to the analysis above, only the copula without the LDF surface and the jump pattern as conditions is applied to the same data as for the backtest procedure in 5.4.3.1. Thus, it is possible to get an understanding on the impact of the different model components. Hereby, 50 splits between training and test data are done with 50 simulations for each single claim. For the pricing procedure, the same layer with EUR 2m xs EUR 1m is considered.

By only applying the copula, the claims' development is not limited by the LDF surface which might lead to higher development steps. Moreover, the claims' development is not controlled by the jump pattern so that the history of the claims is not taken into account. Similar to the above, the impact of the calendar year effect should be minor for the same reason.

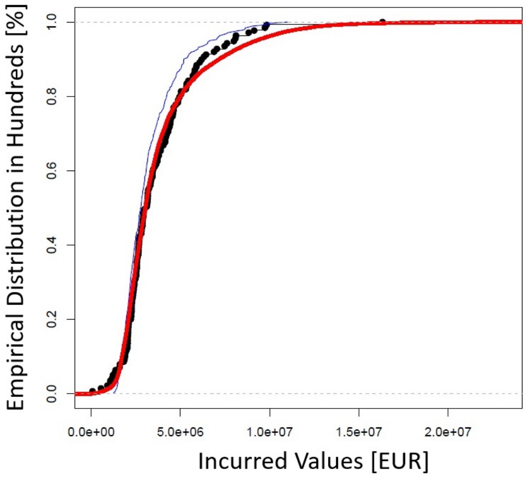
5.4.4.1. Backtest Procedure Results

For the analysis, the actually occurred latest ultimate distribution (black), the newly simulated distribution for the ultimates (red), as well as the starting distribution (blue) of the respective development year are compared for different numbers of known and predicted development years in Figure 5.17. The average squared distance between the actual and simulated distribution over all simulations is shown in Figure A.12 on page 212 in the Appendix. For the distributions, only a selection showing the most important observations are shown.

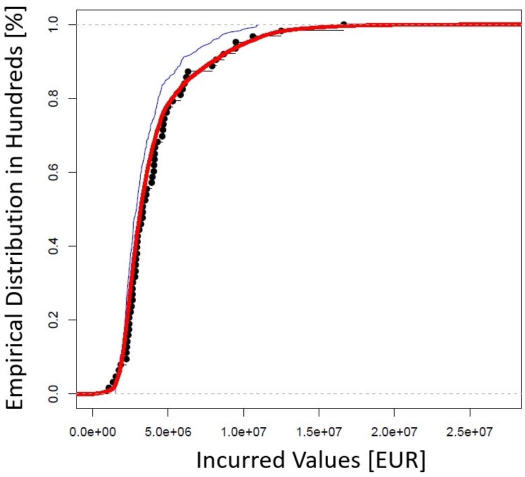
In comparison to the simulations using all features in Section 5.4.3.1 on page 141, it is notable that the simulated distribution fits the actual distribution much better for the upper tail. While the simulated distribution in Subfigure 5.14a stays close to the starting distribution, it fits the actual distribution better if only the copula is used in Subfigure 5.17a. However, the smaller claim amounts are overestimated, e.g. the positive run-off of some claims is not simulated correctly. However, a slight overestimation of the tail also occurs here as shown in Subfigure 5.17b, if 15 to 18 development years are predicted. With more known development years in Subfigure 5.17c, the distribution fits approximate the actual distribution well as it is also the case for the model using all components. However, there are simulations where the fits are not that well. In later development years, shown in Subfigure 5.17d, the actual claims are getting worse for medium sized claims which is not simulated correctly by the copula-based SLD model. This is also related to the sparse number of claims with such a long known period leading to a lack of the prediction power for the SLD model. This can also be seen in the residual distances in Figure A.12 in the Appendix which are smaller overall compared to the model using all components. However, the main prediction scheme in respect of the known and simulated development years remains the same.



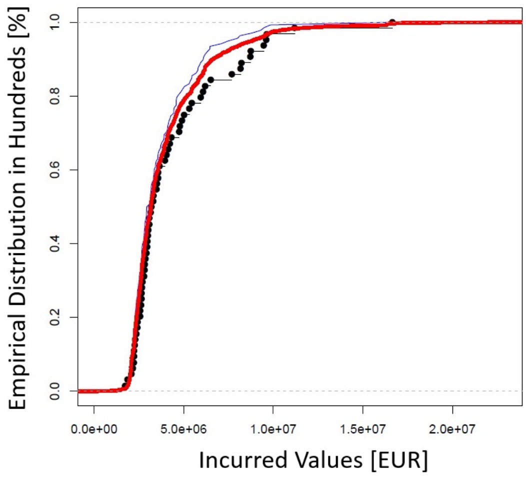
(a) DevYears Known: 3, Predicted: 11.



(b) DevYears Known: 7, Predicted: 16.



(c) DevYears known: 10, Predicted: 20.



(d) DevYears Known: 17, Predicted: 21.

Figure 5.17.: Severity Distribution Comparison Considering Only the Copula.

Overall, the simulated distributions fits the actual distribution in a better way, especially for the tail, which is important for a reinsurance pricing. However, smaller claim amounts as well as developments of claims in later development years are not simulated correctly. The expected tendency towards higher claim amounts does not hold for the whole distribution. Indeed, the distribution fits are good for some ranges of incurred values. However, this does not necessarily account for the ultimate claim size distribution in the end.

5.4.4.2. Pricing Procedure Results

In a next step, the pricing procedure is performed and the average aggregated ultimates per accident year are shown in Figure 5.18. Hereby, the ultimate values per accident year are shown for the aggregated methods and the copula-based SLD method considering only the copula. The related boxplots per accident year as well as the average are shown for the latter.

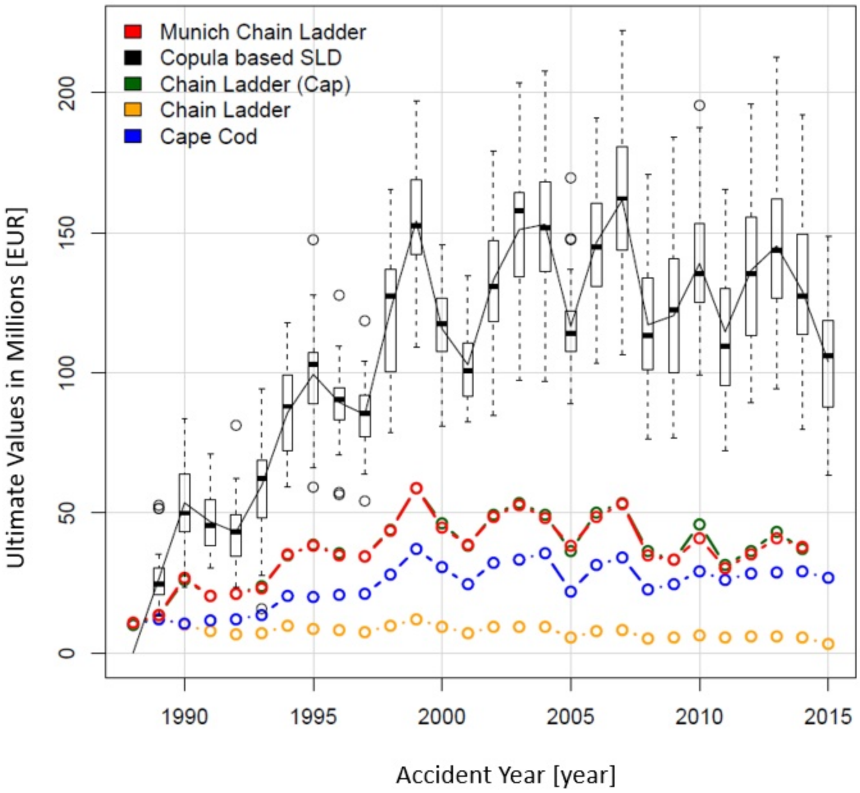


Figure 5.18.: Ultimates of Different Methods Considering Only the Copula.

Since the dataset remains the same, the Cape Cod, the Chain Ladder, the Munich Chain Ladder, and the Chain Ladder without the last diagonal show the same behaviour as before. However, the results of the copula-based SLD model differ. The ultimate values are higher after 1997, the interquartile distances are smaller for earlier years and larger for later years, and the median is closer to the mean values since the distribution of the ultimates per accident year is not so skewed in respect of outliers. This might be due to several effects. On the one hand, the distribution of the single claims as seen above is more severe than when using all model components while the layer automatically limits each single ultimate value at EUR 2 million reducing the volatility of the average ultimate result per accident year. On the other hand, the single claims are developed by only drawing development factors which leads to a higher volatility in each single claim's development compared to the copula-based SLD model utilizing the LDF surface and the jump pattern. After aggregation, this can result in larger whiskers and a larger interquartile distance while the median and mean are closer together. Thus, it affects the skewness of the average ultimate values per accident year, which is less pronounced compared to the results using all model components.

Looking at the as-if frequencies in Figure 5.19, a similar frequency movement can be observed for the copula-based SLD model considering only the copula (red) compared to the model using all components (black).

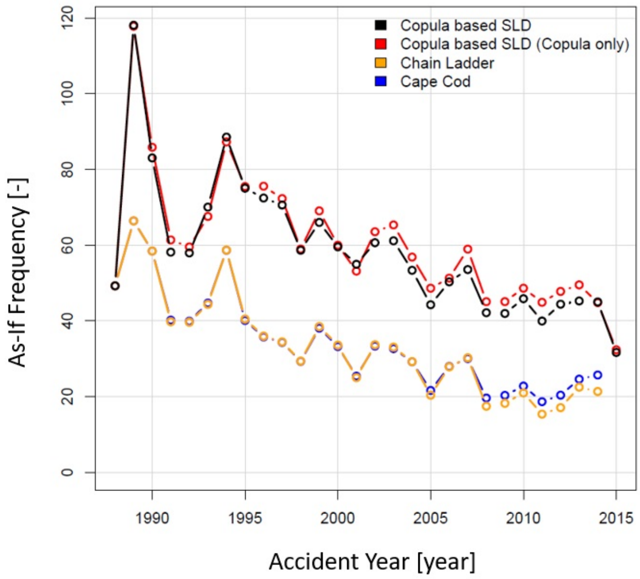


Figure 5.19.: As-If Frequency at EUR 1 Million Considering Only the Copula.

While the as-if frequency of the copula-based SLD model using only the copula at EUR 1 million is again around 20 to 30 claims higher per accident year, it can be seen that after 1995 the differences are larger compared to the frequencies obtained by the full copula-based SLD model shown in Figure 5.16. This also impacts the higher ultimate claim values which is higher for the same time horizon.

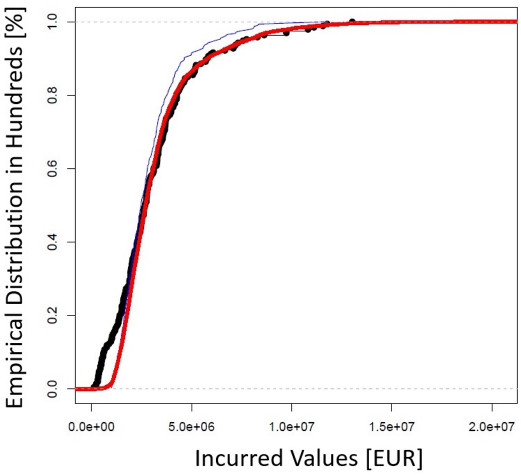
Overall, the ultimate claim values show more volatility and the skewness over the simulations is not as large as for the copula-based SLD model with all components. Again, nearly all simulated ultimates of single claims are hitting the full layer which leads to the higher as-if frequency and the increased average ultimates per accident year as a consequence. Thus, the lag of simulating the positive run-off of single claims in a more reliable and realistic way is noticeable in the results again.

5.4.5. Application of the Copula and LDF Surface

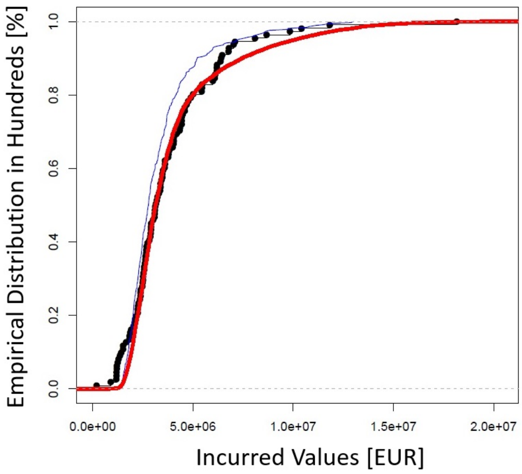
In the next step, the copula as well as the LDF surface condition are considered for the development of single claims. Keeping 50 simulations per single claim, a total of 40 splits between training and test data could be done. Again, the layer EUR 2 million xs EUR 1 million is considered for the pricing procedure. By considering these model components, the LDFs drawn from the copula are limited according to the current payment ratio and incurred value of the claim. This avoids unreasonably large LDFs for the current state and should result in smaller claim amounts and outliers. The impact of the calendar year effect is assumed to be negligible again.

5.4.5.1. Backtest Procedure Results

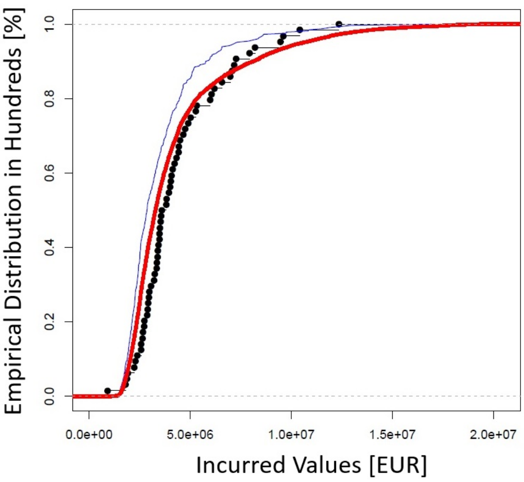
In each simulation, the originally occurred distribution (black), the simulated distribution (red), and the starting distribution (blue) are computed. For an easier assessment of the most important observations, only representative distribution fits are selected for a different number of known and predicted development years and shown in Figure 5.20. Besides that, the average squared distance between the actual distribution and the simulated distribution over all simulations is shown in Figure A.13 on page 213 in the Appendix.



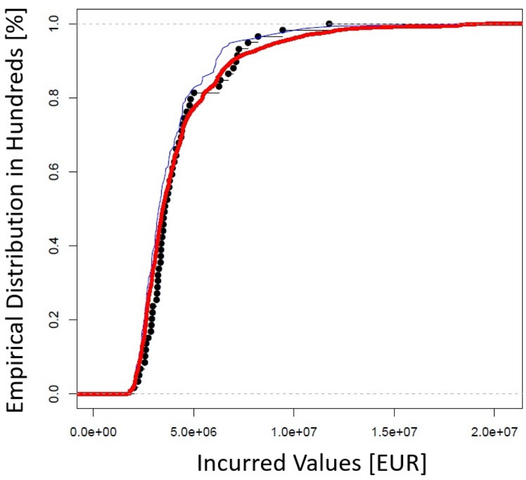
(a) DevYears Known: 1, Predicted: 8.



(b) DevYears Known: 8, Predicted: 18.



(c) DevYears Known: 10, Predicted: 21.



(d) DevYears Known: 17, Predicted: 22.

Figure 5.20.: Severity Distribution Comparison Considering the Copula and LDF Surface.

By applying the copula and the LDF surface, the distribution fits for the upper tails are good if a small number of known developments is given and the prediction period is not too long as shown in Subfigure 5.20a. The goodness is similar to the previously seen simulations by only using the copula. However, smaller claim amounts are not simulated correctly and are overestimated. With an increasing number of known development years, the positive run-off of claims is not projected correctly. As it can be seen in Subfigure 5.20b, the lower tail and the upper tail are overestimated. The overestimation of the upper tail and the larger claim values in particular are problematic for reinsurance pricings. Nevertheless, the middle part of the distribution is fitted well. However, the overall tendency of the simulated distribution is in the right direction as shown in Subfigure 5.20c. The overestimation of the upper tail is still observable and the rest of the distribution is underestimated. This also applies to later development years as shown in Subfigure 5.20d, where claims' development can still be observed. The average squared distance between the distributions displays a similar behaviour as for the simulations by only considering the copula itself. Even if the residuals are overall slightly higher, the predictions are similar if 5 development years are known until the 20th simulated development year. For later years, the data basis is getting too sparse and volatile leading to unreasonable results. Nevertheless, the average squared distance says nothing about the shape of the distribution, which is important for reinsurance pricing and, thus, should be treated with caution.

Overall, the distribution fits are similar to using only the copula by showing the same strength and weaknesses. While the overall shape of the actual distribution is approximated in a better way, the upper tail is overestimated. For the lower tail, the overestimation occurs for a small number of known development years and turns into an underestimation with more known development years. Hereby, the LDF surface has no significant impact on the shape of the distribution.

5.4.5.2. Pricing Procedure Results

The pricing procedure as described in Section 5.3.4 is done next to compare the ultimate values of the copula-based SLD model using the copula and LDF surface with the ones from commonly used aggregated models. The related average aggregated ultimates per accident year are shown in Figure 5.21. Hereby, the related boxplots per accident year and the average shown as line are stated for the copula-based SLD model.

Similar to the previous analysis of the pricing procedure, the ultimate values of the aggregated methods do not differ significantly since the same market dataset is considered. In the case of the copula-based SLD model, the ultimate values are similar to the ones seen when only the copula is considered. However, for more recent accident years, mainly after 2008, slightly smaller ultimate values can be observed. Additionally, there is less volatility and the interquartile distances and whiskers are not as large as when only the copula is considered. This is reasonable since the LDF surface limits the LDFs reducing the probability of outliers.

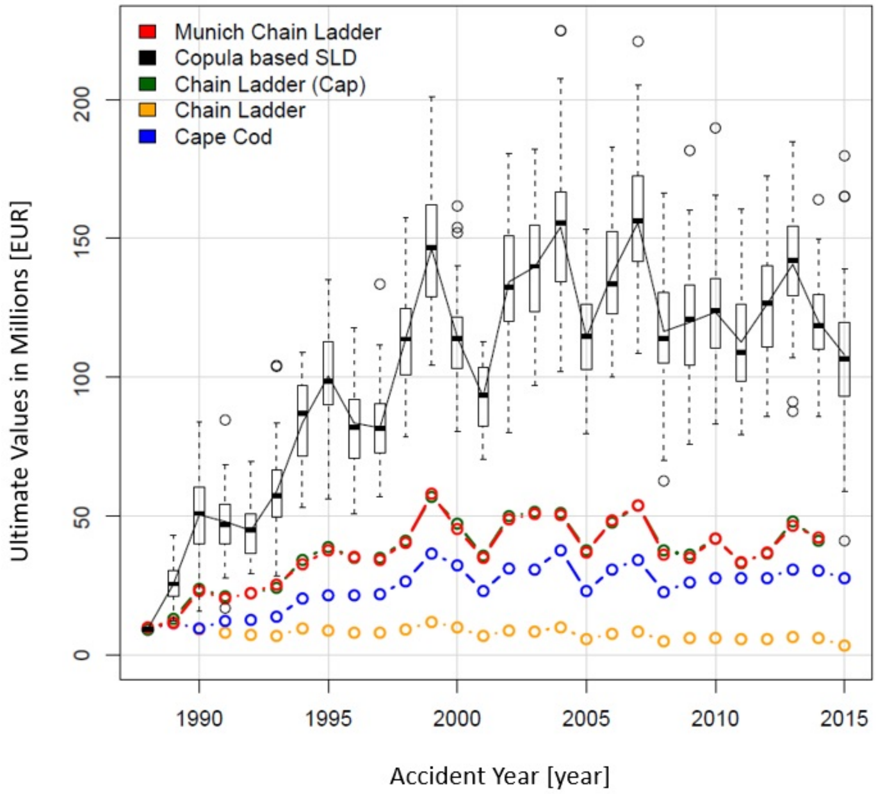


Figure 5.21.: Ultimates of Different Methods Considering the Copula and LDF Surface.

For the as-if frequencies in Figure 5.19, a similar frequency for the copula-based SLD model considering the copula and LDF surface compared to the model using only the copula can be observed.

In the case of the copula-based SLD model considering the LDF surface, the frequency only differs slightly from the frequency seen when only considering the copula. Since the LDF surface only limits the larger LDFs, the impact on the frequency considering a priority of EUR 1 million is minor. However, the frequency for higher layers should be smaller since the claim sizes are also smaller. Nevertheless, this is not investigated further since a comparison for higher layers would lead to complications for the unsupervised usage of aggregated models.

Considering the LDF surface on top of the copula leads to less volatility and skewness since the claims' developments are limited to the top. This leads to slightly smaller ultimate claim values compared to the copula-based SLD model using only the copula. In the case of the frequency, a smaller frequency cannot be observed since the priority is too small to show this effect. Nevertheless, a slightly smaller frequency would be expected as well. However, the lag of simulating the positive run-off of single claims is observable which still leads to a higher frequency and ultimate value per accident year compared to the commonly used aggregated methods. In conclusion, the impact of the LDF surface is minor.

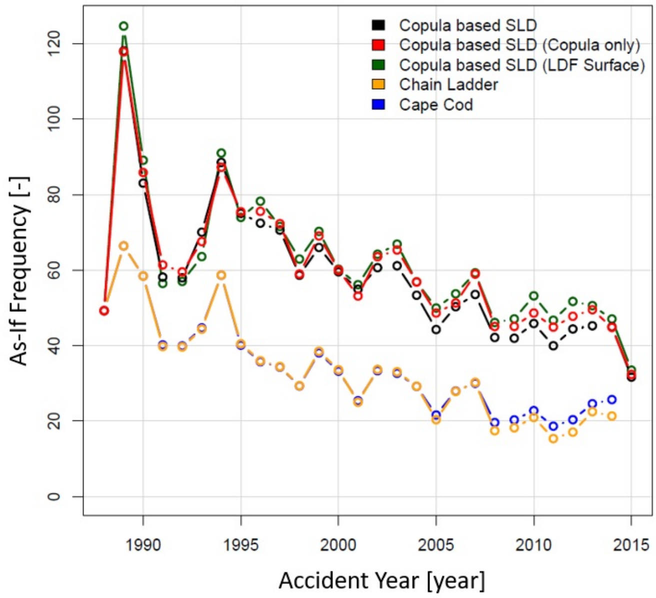


Figure 5.22.: As-If Frequency at EUR 1 Million Considering the Copula and LDF Surface.

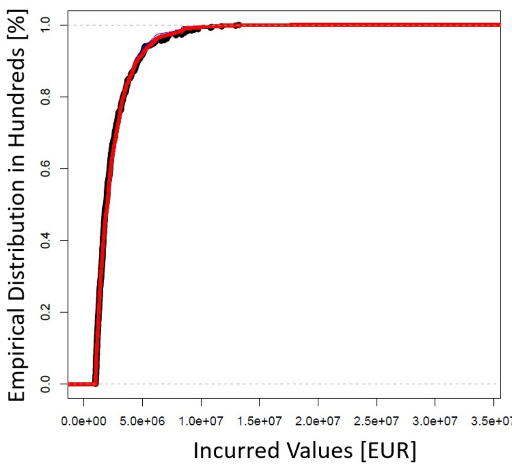
5.4.6. Application of the Copula and Jump Pattern

The last constellation that is analysed is the copula in combination with the jump pattern. Considering 50 simulations for each single claim, 26 splits between training and test data are performed. The same layer EUR 2 million xs EUR 1 million is used for the pricing procedure. Due to the usage of the jump pattern, the historic paths of the claims are considered for the further development. This should lead to a better prediction of the claim values if a higher number of known development years is considered. The impact of the calendar year effect is considered to be negligible again.

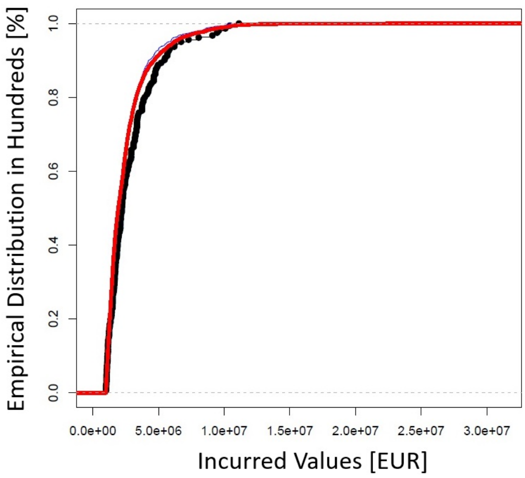
5.4.6.1. Backtest Procedure Results

Similar to the previous analysis, the starting distribution, the originally occurred distribution, as well as the simulated distribution are compared for each set of development years. In order to allow an easier assessment of the most important results, only representative distribution fits for different numbers of known and predicted development years are selected and shown in Figure 5.23. Additionally, the residual distance in form of the average squared distance over all simulations is shown in Figure A.14 on page 214 in the Appendix.

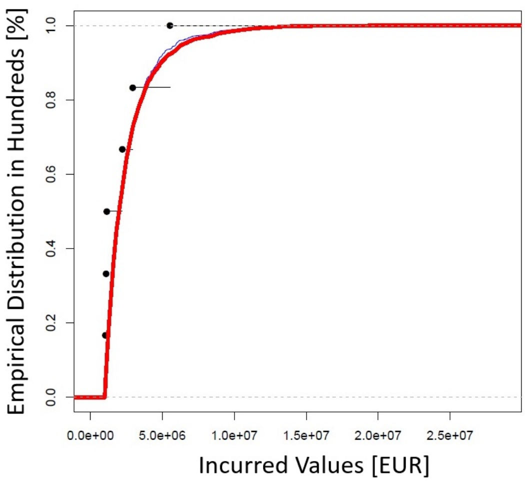
Using the jump pattern in addition to the copula should show an increasing impact with an increasing number of known development years. While there is generally little development in the claims in the first years as shown in Subfigure 5.23a, this changes for later years. Considering four known development years in Subfigure 5.23b and predicting the 20th development year shows that the simulated distribution stays close



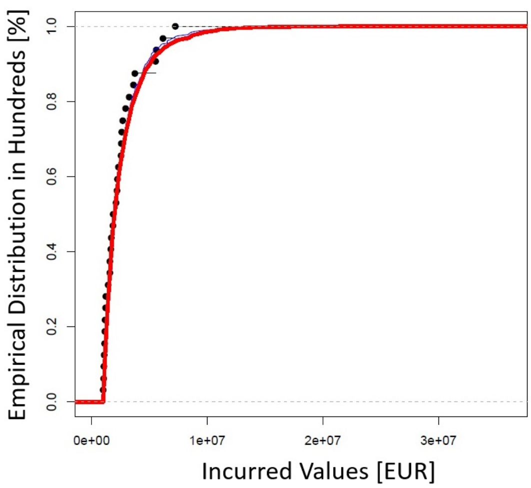
(a) DevYears Known: 2, Predicted: 20.



(b) DevYears Known: 4, Predicted: 20.



(c) DevYears Known: 7, Predicted: 28.



(d) DevYears Known: 11, Predicted: 26.

Figure 5.23.: Severity Distribution Comparison Considering the Copula and Jump Pattern.

to the starting distribution while the actual distribution is getting more severe. The simulated distribution does not show much development and does not fit the actual distribution well as it has been the case for only using the copula or the LDF surface on top. This behaviour does not change if more development years are predicted as done in Subfigure 5.23c, where the claims database is getting sparse. While the actual distribution shows a slight swing towards lower claim amounts and a positive run-off with an increasing number of known development years, this is not the case for the simulated distribution. This is shown exemplarily in Subfigure 5.23d. The simulated distribution does not change much compared to the starting distribution. Having a look at the average squared distance shows that the residuals are slightly smaller compared to the residuals of the German market using all features. However, this is more related to the claims chosen by the split between training and test data and is not related to the missing LDF surface. Since the simulated distribution stays close to the starting distribution, similar sizes of the average squared distances occur.

Overall, the jump pattern has a strong influence on the shape of the distribution and the development of the claims. It ensures that the simulated distribution stays close to the starting distribution even if the real claims show some development. Thus, it restricts the development in a way that is not intended.

5.4.6.2. Pricing Procedure Results

In a next step, the pricing procedure as described in Section 5.3.4 is performed to compare the ultimate values of the copula-based SLD model using the copula and jump pattern with the results from traditionally used aggregated models. The resulting average aggregated ultimate values per accident year are shown in Figure 5.24. For the copula-based SLD model, the related boxplots per accident year as well as the average shown as line are stated.

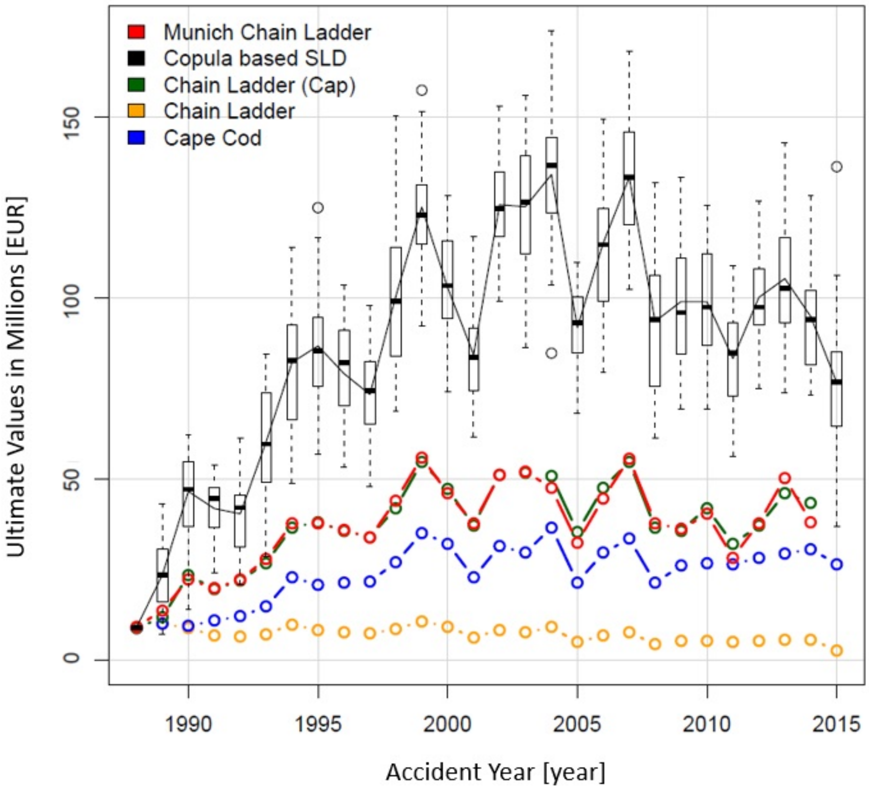


Figure 5.24.: Ultimates of Different Methods Considering the Copula and Jump Pattern.

The aggregated models are similar to the previous analysis of the pricing procedures since the same market dataset was used. In comparison to that, the ultimate values of the copula-based SLD model are smaller than the ones using the copula and LDF surface and are more comparable to the ultimate values using all model components. This also accounts for the interquartile distances and whiskers, which are similar to the ones using all model components. Hereby, the jump pattern controls the LDFs and reduces the claim sizes as well as the volatility which leads to less skewness and that the median values are closer to the means. The higher ultimate values per accident year are a consequence of the high as-if frequency shown in Figure 5.25.

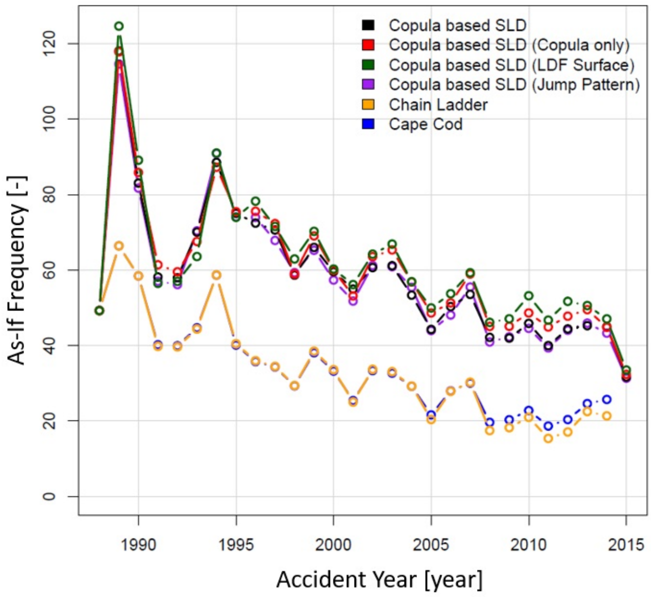


Figure 5.25.: As-If Frequency at EUR 1 Million Considering the Copula and Jump Pattern.

Hereby, the frequencies of the copula-based SLD model considering the jump pattern are similar to the frequencies obtained by the copula-based SLD model using all model components. Since the jump pattern reduces the development of the claims, it limits the possibility of simulating the claims' run-off correctly. Hence, the frequency is not as high as for only considering the copula and LDF surface.

Overall, the jump filter has a large impact on the ultimate results. Due to the control of the single claim's development, it leads to smaller ultimate values per accident year and less volatility. Hereby, the simulated distribution stays close to the starting distribution and does not move with the actual distribution as it should. Thus, it leads to less severe results which reduces the as-if frequency and ultimates per accident year in comparison to the modelling using only the copula and SLD surface. Nevertheless, the positive run-off of the claims is not simulated correctly. Thus, too many claims exceed the priority and hit the full layer which leads to an overestimation of the frequency and ultimate value.

5.5. Summary

In this chapter, the goodness of the copula-based SLD model is analysed based on real empirical data. Hereby, it is important that these evaluations only allow to give a tendency of the model's behaviour and not a general statement about the goodness of the model. After analysing the impact of different claim parameters, the focus is set on the number of simulations. Hereby, the trade-off between the accuracy of the results and the required runtime is discussed. Afterwards, the evaluation methods in form of a backtest procedure and pricing procedure is stated. This includes a comparison of the claim size distributions per development year as well as a comparison of the ultimates and as-if frequencies in comparison to commonly used aggregated models. The chapter concludes with an analysis of the different model components. Therefore, the results using all model components, only the copula, the copula and LDF suface, as well as the copula and jump pattern are analysed. Hereby, it can be seen that the copula-based SLD model tends to not simulate positive claims' run-off correctly leading to an overestimation of the as-if frequencies and ultimate values. Additionally, the upper tail of the severity distributions is overestimated while the usage of the jump pattern reduces the development of the claims. The usage of the copula only, also in combination with the LDF surface, shows a better assessment of the claims' distribution shape. However, the tail problematic, for the upper and lower tails, remains.

6. Additional Markets

Besides the German MTPL market, other countries are analysed as well. While the German market provides a large number of claims with annuity character, the full copula-based SLD model is also tested in respect to a smaller data basis and lump sum payments. The market share of all further countries is again more than 50% [110, 112, 165, 219] of the respective MTPL markets and the claims' information is internal real data from a leading reinsurance company. For the indexation of the claims, the country specific CPI index [261] plus a loading for superimposed inflation is considered for simplicity reasons[1]. The data used follows the form described in Section 2.3 in Table 2.1 and the related market information is taken from Axco Insurance Information Services [7, 9, 10, 11] if not stated otherwise. Plots of the related LDF surfaces are shown in Appendix A.4.

6.1. Example II: Maltese MTPL Market

The second market that is analysed is the Maltesian MTPL market, which is also considered in Section 5.2.2. Overall, 308 claims in the period from 1999 to 2017 are known. Before the model can be applied, the data has to be examined for influential effects in the past and the specific market characteristics influencing the claims' behaviour and development have to be taken into account as well.

6.1.1. Market Characteristics

Legislation in Malta is mainly determined by EU directives which are transposed into national law since Malta joined the EU in 2004. At end of the 2010s, the main focus is set on implementing Solvency II.

In recent years, the Maltese have slowly become more litigious. This was driven mainly by an increasing awareness of their rights due to the publicity of various notable accident cases and an easier access to legal opinion. As a result, more and more cases are going to court. Nevertheless, an ongoing traditional fatalism discourages individuals to take court actions even if the court costs and lawyers' fees are low. Furthermore, there is the fear of bureaucracy and of the potential length of court proceedings. If a claim goes to court, it can take up to four years to conclude and another four years on

[1]The actual index used is not stated further due to confidentiality conditions by the company providing the data.

appeal. A period of four years to reach judgement and an average of six years until settlement is common for companies. However, this might even be extended to a delay of 10 years or more. Thus, both parties are encouraged to seek settlement out of court and a small claims' court was introduced to speed up court decisions involving small claim amounts. Overall, most claims are settled out of court.

Compared to other EU countries, compensation in Malta has traditionally been low, although there has been an upward trend in recent years. This is related to an increase in salaries, longer life expectancy, discounts for non-dependants, and a sympathetic approach taken by some judges. This sympathetic and inconsistent approach was considered problematic since the court rulings were less predictable as a result. Consequently, this was reviewed and led to new guidelines including moral damages and a draft law in 2011. Until 2019, there were no further developments for new court decisions guidelines and the courts are regarded as being unaffected by external influences.

The insurance market in Malta is small and motor insurance is the largest segment largely determining the profitability of Malta's insurers. This is due to the fact that MTPL insurance is compulsory since 1939 and that no tariffs are in place. Thus, the competition in motor business is strong since it is the easiest class to sell and even if it is considered a loss leader to some extent, it is also a way to upsell other classes of business. This competition was particularly evident during the time when Allcare Insurance was active, affecting the rates significantly in all classes up to the point of their bankruptcy in 2013. Since 2016, the market has recovered to some extent and returned to a healthier rating level.

Significant reasons for the poor results in MTPL business is the local driving environment. The roads are considered overcrowded and of poor quality. Additionally, the standard of driving is often poor, especially in young and inexperienced drivers using their cell phones while driving. The accident frequency for young drivers under 25 years is reported to be 50%, with half of the young drivers expected to have at least one accident. Thus, premium loadings are common, resulting in telematic tariffs or the registration of the vehicle in someone else's name to obtain MTPL insurance. Hence, this results in many accidents per day and a large number of deaths, injuries and MTPL claims. This can be seen in Figure 6.1 for Malta since 2004.

The figure shows the development of the overall number of motorised vehicles since 2004, the number of accidents, and light and severe injuries[2] [165]. While the number of motorised vehicles increased steadily over time, the number of total accidents remained stable until 2006 and then decreased significantly until 2010. This was the result of a road safety campagne including a newly revised driver test, seat belt campaigns, the installation of speed cameras, campaigns on the usage of child restraints,

[2] No accident statistic exists prior to 2004.

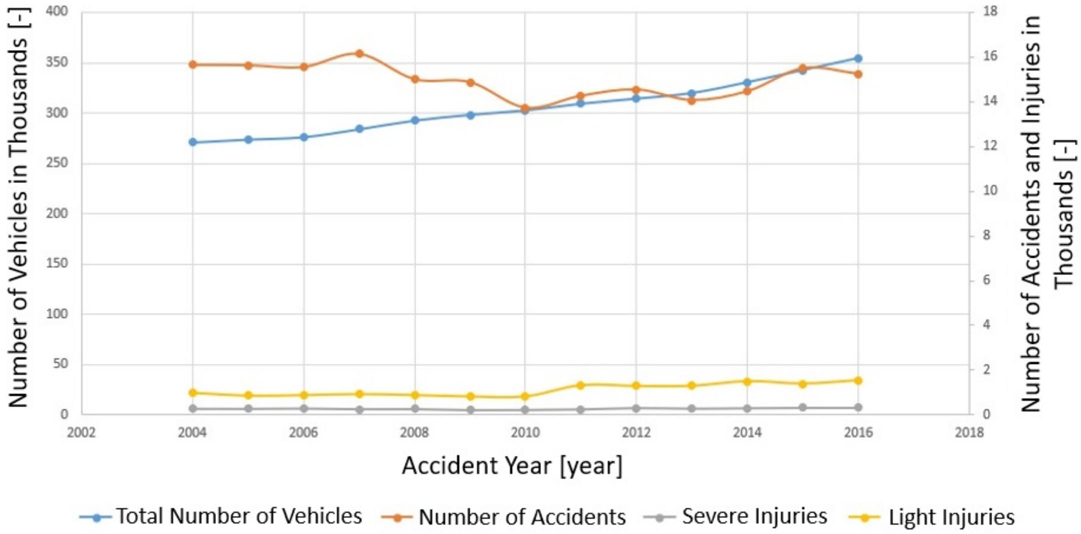


Figure 6.1.: Market Development for Malta.

and the implementation of traffic management [158]. However, this mainly affected the number of accidents with property damages since the number of light and severe injuries remained stable until 2010. In 2011, the number of light injuries in particular increased by a total of 50%. This was driven primarily by drivers older than 60 years. The amount of light injuries increased by 157% while all others increased by are around 50% [165]. However, no reason is stated for this increase. A possible explanation could be a change in the methodology or in the road traffic laws. As of 2010, the European Union and more specifically the United Nations General Assembly forces the member countries to increase their road safety management, road infrastructure, vehicle safety, road user behaviour, road safety education and post-accident response [235]. While the number of total accidents is increasing proportionally to the number of registered vehicles, the number of injured persons has been stable since 2010. This increase in the number of registered vehicles is connected to the continuing growth of the Maltese economy that is also reflected in the development of the insurance market. Overall, the situation in Malta leads to many road accidents per day and a large number of deaths and injuries compared to the experience of most other EU member states. Approximately 60% of the claims involve third party damages and some of the largest losses in recent years involved claims for death or injuries caused to citizens of other EU member states. This is expected to continue in the future due to the fast ferry links between Malta and Sicily making it relatively easy to bring the own car to Malta.

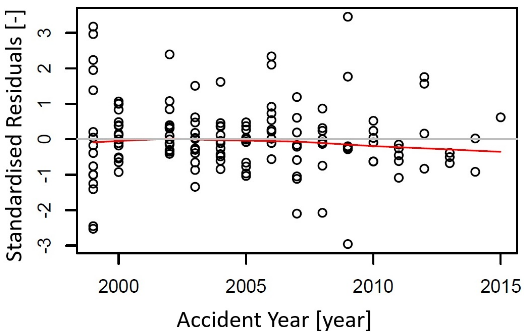
Overall, the market is chosen for following reasons:

- There is a lot of data available with a seen market share of over 50% of the total MTPL market.

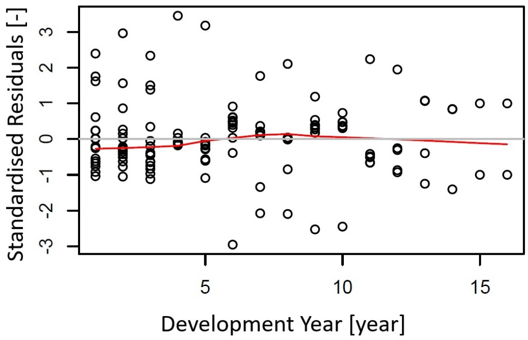
- Compared to other countries, the data basis is small but grands a great overview over the Maltese MTPL market.

6.1.2. Data Analysis, Trends, and Influential Effects

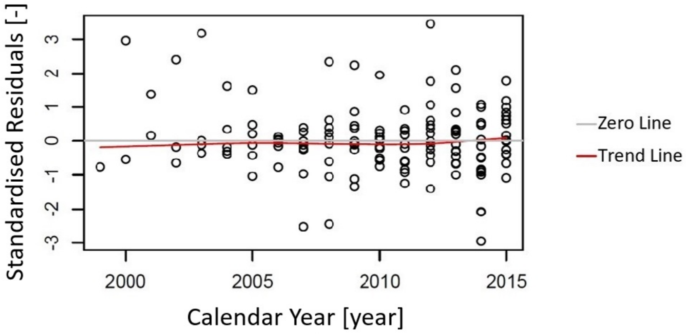
In order to analyse possible influential effects and trends, the standardised residuals of the Chain Ladder method are used as described in Section 2.8.1 and shown in Figure 6.2 for the used MTPL data of the Maltese market. Therefore, it has to be kept in mind that possible effects cannot be eliminated from the data on a single loss basis. Nevertheless, possible effects and influences can be identified. Hence, the analysis of the standardised residuals is done with respect to the accident, development and calendar year to identify market changes towards these time lines. This analysis is conducted for the whole market and does not necessarily account for every single company.



(a) Accident Year.



(b) Development Year.



(c) Calendar Year.

Figure 6.2.: Standardised Residual Plot for the Maltese MTPL Market.

No trend can be observed when analysing the standardised residuals with respect to the accident years in Subfigure 6.2a. Even if the trend line for the residuals decreases in more recent years, this cannot be considered significant[3]. Thus, the data is considered to be trend free with respect to the accident year. It can also be seen that there is a loss free year in 2001 and that the residuals are volatile for 2009. Last one is a result of the dominance of a single company for this year.

[3]The analysis of those residuals in general is not based on small changes but on more pronounced shifts and trend line changes.

The standardised residuals with respect to the development year are shown in Subfigure 6.2b. As seen above for the accident year, there is no trend for the development year either. The same is observed with regard to the calendar year in Subfigure 6.2c. For both, only small shifts can be observed which are too small to be counted as a trend effect. Since no distinction between a trend effect and normal volatility can be made, the data is considered to be trend free. However, and according to Simpson's Paradox, trends in the data might cancel out.

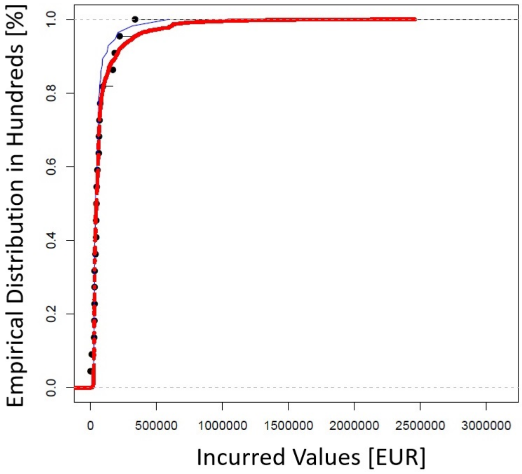
One of the reasons for not seeing any trend effects might be the Malta Insurance Repair Efficiency (MIRE) that helps to contain inflation in repair costs. Therefore, garage repair shops have been classified into five categories which helps to increase the efficiency of repairs later on. However, this only accounts for the property damage part and not for the bodily injury part that usually dominates the claims' amount. Thus, it seems like the court rewards and lump sum payments for larger MTPL claims have been stable and consistent over the years.

6.1.3. Backtest Procedure Results

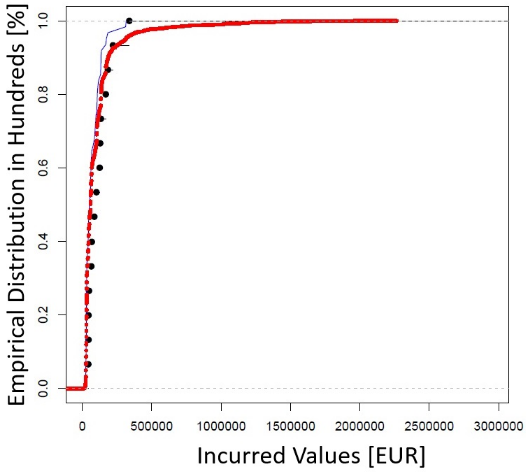
With this information, the results of the backtest procedure described in Section 5.3.3 are discussed for the Maltese market. Hereby, the starting, actual, and simulated distribution in Figure 6.3 on page 162 and the average squared distance between the simulated and the actual distribution in Figure A.15 on page 215 are considered. For simplification, only a representative selection that shows the important characteristics of the distribution fits is shown in Figure 6.3.

In comparison to other markets, Malta has a small MTPL market with a more sparse claims database where many claims are settled out of court as lump sums. Thus, the claims' development is rather limited as it can be seen in Subfigures 6.3a and 6.3b.

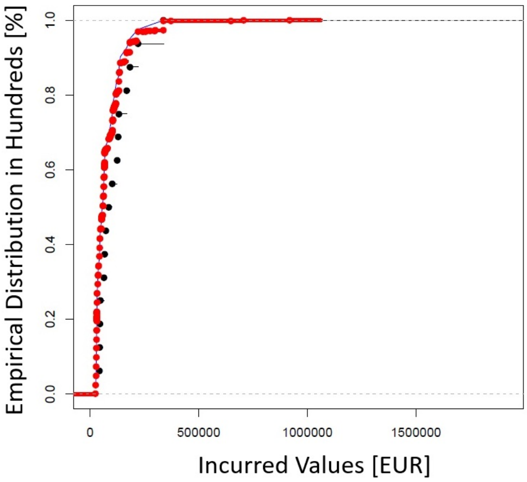
While the actual distribution in black is close to the starting distribution in blue and only shows a slight tendency towards more severe claims, the simulated distribution is more severe in the tail. However, the shape of the distribution for the smaller claim amounts is fitted well. For later simulated development years, the severity of the tail is increasing more since many claims are already closed at this point in reality but not in the simulations. With 13 and more simulated development years, the actual distribution is more severe for medium and larger sized claims as shown in Subfigure 6.3c. The shape of the distribution above 50% probability is more severe and approaches the actual distribution for the tail. This behaviour is not simulated correctly by the copula-based SLD model which stays close to the starting distribution. Since seven development years are already known, many claims are already compensated as lump sums and mostly claims that went to court and are decided later on are left which leads to these increased claims' amounts. However, there is still not much development in



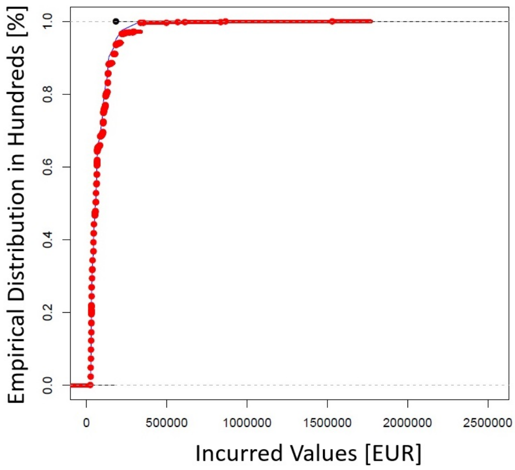
(a) DevYears Known: 2, Predicted: 9.



(b) DevYears Known: 4, Predicted: 13.



(c) DevYears Known: 7, Predicted: 13.



(d) DevYears Known: 7, Predicted: 16.

Figure 6.3.: Severity Distribution Comparison for the Maltese MTPL Market.

the claims compared to other countries that are more based on annuity payments. If 16 or more development years are predicted, the number of claims gets too sparse for a meaningful analysis as shown in Subfigure 6.3d. Thus, there are no simulations done with 16 or more known development years since only one claim is left there. This can also be seen in the average squared distances in Figure A.15 which is small for the upper left triangle until the 12th development year. Afterwards, the residuals are increasing strongly and are worst for the 16th development year. The improvements afterwards are driven by the short lifetime of most claims and the longer development period.

Overall, the simulated distribution is overestimating the tail of the actual distribution for all development years while the rest of the shape is fitted in a good way. For older development years, the claims data is too sparse to make a meaningful statement. Since there is little development of the claims in general, the copula-based SLD model also shows little development. Thus, the original structure of the dataset automatically ensures that the simulated claim amounts are not too severe and limits the volatility.

However, it can also be observed that a correct distinction between lump sum payments in order to avoid courts proceedings and payments as a result of court decisions is not possible so far. This leads to cases where the shape of the actual distribution is not fitted well as shown in Subfigures 6.3b and 6.3a.

6.1.4. Pricing Procedure Results

The Maltese MTPL market is also analysed by performing the Pricing Procedure as described in Section 5.3.4, comparing the ultimate values of the copula-based SLD model with the results of currently used aggregated models. The 50 simulations per single claim result in 318 simulation runs and split between training and test data. A layer of EUR 2.825 million xs EUR 0.175 million is used for this market. It has to be mentioned that a calculation of the results for the aggregated methods is not possible for all simulations depending on the claims drawn for the test dataset.

The comparison of the ultimate aggregated claim values per accident year are shown in Figure 6.4 for the aggregated methods and the copula-based SLD model. The latter includes the related boxplots per accident year and the average values as line.

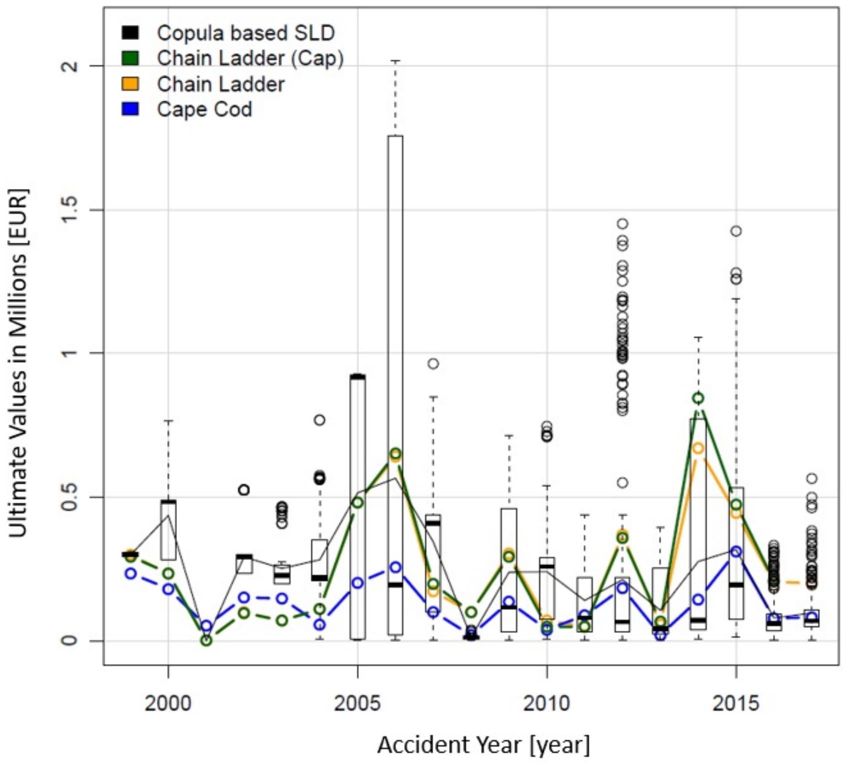


Figure 6.4.: Ultimates of Different Methods for the Maltese MTPL Market.

First of all, a comparison with the Munich Chain Ladder method is not possible due to the incompleteness of the test data triangle. Since the last diagonal of the triangle is completely filled, both Chain Ladder approaches show similar results. In comparison to both, the ultimate values of the Cape Cod method are smaller for most years. However, there are some years for which the Chain Ladder method gives an ultimate value of 0. Here, the strength of the Cape Cod method is observable since it uses the exposure values per accident year for the estimation of an ultimate value. However, in total the results of the Cape Cod method are smaller than the ultimate values of the Chain Ladder method. In case of the copula-based SLD model, the results for older accident years in form of the average values tend to be slightly above the results of the Chain Ladder method. For more recent development years, they lie between the results of the Chain Ladder and the Cape Cod method. However, the interquartile distances vary a lot. While there are some years with a small interquartile distance, there are also years, e.g. 2005 and 2006, where the interquartile distances are large. Additionally, there are years with many outliers like 2012. Since the number of claims is small for Malta and the split between training and test data is done randomly, there are two scenarios. In the first one, the test data only has one small claim in 2012 while in the other scenario there is only one large claim. Hence, there is a split of the overall ultimate results over all simulations leading to the seen large number of outliers. Overall, it can be stated that the results of the methods are highly influenced by the number of claims in the test dataset which can result in a high volatility. A similar effect can be seen in the as-if frequencies in Figure 6.5.

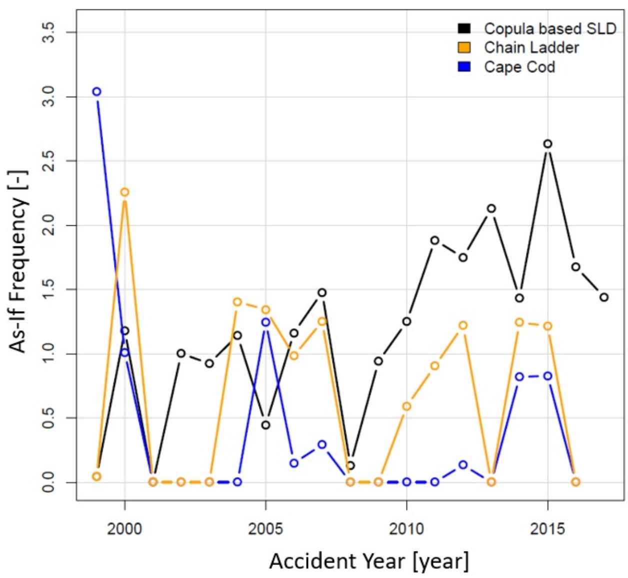


Figure 6.5.: As-If Frequency at EUR $0,175$ Million for the Maltese MTPL Market.

The Cape Cod method gives a smaller frequency for all accident years except for 1999 compared to the Chain Ladder method. While the copula-based SLD model frequency is similar to the frequency of the Chain Ladder method for older years before 2008, it shows a higher frequency for more recent accident years. However, this does not show up in higher average ultimate claim values for those years as it is the case in the German MTPL market. This is also related to the small priority which leads to an increased frequency but not to an increased average ultimate value per accident year.

The results of the copula-based SLD model are similar to the results of the aggregated methods. However, it has to be kept in mind that the dataset is sparse and that the results for the unsupervised aggregated methods could not be estimated for all kind of test datasets. Thus, the results have to be treated with caution.

6.2. Example III: Italian MTPL Market

The third market that is analysed is the Italian MTPL market. Overall, $7,729$ claims in the period from 1998 to 2016 are considered. The related influential effects and market specific characteristics that influence the claims' behaviour and development are further analysed.

6.2.1. Market Characteristics

Italy is one of the founding members of the European Union and has to implement EU directives into national law. The Italian legal system is based on the Napoleonic Code and a written and codified legal system consisting of abstract rules that have to be applied to particular cases. Thus, Case law precedents are not binding and have only a persuasive value. At the end of the 2010th, the Insurance Code 'Codice Delle Assicurazioni Privat' [114] has been in effect since 2006. It represented a complete overhaul, improvement and codification of the existing insurance laws aiming at an improvement of competitiveness within the insurance industry and the protection of the insured. It was also consistent with EU laws targeting the intermediation, reinsurance, international accounting, and solvency standards. In 2007 and 2008, various EU directives were implemented ending multi-year policy contracts among other things. The EU directive known as Solvency II came into effect more recently in January 2016. Thus, all EU directives have amended the Insurance Code from 2005 so far.

Italy has more lawyers than any other EU country and is considered to be one of the most litigious countries in Europe. In more recent years, litigiousness increased even further due to an increase of consumer awareness along with numerous but not always scrupulous lawyers[4]. The increasing number of consumer associations and publications did raise the public's awareness of their legal rights and abilities to claim redress. Additionally, if the opportunity to 'profit' from the legal system arises, it is most likely taken since many Italians hold the view that if insurance is purchased, some form of 'payback' is required. This is supported by the relatively low cost to initiate a legal action. However, the length of time that is required to obtain settlement, and the potential legal costs may deter many litigants from taking legal actions. Some civil cases might

[4] Meaning the exaggeration of injuries sustained in motor accidents.

take up to eight years until settlement and there is a backlog of several million cases waiting to be heard. Hereby, around 20% of all outstanding court cases are in respect of motor and watercraft liability in 2019. The majority of about 94% [117] of the overall pending claims relates to the first instance district court hearings. In general, there are different limitations given by the tort system and contracts. A legal action with respect to the tort system has a limitation period of two years with respect to road traffic accidents[5], which also applies to a direct action against an insurance company.

Since the mid-1990s, compensations in Italy are premised on a tabular system that is based on the degree of permanent disability suffered by a victim. Earlier, the amount of compensation was based on the judges precedent cases or subjective assessment of the facts of the case. The best known approach are the Milan tables. Their first edition was released in 1995 and received a remarkable response and positive feedback by the claim handling community. This tabular approach made it possible to predict the amount of compensation which had a considerable effect on reducing litigation while establishing some consistency and equality. However, there were still regional differences and inconsistencies. The tables included compensation for biological damages and moral damages. A modification in 2004 defined minimum and maximum amounts payable as well as compensation standards for relatives. However, until 2009 there were still different tables in use making a sufficient prediction of compensations difficult and inconsistent. Even though many other courts already adopted the Milan tables after 2005/2006, it needed a gridlock in the legal system until the Milan tables were accepted countrywide in 2011. This was initiated by guidelines issued by the Supreme Court of Cassation in 2009 that were quickly implemented into the Court of Milan after a gridlock of the legal system to find a suitable solution. Due to the newly implemented principles, a clear division of the compensation into economic and non-economic damages like biological, moral and existential damages was introduced. After the decision of the Supreme Court of Cassation in 2011, indicating that the Milan tables were the most compliant with its own ruling, many other courts adopted them. Today, the Milan tables are used in all compensation claims in every sector of business except for liability claims after a road accident and medical malpractice claims involving injuries between 1% and 10%. Therefore, a table is used that is compliant with the Italian Insurance Code, Article 139 [114] and that is updated annually, settling the claims lower than those obtained using the Milan tables [256]. Thus, the compensation of a permanent disability between 1% and 9% is based on the existing award tables while disability claims between 10% and 100% are based on the Milan tables.

Apart from this important legal change, the Bersani Decree[6] signalled the end of the Italian tied, or sole agency system. It ensures that insurance agents are allowed to offer

[5] Article 2947 of the Italian Civil Code [116].

[6] Law No 248 of fourth of August 2006 affecting Article 131 of the Italian Private Insurance Code [114].

MTPL policies from different insurance companies. This increased the competition in the insurance sector and the distribution of MTPL policies. Additionally, a decree from 2012 implemented stricter measures for the compensation of minor personal injuries by requiring an independent medical decision.

Focusing on the legal changes mentioned above, the compensation for non-economical damage is substantially higher than in other EU countries [255]. In total, about EUR 6.9 billion were paid in 2017 in respect of motor injury claims including property damage components. However, 20.4% of this was used for personal injury compensation for cases with permanent disability of 1% to 9% and 43.5% for cases with more than 9% permanent disability [2]. Thus, non-economical compensation is an important factor in the Italian MTPL market. Additionally, around EUR 6.2 billion were reserved by insurers for pending lawsuits. With regard to the last seven years beginning 2019, less than 20% of the legal claims were closed in favour of the insurer, 22% of the cases were settled in favour of the claimant, and 44% of the cases were settled by mutual agreement [117] with the tendency to favour the claimants overall. In 2015, the Court of Milan rewarded a substantial amount of compensation as an annuity for the first time. This was somewhat revolutionary since lump sums had been customary for severe permanent disability claims so far. This might result in a change of compensation behaviour in the future.

Overall, MTPL is the key driver of profitability for insurance companies and accounts for roughly 40% of the total non-life premium in 2018, reflecting the fact that this is a compulsory insurance class. Thus, there is a strong competition that also affects commercial lines in general. With respect to the reinsurance business, excess of loss contracts are the main method requested. Due to EU directives, there is no tariff applicable and each company establishes its own rating. However, the rates are still subject to outside scrutiny from the government, consumer associations and the anti-trust authority.

Multiple motorway crashes are not uncommon in Italy and there is a market agreement to avoid lengthy debates or court cases concerning the question of liability. In the case of an accident involving more than 40 vehicles, each insurer pays the loss of its own insured irrespective of the liability. However, the accident frequency in general seems to improve in more recent years, which is mainly driven by a reduction of minor injury claims according to the National Institute of Statistics [113] shown in Figure 6.6 for Italy since 2004.

Before 2012, there was no consistent definition of severe injuries in Italy and no split between severe and light injuries. Since then, a classification of the seriousness of injuries has been done according to the MAIS3+ classification and Italy is one of the first EU countries to do so [6, 113, 115]. The number of total vehicles increased steadily

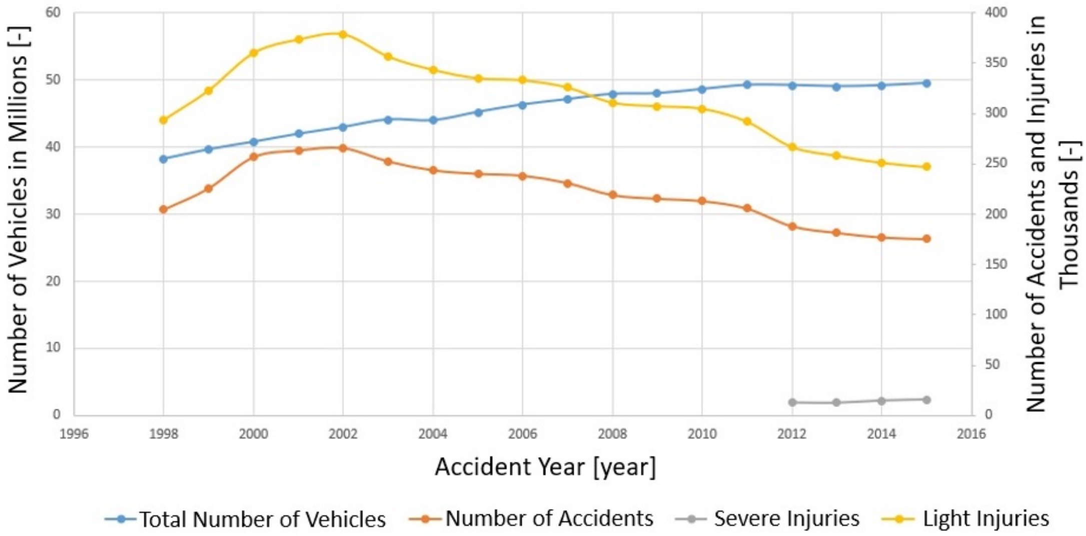


Figure 6.6.: Market Development for Italy.

until 2011 to nearly 50 million and remained at that level until 2015. This can be explained by the financial crises and the related economical problems afterwards[7]. Until 2002, the number of accidents and the number of injured persons increased simultaneously. In order to reduce the number of accidents, Italy adopted a National Road Safety Plan, the 'Piano Nazionale per la Sicurezza Stradale' [191]. Focusing on improvements in the infrastructure and urban area safety, as well as road accidents at work, high risk drivers, and an improved teaching of driving practice for novice drivers and foreign citizens, a reduction of the accident frequency of 40% until 2010 was planned [111, 191]. Even though this target was not achieved fully, the number of accidents and injuries decreased by about 30% on an as-if basis while the number of vehicles in the Italian market is still increasing. Since the first road safety plan, Italy accomplished multiple road safety plans and also adopted the EU directive 2008/96/EC on road infrastructure and safety management in order to continue the positive development [111, 191]. Furthermore, the reduction of the accident frequency also favours the profitability in the MTPL business. Factors like a changed driving pattern by using car sharing, a reduced usage of vehicles, stricter criteria for minor personal injury claims, tougher road traffic penalties, higher fuel charges, a reduction of trucks on roads, and the increasing usage of telematics are important factors here. While there is no information before 2012, the overall number of MTPL claims has fallen since 2012 from 5.59% to 5.4% in 2018. Simultaneously, the number of larger losses increased [2].

However, the average MTPL premiums in Italy are higher compared to other European markets, which is why the Italian insurance association (ANIA) has isolated some reasons for this in relation to the still high accident frequency [111]. Firstly, there is greater traffic congestion increasing the likelihood of an accident. Even if this is slightly set off by a lower average mileage, it still increases the accident frequency.

[7]Even though the population increasing slightly during that time.

Secondly, poorer road safety and a large number of motorcycles and scooters are impacting the claims' frequency as well. Thirdly, a higher rate of undetected frauds due to organised crime and less effective law enforcements increases the claim sizes. Furthermore, the repair costs are higher in Italy due to lower efficiency, higher fixed costs and a lower productivity rate [111]. In order to counter that, some larger insurers established their own approved repairer network, offering premium discounts, and a faster access to repairer parts boosting the efficiency. However, this is not fully offset since not all insurance companies have these kind of repairer networks.

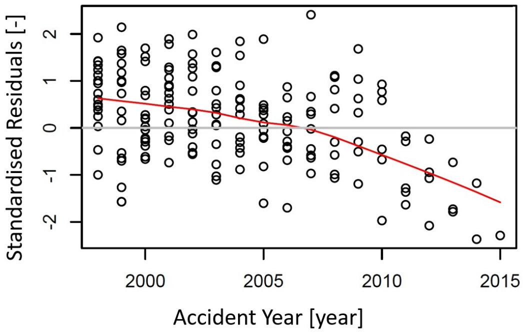
Conclusively, this market has been chosen for several reasons:

- There are many claims available with a seen market share of over 50% of the total MTPL market.
- Italy is a market where mainly lump sums are paid. Thus, the modelling of short claim lifetimes can be analysed.
- It is a market with many legal changes and efforts to improve the road safety.
- Court compensations include a high amount of non-economical compensation.

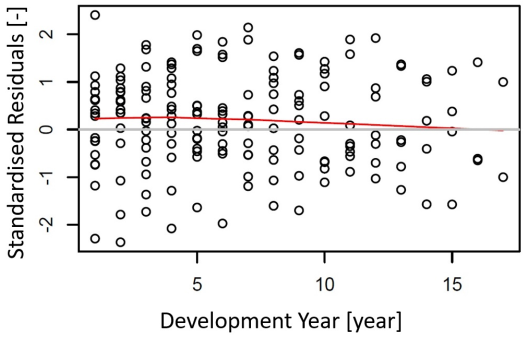
6.2.2. Data Analysis, Trends, and Influential Effects

As done for the previous markets, the analysis of possible influential effects and trends is done by calculating the standardised Chain Ladder residuals as described in Section 2.8.1. These residuals are shown in Figure 6.7 with respect to the accident, development, and calendar year. It has to be kept in mind that possible effects cannot be eliminated from the data on a single loss basis and that this analysis focuses on identifying possible influences on the Italian MTPL market. Hereby, the whole market data available is analysed.

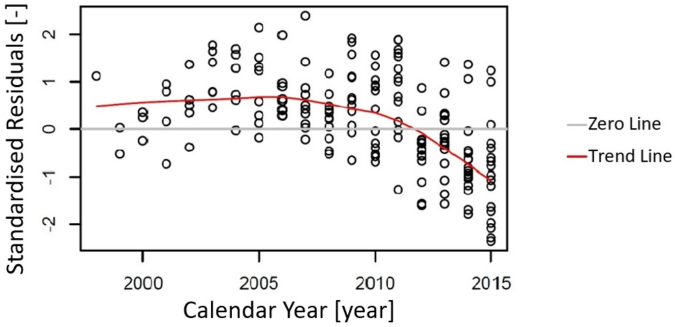
Firstly, the standardised residuals with respect to the accident years are considered in Subfigure 6.7a. This can be divided into three sections. The first section covers the time until 2004 where the standardised residuals are mainly positive. The individual development factors in older accident years are higher than the respective average of their development year. The second section includes the years from 2005 until 2010. Here, the residuals vary around zero and are distributed more randomly. The last section starting in 2011 only contains negative standardised residuals. Here all individual LDFs are smaller than the respective averages. Thus, the older years show a higher development compared to more recent accident years. This can be explained by the Milan tables. In 2005, the Milan tables were adopted by some other courts and were also accepted by the claims handling community. Before that, every court had its own rulings. Thus, there was more uncertainty in setting the reserves before 2005 leading



(a) Accident Year.



(b) Development Year.



(c) Calendar Year.

Figure 6.7.: Standardised Residuals Plot for the Italian MTPL Market.

to higher loss developments. The second breakpoint was the acceptance of the Milan tables in the Supreme Court of Cassation, stating that these tables were most compliant with its own ruling. Thus, the Milan tables were adopted countrywide, leading to more certainty and smaller individual loss developments as well. Overall, this explains the red trendline that is decreasing continuously.

Secondly, the influence in the development year is analysed in Subfigure 6.7b. Here, no specific trend can be observed. The standardised residuals vary around zero and are distributed randomly.

Thirdly, the standardised residuals plotted against the calendar year are shown in Subfigure 6.7c. Until 2005, the standardised residuals increased while the trend line indicates a constant development above zero. Since the higher residuals are located in the upper part of the claims triangle, those are plotted first leading to this increase until 2006. Since then, the Milan tables are applied and smaller residuals are included for later calendar years leading to a decreasing trend line due to more negative standardised residuals. Additionally, the improved competition between the insurance companies due to the end of the sole agency system comes into play as well. After 2011, the Milan tables were adopted countrywide and more negative standardised residuals are included due to more recent years in the diagonal region.

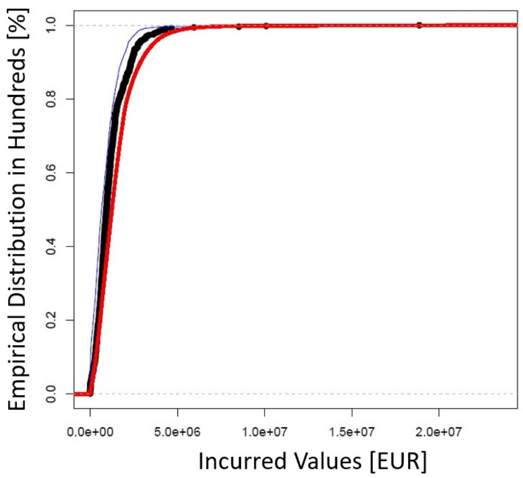
Overall, the changes of the court tables and a better estimation of possible court compensations drive the claims in Italy from 1998 till 2015 for the available dataset. Due to more recent court decisions, a development towards annuity compensations seems more likely. Thus, the risk of legal changes is still high in Italy. This would most likely result in an accident year effect since more recent accident years would contain claims with annuity payments. Thus, the impact of an accident year effect is of great interest in the Italian market. Besides that, it has to be mentioned that the seen calendar year effect most likely stems from the seen accident year effect in Subfigure 6.7a.

6.2.3. Backtest Procedure Results

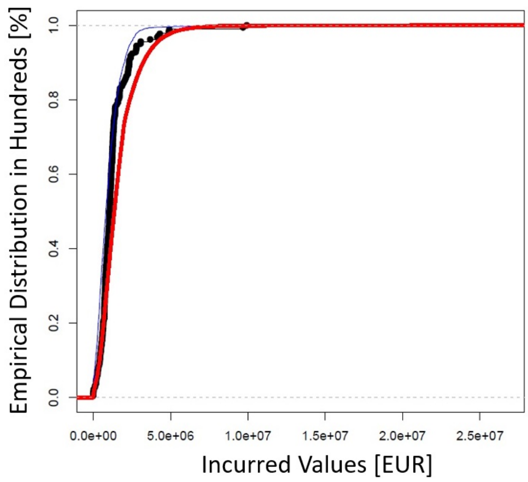
Under these circumstances, the results of the backtest procedure described in Section 5.3.3 are discussed for the Italian market. Compared to the other countries, Italy is special due to the common usage of lump sum payments and the accident year effect. In order to analyse the simulated distributions, the impact of the accident year effect has to be clarified. Hereby, two different impacts have to be considered. On the one hand, there is the impact for an increasing number of simulated development years and one the other hand the impact for an increasing number of known development years. Therefore, it has to be kept in mind that the average LDFs in more recent accident years are smaller compared to older accident years. Starting with a small number of known development years this leads to a mixture of larger and smaller LDFs which increases the volatility of the simulated outcomes. This is strengthened even more by the reduced impact of the claims' history for those claims. With an increasing number of simulated development years, the impact of earlier accident years is increasing since only single claims with the respective number of development years can be compared. While this can be offset partly by the copula, which is calibrated on a mixture of smaller and larger LDFs, it still leads to deviations.

With an increasing number of known development years, the impact of smaller LDFs is reduced. More recent accident years with smaller LDFs are not considered then and the claim paths of older years are the basis for the simulations. Thus, open claims at that stage are developed on the basis of higher LDFs. However, the impact should be minor since many claims are already settled by then. Overall, the impact of the accident year effect should lead to minor deviations. This should result in slightly reduced claim sizes for a smaller number of known development years. With an increasing number of known development years, this impact should reduce. However, it is expected that the deviations resulting from the accident year effect might also disappear in the normal volatility of the simulations.

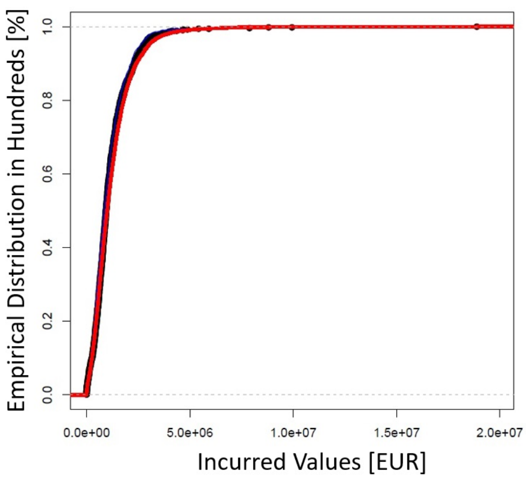
Keeping this in mind, the starting, actual, and simulated distribution is analysed in Figure 6.8 for a representative selection of the distribution fits. Furthermore, the average squared distance between the simulated and the actual distribution is shown in Figure A.16 on page 216 in the Appendix.



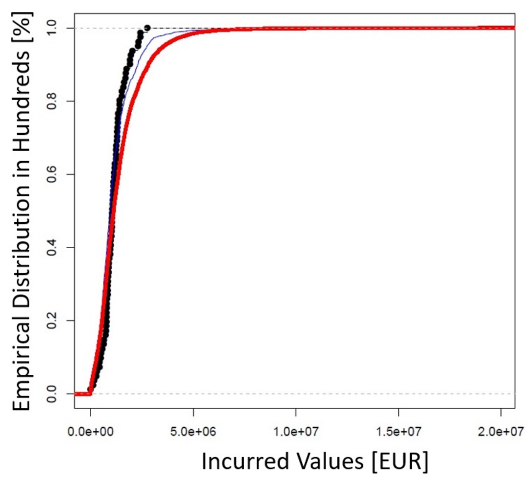
(a) DevYears Known: 1, Predicted: 10.



(b) DevYears Known: 2, Predicted: 14.



(c) DevYears Known: 4, Predicted: 8.



(d) DevYears Known: 8, Predicted: 16.

Figure 6.8.: Severity Distribution Comparison for the Italian MTPL Market.

In earlier years, the actual distribution in Subfigure 6.8a is close to the starting distribution. Over the developments there is a slight shift towards higher claim amounts which is most notably for the tail. While smaller and medium sized claims are predicted well by the simulated distribution, the difference towards the actual distribution increases for larger claim amounts. However, above EUR 5 million the distributions are again close to each other. This is also shown in the average squared distances shown in Figure A.16 for the first known development. While there is a slight increase with more projected development years, this increases faster for the latest ones. The claim sizes projected are too high and the copula-based SLD model still simulates claim movements. This also accounts for two known development years in Subfigure 6.8b. However, the actual distribution is even closer to the starting distribution in this case and shows a slight increase for higher claim amounts. The simulated distribution fits the smaller claim amounts but cannot match the claims' behaviour for larger claim amounts.

While there is still some claim movement towards larger claim amounts in early years, this changes if four or more development years are known. As seen in Subfigure 6.8c, all distributions are close to each other and nearly identical. However, this is not necessarily related to the power of the copula-based SLD model, but to the large amount of settled claims showing no further development. Due to the lump sum payments, many claims are closed within the first four years leaving fewer open claims to be projected. This can also be seen in the residual squared distances in Figure A.16 which are smaller after three development years are known. The distance between the actual distribution and the simulated one is not increasing as much as for less known development years. However, this does not account for later simulated development years. Therefore, the distribution fits the smaller claim amounts well but overestimates larger amounts, resulting in larger distances.

An additional effect appears if eight known development years are considered in Subfigure 6.8d. While the simulated distribution shows more severe claims than the starting distribution, the actual distribution shows a positive run-off. However, this effect appears only for the 16th development year which is related to a different development pattern of these older years. Afterwards, the fit of the distributions is more similar to Subfigure 6.8c. Here, the data basis is getting to sparse and the impact of a better or worse single accident year on the related distribution can be observed.

Overall, the actual distribution stays close to the starting distribution and only shows a slight increase in claim sizes for the tail, which is not projected correctly for the simulated distribution. It stays close to the actual distribution until the 50% quantile but is more severe in the tail. For later development years, the lump sum payments are driving the better distribution fits. Due to the combination of the accident year effect, lump sum payments, and volatility of the simulations the impact of each single component cannot be assessed correctly.

6.2.4. Pricing Procedure Results

As next step, the Pricing Procedure as described in Section 5.3.4 is applied for the Italien MTPL market. Overall, the 50 simulations per single claim result in 19 simulation runs and split between training and test data. A layer of EUR 2 million xs EUR 0.5 million is used for this market. The comparison of the ultimate aggregated claim values per accident year are shown in Figure 6.9 for the aggregated methods and the copula-based SLD model. Hereby, the related boxplots per accident year and the average values shown as line are stated.

The ultimate values of the aggregated methods are close to each other until the accident year 2010. Then the Chain Ladder method based on the full triangle and the Cape Cod method are significant smaller than the Munich Chain Ladder method and the Chain

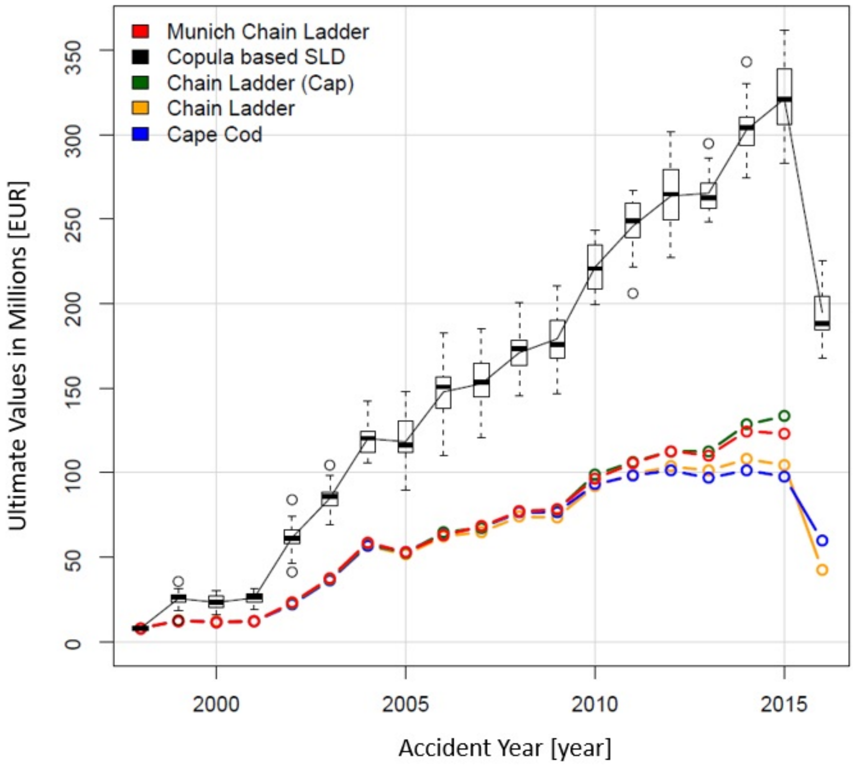


Figure 6.9.: Ultimates of Different Methods for the Italian MTPL Market.

Ladder without the last diagonal. Here, the last diagonal includes some positive run-off of single claims, leading to smaller loss development factors for later development years. However, the ultimate values are still close to each other if the results of the copula-based SLD model are compared to them. The interquartile distances and the whiskers for all years are small while the ultimate values increase significantly over the years. Only for 2016, the ultimate value drops similar to the ones of the Chain Ladder and Cape Cod since the information of the most recent accident year is not complete. The increased ultimate values over the accident years are related to the lump sum payments which are not handled correctly by the copula-based SLD model. This combined with the lag of simulating the positive run-off of claims more adequately leads to an overestimation of the ultimate values since the claims are not settled in time. Hence, the ultimate values per accident year increase steadily over time depending on the number of simulated development years. This should also result in higher frequencies at the priority which are shown in Figure 6.10.

The frequency of the Cape Cod model is close to the one predicted by the Chain Ladder model without the last diagonal and drops slightly for more recent accident years due to the smaller latest diagonal values. Apart from the large over prediction of the frequency for the oldest years, the copula-based SLD model constantly predicts around 50 claims more than the other models. This also shows that the claims are not settled in time and that the positive run-off is not modelled correctly.

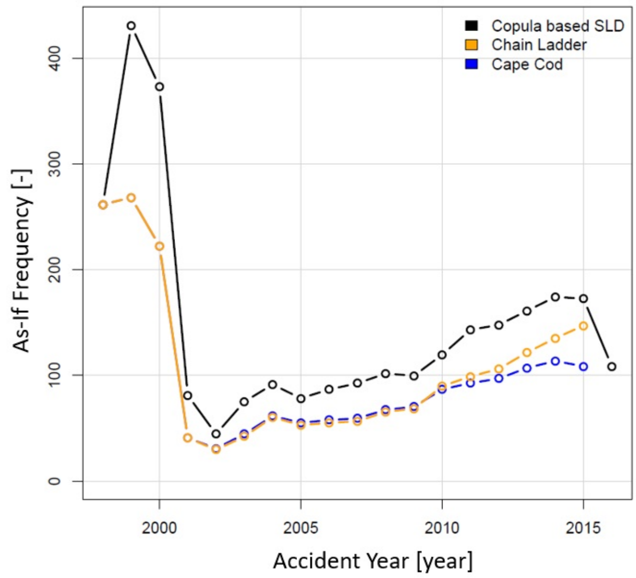


Figure 6.10.: As-If Frequency at EUR 0.5 Million for the Italian MTPL Market.

Overall, it can be seen that the copula-based SLD model does not simulate the lump sum payments correctly and cannot deal with a fast closing of the claims related to a positive run-off. It cannot predict this correctly, does not close the claims according to the pattern seen in the market and overpredicts the frequency as well as the ultimate claim values per accident year.

6.3. Example IV: Swedish MTPL Market

The fourth market that is analysed is the Swedish MTPL market with 766 claims in the period from 1979 to 2015. The related market characteristics that can influence the claims' development are analysed further.

6.3.1. Market Characteristics

Sweden has been a member of the European Union since 1995, which mainly influenced the development of insurance legislation. As such, EU insurance legislations and directives have been implemented in a timely manner like the Solvency II directive which was amended in 2009 with effective date January 2016. In contrast to most other European countries, there are no extensive legal codes and the Swedish laws are based on written legislation with case law playing a small but important role.

The Swedish society is not particularly litigious. Nevertheless, the number of legal actions did increase recently reflecting the economic cycle. As such, more cases effecting individuals or companies that suffered some form of financial losses are brought to court in less prosperous times. However, compared to other countries, the number is comparatively small and court actions are similar to other actions in other jurisdictions. Overall, there have been some important court actions resulting in extensions to case law, which might increase the likelihood of potential claims against local municipalities and authorities. However, there are no signs of a significant trend in this direction yet. Furthermore, the legal costs are high but do not necessarily deter private litigants since costs can be recouped with a favourable verdict. There is a limitation of ten years in which legal action for bodily injury must be brought to court, beginning with the plaintiff's first notice of the injury. Class actions are not allowed in Swedish law which has not resulted in a significant deterioration in the legal environment around claims' issues. For motor third party liability, strict liability is in force meaning that the claimant only needs to prove that the claim occurred and that the defendant was responsible for this. In general, injured parties have the right to appeal to court if there are disagreements over the compensation level but this rarely happens.

The compensation levels are low by international standard since the insurance market is used primarily to supplement state provision. The reason for this is that Sweden has an excellent social security system that covers a substantial part of the costs of any medical services or compensation payabel in the event of any personal injury. Thus, the insurance market has not been affected by any large individual claims and the largest MTPL claims have settled between EUR $0,92$ million and EUR $1,38$ million. However, there are plans for the private insurance market to take on more burden of motor insurance claims, but proposals to introduce this have been postponed so far. For MTPL death claims, awards vary considerably depending on whether there are any dependants involved[8]. The overall basis for compensations is the current status and the age, e.g. salary and the future expectations of salary and dependants. Compared to death claims, disability claims are more expensive and based on the percentage of disability[9]. For MTPL claims, the amounts awarded are calculated by the Road Traffic Injuries Commission and the loss of future income is paid as annuity while non-financial compensation is paid as a lump sum. Compensation for pain and suffering is therefore based on detailed tables. Differences in the paid amounts between insurers are based on the compensation of whiplash claims that may vary[10].

The insurance market in Sweden is well developed with new participants choosing niches and trying to get the best risks by making use of price comparison websites. This makes the MTPL market highly competitive which is, nevertheless, still profitable since the rates tend not to reduce. MTPL has been the main compulsory class in Sweden since 1975 and there are no tariffs in place. Therefore, insurers have to offer insurance on request, even if there are good reasons to refuse. It is estimated

[8]The partner of a deceased victim is usually not considered to be a dependant if this person is working. In the case of a child, the needs of the whole family are considered.

[9]The resulting percentage is applied to the victim's salary, multiplied by a factor to produce an amount for loss of future earnings.

[10]Note that whiplash claims are open to moral hazard.

that between 90% and 95% of motorists have at least a coverage for MTPL and limited property damage providing cover for up to EUR 27.5 million. In recent years, some insurers are making use of telematics by lowering the premium for safe driving or people driving less than 15,000 kilometres. Since the paydriving technical unit is connected to the vehicle, stolen cars can easily be tracked in those cases.

Despite the good market environment, the profitability of motor accounts is depending on how the motorists deal with the first snow of the winter. Many may not be prepared, still driving with summer tyres, which results in a considerable increase in motor accidents. A second issue in the market is that parents buy MTPL insurance for their children, pretending to drive themselves to lower the premium. Another problem affecting MTPL business are whiplash and fraud claims. The points mentioned above are drivers for the number of accidents that are shown in Figure 6.11 for the Swedish MTPL market since 1975.

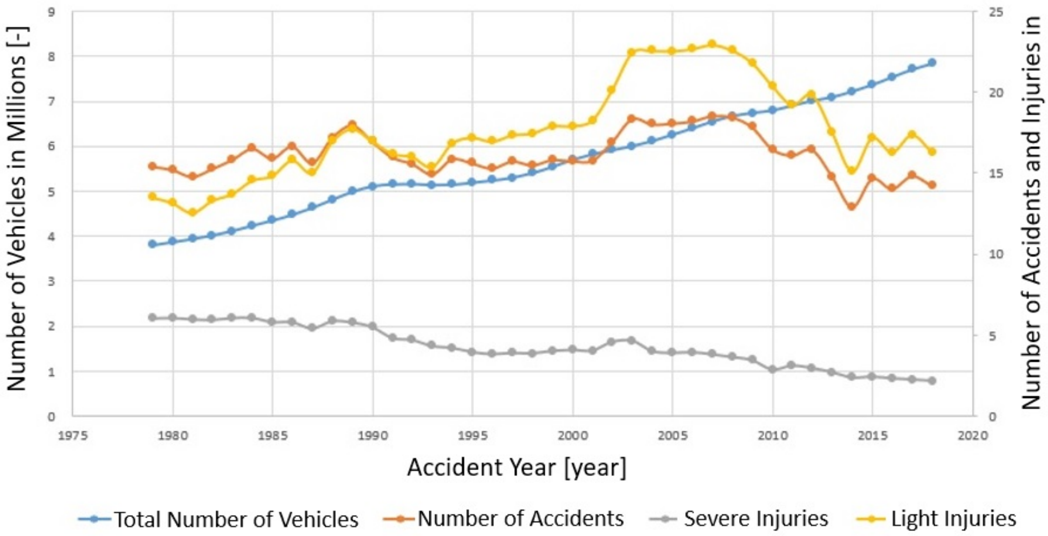


Figure 6.11.: Market Development for Sweden.

Besides the number of accidents, severe and light injuries [234], the figure also shows the overall number of motorised vehicles in use since 1979 [195][11]. The total number of vehicles grew nearly the whole time since 1979 except for 1993 where it remained constant. Overall, around 70% of the vehicles are passenger cars, reducing to 60% in recent years due to an increasing market share of trailors [236]. While the number of vehicles increased until the year 2000, the number of accidents remained constant while the number of light injuries showed a slight positive trend despite some volatile peaks. For the severe injuries, the number was constant until end of the 80's and decreased until mid of the 90's due to the implementation of airbags. In 1997, the 'vision zero' plan was decided, with the aim to reduce the number of persons killed or severly injured in motor accidents [233]. In order to achieve this, three main factors were

[11] People in Sweden can make an off road notification to mark their vehicle as not in use. Hence, they do not have to pay vehicle tax or traffic insurance. This leads to a large number of vehicles not in use compared to other countries [236].

agreed on. First of all, road traffic accidents should not lead to serious health loss. Accepting minor injuries, this is determined by the protection of the road user and the speed at which the accident happens. Secondly, the human capability and limitations has to be considered. Thus, the safety system has to be planned for mistakes and accidents that will most likely occur. The last factor is that the safety system is everybody's responsibility. The road operators, vehicle manufacturers, transport companies, legislators, the police, as well as the drivers have to work together to achieve this. Overall, this resulted in the construction of median barriers, speed limit reductions, road safety cameras, roundabouts, improved car safety, and speed bumbs in front of pedestrian crossings [233]. As a result, the number of severe injuries has decreased steadily since 2003 and today, Sweden has one of the safest traffic systems in the world.

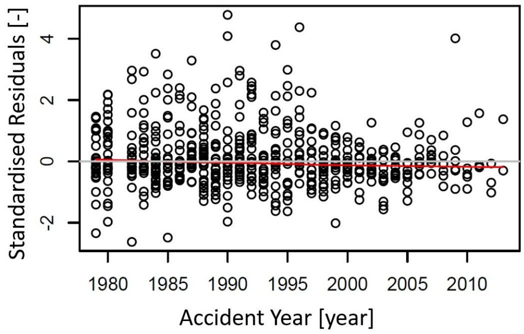
In respect of the number of accidents and light injuries, both increased drastically during the late 1990 and early 2000s. This is related to the 'vision zero' plan. Since minor injuries were accepted in order to reduce the number of severe injuries, this can be regarded as a trade off. However, both key numbers have reduced again since 2008. Another factor is the number of whiplash claims that increased dramatically due to stronger car seats in modern cars leading to around 60% of all claims disability causes. However, the high number of whiplash claims have alleviated since 2015. It was made more difficult for individuals to be certified for having a degree of disability. This led to a reduction in reported claims' frequency. However, this has also extended the tail on motor claims. Currently, the outlook for the Swedish motor market continues to be positive as new vehicles have become affordable bringing more modern vehicles on the roads.

Overall, the market has been chosen for the following reasons:

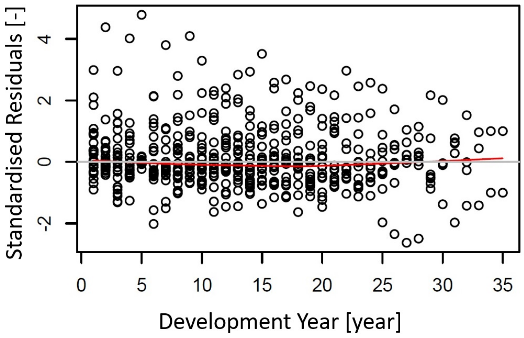
- There is a limited number of large losses reported to reinsurers.
- The known reinsurance claims are long tailed due to the payments of annuities.
- There are many IBNYR claims, especially in later development years.

6.3.2. Data Analysis, Trends, and Influential Effects

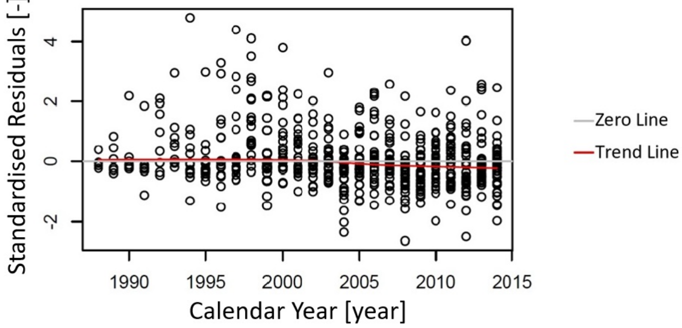
As described in Section 2.8.1, the standardised Chain Ladder residuals are analysed to show possible influential effects and trends for the Swedish MTPL business. In Figure 6.12, the standardised Chain Ladder residuals are shown in respect of the accident, development, and calendar years. However, it has to be kept in mind that possible effects cannot be eliminated from the data on a single loss basis. Thus, this analysis focuses on identifying possible influences for the later analysis. Therefore, the whole Swedish MTPL market data available is considered. It has to be noted that there is a large number of IBNYR claims, especially for older accident years leading to a different number of years shown for the accident and the calendar years.



(a) Accident Year.



(b) Development Year.



(c) Calendar Year.

Figure 6.12.: Standardised Residual Plot for the Swedish MTPL Market.

Firstly, the standardised residuals with respect to the accident year are shown in Subfigure 6.12a. The residuals vary around zero and are distributed randomly. The overall trend line is nearly zero for the whole period. However, there are more higher residuals until 1998 than afterwards. This is related to the 'vision zero' plan reducing the volatility and making the development of the claims more stable. However, it has to be kept in mind that the MTPL business is long tail based, especially for the Swedish market, and that further run-off of the claims can still be expected. Moreover, there is a small tendency to higher average residuals in more recent years that can be explained with an overall increase of the paid amounts per claim leading to higher development factors.

Secondly, the influence on the development years is analysed in Subfigure 6.12b. Similar to the accident years, the residuals do not show a significant trend. There are more higher residuals in early development years and more volatility can be observed after 25 years. However, this is most likely due to sparse claims data for such a long period.

Thirdly, the standardised residuals plotted against the calendar years are shown in Subfigure 6.12c. Similar to the other analysed periods, no specific trend can be observed for the calendar years either. The residuals are distributed randomly and the trend line does not show a significant trend. However, there are some years with a tendency towards higher residuals in 1993 and 1998 as well as towards more negative residuals in 2004 showing the market reactions with respect to the development of the number of accidents and injuries. In 1993, this can be seen as a reserve increase and, thus, higher LDFs compared to the other years. Similarly, the residuals in 1998 is a safety reaction to account for the volatility of the accidents and the increasing number of light injuries. In 2004, the residuals are smaller compared to the other years signalling a release of reserves towards a more positive future.

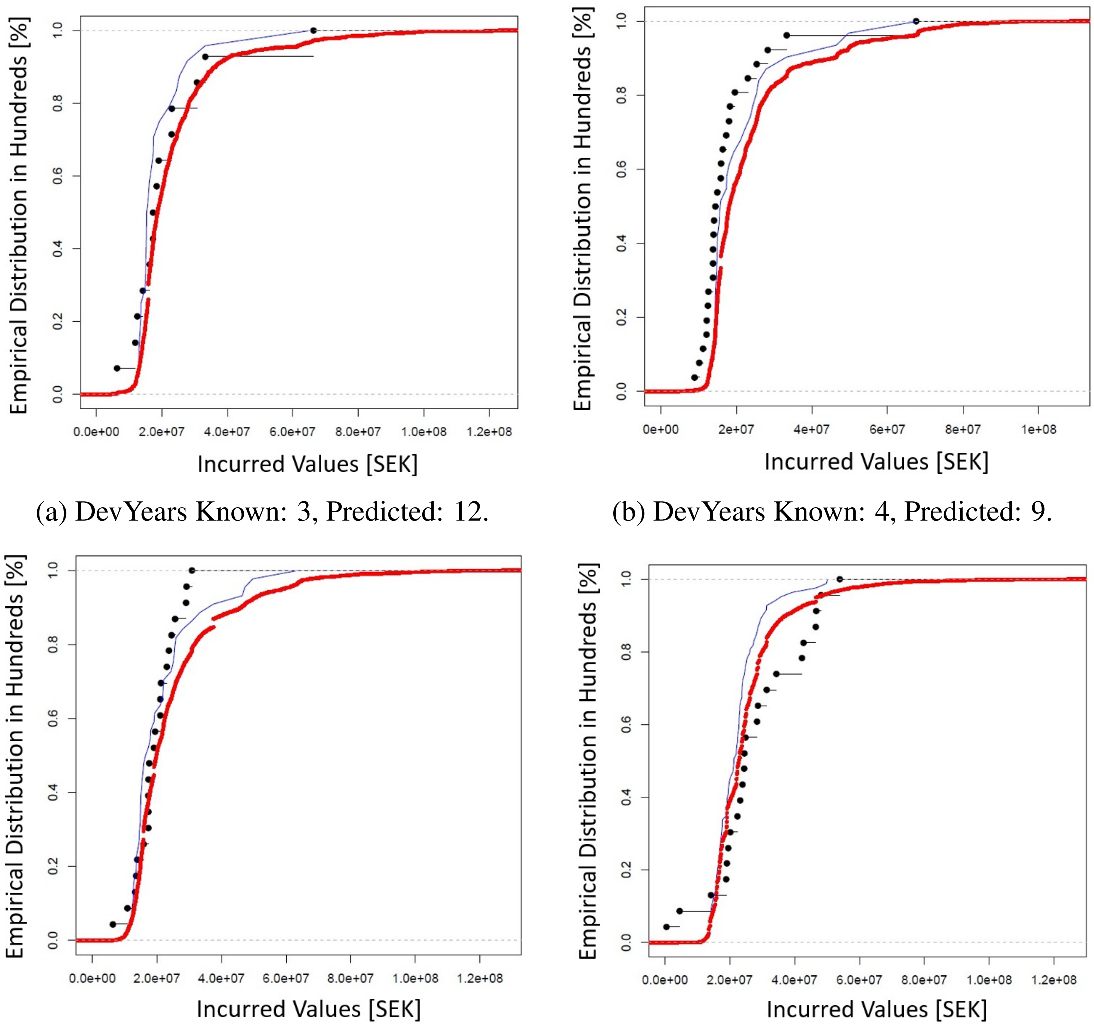
Overall, the Swedish MTPL market does not show significant trend effects for any of the analysed periods. However, this does not mean necessarily that there are no trend effects for single claims since multiple effects might cancel out.

6.3.3. Backtest Procedure Results

In this context, the results of the backtest procedure described in Section 5.3.3 are analysed for the Swedish market. Therefore, the focus is set on the starting, actual and simulated distribution in Figure 6.13 showing a selection of the distribution fits that is representative for the seen observations. Additionally, the average squared distance between the simulated and the actual distribution are shown in Figure A.17 on page 217 in the Appendix.

Sweden is a country that has many IBNYR claims. Only around 100 of the 766 claims in the market portfolio are known in the first development year. All others are IBNYR claims that can have a long time delay until they are reported. Thus, the database across the development years is sparse. One reason for this is the 10 year delay in which legal action for bodily injury can be brought to the courts. Furthermore, the claims can have a long runtime due to the annuity character. In combination with the reducing number of claims due to a higher development year, this automatically leads to a higher volatility of the claims which can be seen in Subfigure A.17 for the first known development year. Here the average squared distance increases strongly until the 19th development year and then reduces to one third. This results from a reduced number of claims having no 20th development year yet. This behaviour can be observed for all later years and can also go in a positive direction, e.g. around the known development years 3 to 10 and 19 to 26. Thus, those years are not appropriate for a sophisticated assessment of the model.

However, this effect also influences the other distribution fits for more known development years. In Subfigure 6.13a three development years are assumed to be known and the distributions in the 12th development year are compared. The number of claims that can be used for this analysis is already small. While the actual distribution is still close to the starting distribution, a positive run-off towards smaller claim amounts



(a) DevYears Known: 3, Predicted: 12.

(b) DevYears Known: 4, Predicted: 9.

(c) DevYears Known: 5, Predicted: 14.

(d) DevYears Known: 16, Predicted: 25.

Figure 6.13.: Severity Distribution Comparison for the Swedish MTPL Market.

can be observed for 20% of the claims and a negative run-off for around 50% of the claims. While the positive run-off is not simulated by the copula-based SLD model, the negative run-off leading to higher claim amounts is fitted well in this case. For more simulated development years, the fits worsen which also leads to a higher average squared distance between the distributions shown in Subfigure A.17.

Adding one more known development year, as shown in Subfigure 6.13b, the previous observation changes completely. The actual distribution shows a positive run-off for all claims in comparison to the starting distribution in blue. The simulated distribution of the copula-based SLD model is close to the starting distribution up to 30% probability and shows a negative run-off afterwards. The tail of the simulated distribution is heavier compared to the starting and actual distribution. Thus, the copula-based SLD model overestimates the claim amounts. This also accounts for the distributions shown in Subfigure 6.13c where another known development year is added. While the actual distribution shows fewer higher claim amounts compared to the starting distribution, the simulated distribution follows the shape of the starting distribution and is just shifted slightly towards a heavier tail. Overall, the reduced claim sizes of the actual distribution and the tendency towards a positive run-off is not projected correctly. This can also be seen in the average squared distances that are small if few years are simulated. For more simulated development years, the shapes and different claim developments are not projected well.

The last Subfigure 6.13d shows the distributions if started with sixteen known development years. While all distributions are close to each other for medium sized claims, the positive run-off towards small claims is not projected correctly. In the case of large claims, this differs compared to the previous observations. Here, the actual claims are getting more severe in later development years which is not projected correctly by the simulations. Moreover, the simulated distribution in Subfigures 6.13c and 6.13d have a different shape compared to the actual distribution. Here, the claims limiting character of the Swedish market is not applied correctly by the copula-based SLD model for the actual distribution. However, due to the legal environment and the annuity character of the claims, there is still a lot of development in the actual distribution even for 16 known development years.

Overall, the fit of the simulated distribution to the actual one is not satisfactory. This is driven by the different behaviour of the claims which can change by adding one more known development year. This has a huge impact on the distribution and the shape and might lead towards a positive run-off for smaller claim amounts or a negative run-off towards higher claim amounts. In general, it appears that the simulated distribution is more severe and has a heavier tail if few development years are known. This is the opposite if more development years are known and strongly influenced by the large number of IBNYR claims and their long reporting delay. Here, the shape of the simulated distribution is not in line with the actual distribution and does not catch the slower claims' development within the Swedish healthcare environment.

6.3.4. Pricing Procedure Results

As next step, the pricing procedure as described in Section 5.3.4 is applied to the Swedish MTPL Market and the ultimate values per accident year of the copula-based SLD model and commonly used aggregate models are compared. Considering 50 simulations per single claim resulted in 212 simulation runs and splits between training and test data. A layer of SEK 80 million xs SEK 20 million is used for this market[12]. First, the ultimate aggregated claim values per accident year as shown in Figure 6.14 are compared. The related boxplots per accident year and the average values, shown as line, are stated for the copula-based SLD model.

[12]For a rough estimation between SEK and EUR use an exchange rate of 10 to 1.

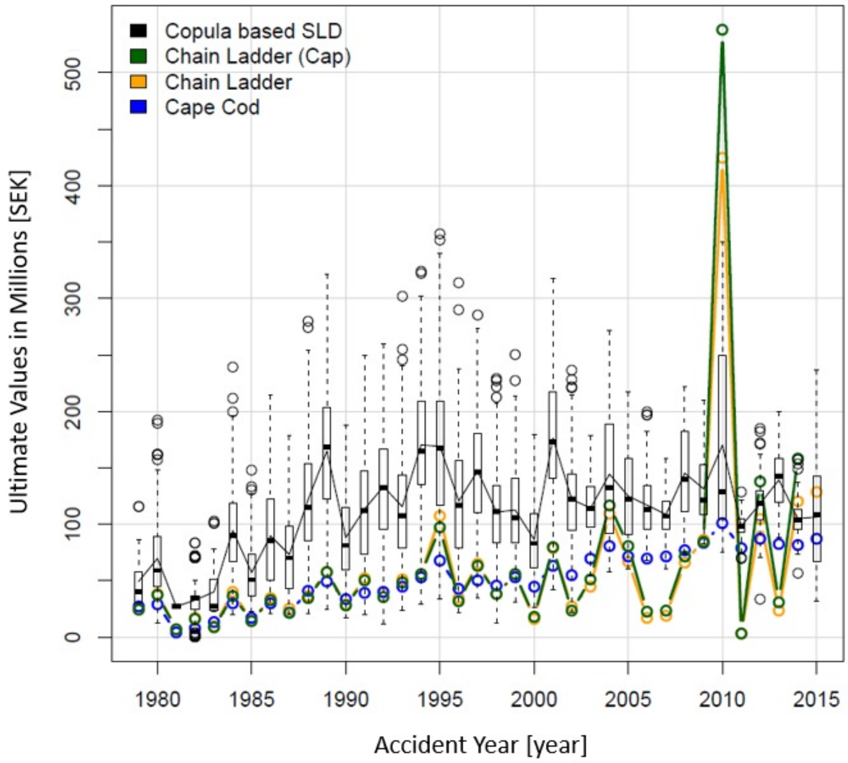


Figure 6.14.: Ultimates of Different Methods for the Swedish MTPL Market.

The ultimate values of the Munich Chain Ladder model cannot be estimated since the test triangles are too incomplete. The ultimate values of the other aggregated models are close to each other again. The impact of the last diagonal of the test data triangle is minor here. However, the results of the Chain Ladder methods show a higher volatility compared to the results of the Cape Cod method that is smoother over the years. In 2010, the ultimate values predicted by the Chain Ladder method are high compared to the values of other accident years. The reason is that there are some large outstanding claims which are indexed and then predicted further leading to these high results. While a large amount of the claim value is still outstanding, this is predicted differently by the copula-based SLD model leading to a smaller average ultimate value in that year. For years before 2010, the copula-based SLD model predicts higher ultimate values per accident year. Hereby, the interquartile distances are small and the full length of the whiskers is usually reached. There are simulations where the ultimate values are more than 3 times higher than for the aggregated models and other simulations where they are smaller. This shows a large volatility in the claim results which is driven by the long tail character of the claims in this market in combination with larger outstanding values due to the annuity character. In more recent years, the results of the copula-based SLD model are getting closer to the results of the aggregated models since the IBNYR claims are missing for those years. This can also be seen in the frequency per accident year at the priority, which is shown in Figure 6.15.

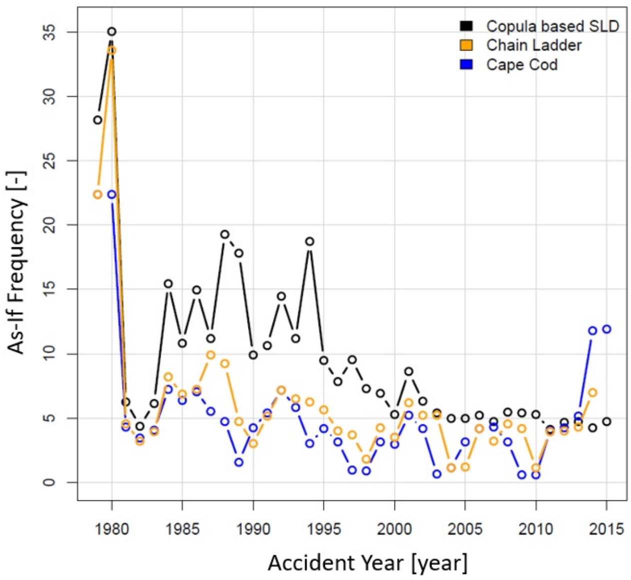


Figure 6.15.: As-If Frequency at SEK 20 Million for the Swedish Market.

The high frequency in 1979 and 1980 is driven by the as-if view of the frequency which is scaled upwards due to the small exposure values in those years. Afterwards, the copula-based SLD model shows a slightly higher frequency until 2000 than the aggregated models, which are close to each other. Afterwards, the frequency prediction is more similar which is also in line with the ultimate values for those years. In 2010, the frequency of the aggregated models is small which shows that there are only a few but large claims. Additionally, the frequency is decreasing slowly since 2001 which is related to the improvements in driving safety but also to the IBNYR claims. The weaknesses of a lag of positive run-off is not so pronounced for the Swedish market due to the long tail character and the appearance of mainly a few but larger claims in the market.

Overall, the ultimate values show a large volatility per accident year due to the high outstanding amounts of the claims, which comes from the long tail character of the Swedish market. This results in a higher frequency compared to the aggregated methods for older accident years, which is also driven by the lag of handling the run-off correctly. In more recent accident years, the predictions of the models are more similar for the ultimate values as well as for the frequency. The disadvantages seen for other markets, e.g. Italy, are not as pronounced for the Swedish market since there are less claims and also less development in the claims.

6.4. Example V: Danish MTPL Market

The last market that is analysed is the Danish MTPL market. Here, $2,580$ claims in the period from 1995 to 2016 are available and the market specific characteristics and influential effects on the claims' developments are analysed further.

6.4.1. Market Characteristics

Denmark is a member of the European Union and had to implement all relevant directives on insurance. This has been the main driver for new insurance legislation in recent years and major changes in the insurance-related legislation only occur intermittently.

The Danish society is not particularly litigious even though this has increased in recent years, especially in order to seek legal assistance for insurance cases where bodily injury is involved. Even if the legal fees are expensive, it does not necessarily deter litigants. A complaints board was set up by the insurance industry and the Danish Consumer Council to settle disputes between policyholder and insurers. However, an insurer may decide not to follow the board's decision but if he does so, the policyholder may take the claim to court. In practice, the board's decision is changed rarely and only a few decisions have been changed. Generally, this is only done to establish blame to a party and rarely to change the payable compensation amount. Additionally, it can take years to go through the whole court system. Besides that, strict liability is in place of MTPL, irrespective of the driver's fault. While class actions are possible in Denmark for claims too small to be litigated separately, there have only been a few major cases in the past none of which had any lasting impact on the legal environment. Ultimately, around a quarter of insurance cases are reported to be whiplash or whiplash-like injuries making up 30% of personal injury claims following road accidents in 2019.

The awards rewarded for damages in liability cases are set according to the law on compensation while the degree of disability is set by the Labour Market Insurance. Even if the rewards for bodily injury have increased in the early years of the 21st century, the awards are still modest compared to international standards since hospitalization and care costs are paid by the Danish public security and welfare system [123]. However, moral hazard has increased the claims' burden within MTPL to some extent in recent years. Serious disability claims are usually paid as lump sums, are subject to annual, inflation-linked increases, and are more expensive than cases involving death[13]. Overall, disability awards consist of several components. Besides the medical cost, the loss of current earnings, pain and suffering, permanant disability, loss of futures earnings, and loss of provider can be claimed [123]. Overall, the maximum amount

[13] A few death awards exceeded DKK 2 million ($268,000$ EUR; 01.06.2021).

payable for an individual disability claim could be DKK 10 million[14]. However, such high amounts are not granted commonly. Larger claims up to DKK 50 million might be payable for Green Card claims[15]. Additionally, there are interest payments on the compensation that are set when all information required to ascertain the claim is received, which might take several years. Overall, the cost of the wages has increased a lot over the years and has to be paid until the case is settled rather than until stabilisation of the disability. Thus, the statutory limits are indexed annually and were DKK 121 million for third party bodily injuries and DKK 24 million for property damage in 2019. There are high costs of purchasing and repairing motor vehicles and it is possible to get an extra compensation of the repair costs. Claims can be re-opened within five years under the condition of a worsening disability.

The Danish insurance market is well developed and pretty much saturated since MTPL insurance is compulsory for all motor-powered, wheeled vehicles above five horsepowers[16]. Due to the weak economy in recent years, the motor segment is under some pressure. The motor portfolios have changed towards smaller, less powerful, safer and cheaper vehicles, resulting in lower risks and lower premiums. Additionally, competition is driving the decrease of the motor rates as well. Thus, extra services such as road assistance is being packed with insurance cover, creating the market fear of underpriced insurance elements. Nevertheless, the results are still positive over the last years and the number of vehicles has been increasing steadily as shown in Figure 6.16. Here, the market developments, according to official police statistics, are stated [211] for the total number of vehicles, accidents, and injuries in Denmark since 1996.

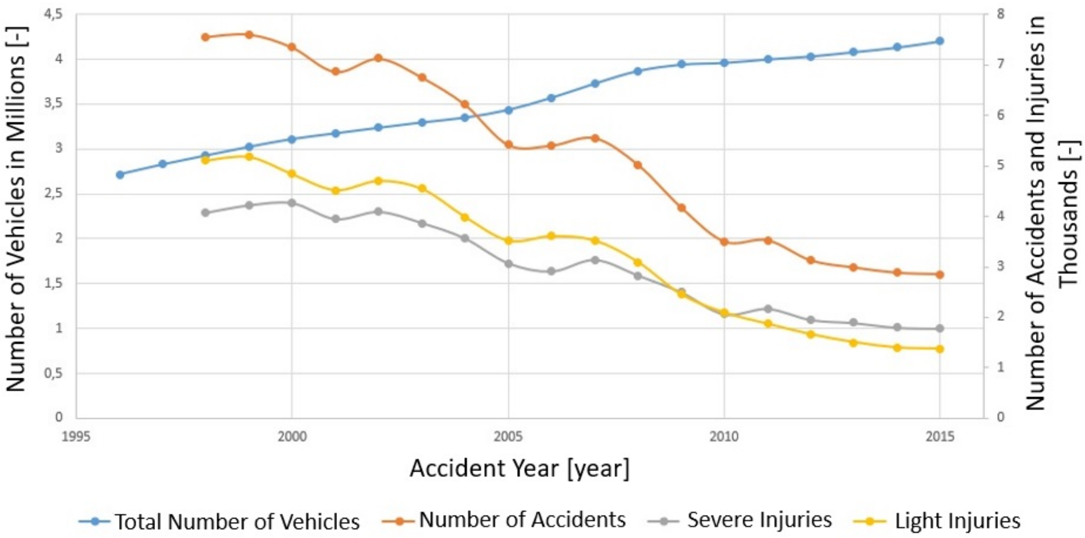


Figure 6.16.: Market Development for Denmark.

[14]EUR 1,345 million; 01.06.2021.

[15]The Green Card system allows a person's MTPL insurance to be valid in another country that is part of the Green Card system. However, the compensations that have to be paid are set out according to the law of the country where the accident occurred.

[16]There is no obligation to insure trailers.

Besides the number of vehicles [210], the total number of accidents recorded by the police [211] as well as the number of severe and light injuries [212] are stated here[17]. The increase in the total number of vehicles is mostly driven by the private car segment followed by trailers which do not necessarily need to be insured. Meanwhile, the number of accidents and the number of injuries decrease, showing a simultaneous and cyclic behaviour. This is driven by several factors. First of all, the motor insurer's loss experience is strongly driven by the harshness of the winter. Hereby, most claims occur on the first days of winter when there is significant snowfall. In those times, the claim frequency tends to double. However, most traffic accidents during the winter month are less severe and cause less serious injuries than accidents during better weather conditions. Since the Danish winters have been mild and relatively snow-free since 2007, the number of accidents has improved greatly over the years. Secondly, Denmark has strategy and action plans to improve the road safety since 1988 set up by the Road Safety Commission [184]. Currently, their fourth plan has been in action since 2013 following the second plan in 2001 and the third one in 2006. After the implementation of those plans, there was a slight increase in accidents followed by a major improvement. Overall, it can be said, that the cyclic behaviour is related to the different strategy and action plans. Overall, those plans focus on improving the road safety and reducing the number of deaths and severe road accidents by focusing on speeding, alcohol and drugs, inattention, usage of seat-belts and helmets, young drivers, as well as rural junctions [53]. Additionally, this is strengthened by changing legislation, vehicle safety, and road engineering which resulted in a reduction of accidents and injuries. However, since 2010, the number of severe injuries has been larger than the number of light injuries. This is a result of the classification of severe injuries in Denmark. Every person that has an injury other than 'minor injuries only' counts as seriously injured [53].

Conclusively, the Danish market has been chosen for following reasons:

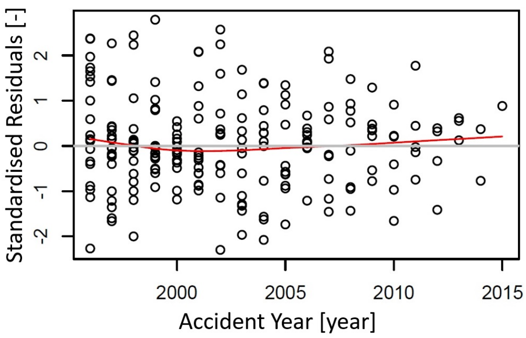
- There is a similar compensation level as in Sweden.
- Sweden and Denmark both have a good road safety.
- However, the motor claims in Denmark have a shorter settlement time than claims in Sweden due to lump sum payments in disability cases.

6.4.2. Data Analysis, Trends, and Influential Effects

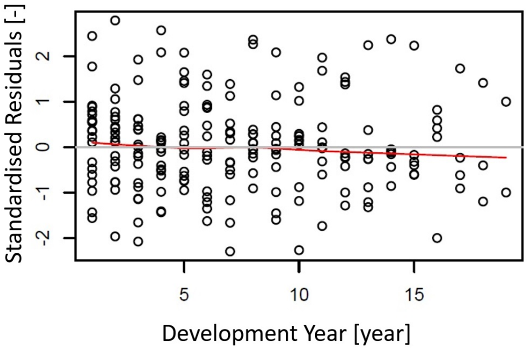
The standardised Chain Ladder residuals, as described in Section 2.8.1, are analysed for the Swedish market to show possible influential effects and trends for the MTPL business. Those residuals are shown in Figure 6.17 for the accident, development, and

[17] No earlier data is available for the number of accidents and injuries.

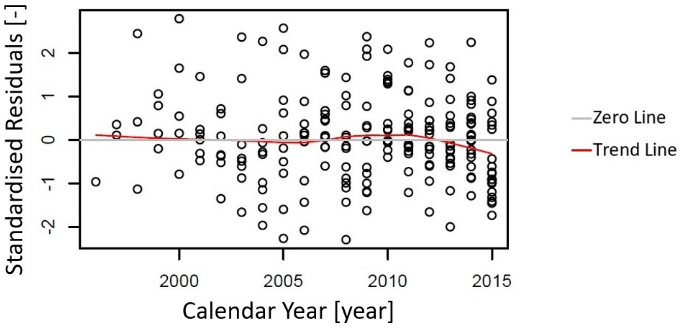
calendar years, focusing on identifying possible trends in the market. Therefore, the whole Danish MTPL market data available is considered for this analysis.



(a) Accident Year.



(b) Development Year.



(c) Calendar Year.

Figure 6.17.: Standardised Residual Plot for the Danish MTPL Market.

In Subfigure 6.17a, the standardised residuals with respect to the accident year are shown. The residuals do not show a significant trend and the trend line is stable around zero. However, there is a slight tendency towards clusters of smaller residuals before 2001, which cannot be observed afterwards again. Additionally, the volatility of the residuals decreases over the years which is most likely related to IBNYR claims.

The influence of the development years is analysed in Subfigure 6.17b. Similar to the accident years, there is no trend effect observable in the residuals. However, in later years after the 12th development year, the residuals tend to be below zero while there are some higher residuals keeping the red trend line around zero. However, this is driven by the increases of outliers in a sparse claims environment leading to a higher volatility of the residuals.

With respect to the calendar years shown in Subfigure 6.17c, the residuals show no trend effect either. In earlier calendar years, there is a slight tendency towards smaller residuals, especially in the early 2000s, balanced due to some higher residuals in respect of the trend line. This might be related to the decreasing number of injuries and accidents leading to a more optimistic reserving behaviour of the primary insurance companies. However, the years after 2004 do not show this behaviour and the residuals are spread around zero. In the last years, after 2012, the trend line goes down due to more negative residuals on the latest diagonals. However, the impact is minor and driven mainly by the latest diagonal related to the calender year 2015.

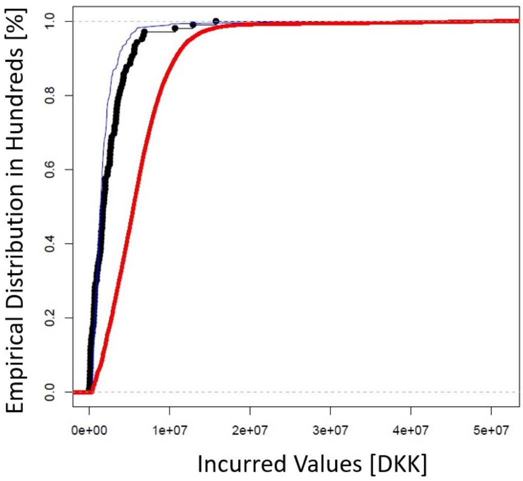
Overall, the Danish MTPL market does not show significant trend effects for any of the analysed periods. However, the residuals are affected strongly by outliers in the data, especially for later development years where only sparse claims data is available. This ultimately makes the residuals volatile. Finally, it has to be kept in mind that this does not necessarily mean that there are no trend effects for single claims since multiple effects could cancel each other out.

6.4.3. Backtest Procedure Results

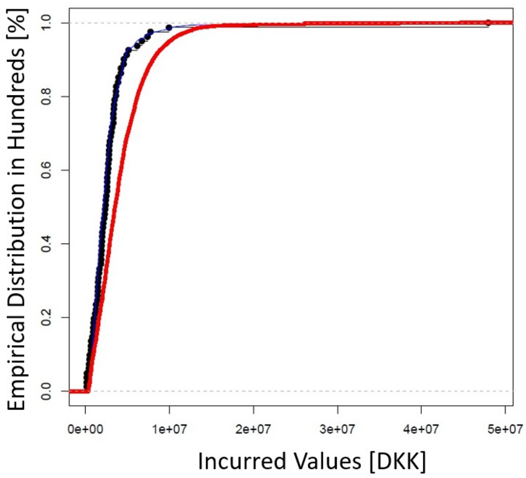
In this section, the results of the backtest procedure described in Section 5.3.3 is discussed for the Danish market. Therefore, the starting distribution, the actual distribution, and the simulated distribution are compared in Figure 6.18 and the average squared distance between the actual distribution and the simulated distribution is shown in Figure A.18 on page 218 in the Appendix. Since the backtest procedure is done sequentially over all development years, only a representative selection is shown.

In Subfigure 6.18a, the actual distribution in black is getting slightly more severe compared to the starting distribution in blue, representing increasing claim amounts between the first and the eleventh development year. The shape of the distribution is close to the starting distribution up to a probability of 60%. The number of smaller claims and their claim sizes are similar to the starting distribution, while 40% of the claims show an increased claim amount. In comparison to that, the simulated distribution is far away from the actual distribution and all claims increase. Furthermore, there is a high probability of simulating claims above DKK 10 million while those claim amounts are not common. This can also be seen in the average squared distance that is increasing strongly during the first few years and shows a clear deviation between the actual and the simulated distribution. Overall, the simulated distribution does not fit the actually seen distribution and is too severe, even by considering the impact of missing IBNYR claims.

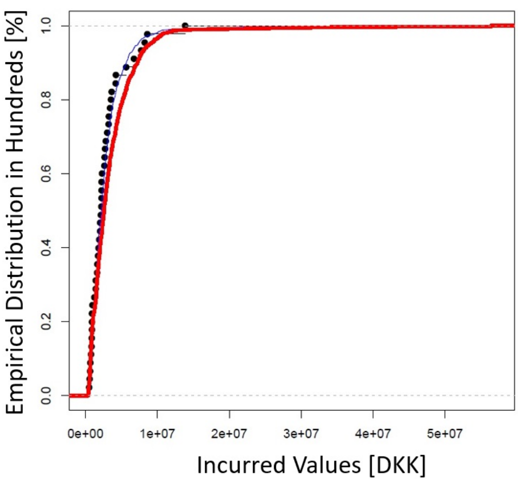
The behaviour of the distributions as seen above continues until three to four development years are known, shown in Subfigure 6.18b. The actual distribution in year 14 does not differ much compared to the starting distribution. Most claims are already closed after four years since the court cases usually take up to three years and lump sums are usually paid for disability claims. Thus, the majority of claims is already closed and only a minority of the claims has the possibility to be re-opened or are paid as annuities with a limit claim amount increase. However, the simulated distribution in red still overestimates the claim amounts as can be seen in the average squared



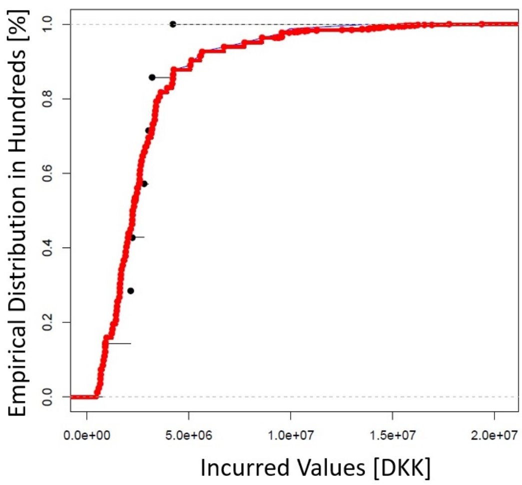
(a) DevYears Known: 1, Predicted: 11.



(b) DevYears Known: 4, Predicted: 14.



(c) DevYears Known: 7, Predicted: 15.



(d) DevYears Known: 14, Predicted: 19.

Figure 6.18.: Severity Distribution Comparison for the Danish MTPL Market.

distances, which is clearly smaller for higher development years if four or more development years are known. Even if the distribution fits are better up to 30% probability, the shape afterwards does not fit the actual distribution. This does not change with more known development years as shown in Subfigure 6.18c. There is no significant improvement for the average squared distance for more known development years. The simulated distribution in red is closer to the actual and starting distribution due to more closed claims and less claims that can have a development.

In later development years, the claims' database is too sparse to make a sophisticated statement and to do a meaningful analysis as shown in Subfigure 6.18d. The residuals are higher if twenty or more years have to be developed no matter how many years are already known. Overall, the claims' development and the simulated claims' distributions by the copula-based SLD model do not fit to the actually seen claims' distribution and are not sufficient. The fit is improving with more known development years which is due to the fact that more claims are closed and less are left to be developed. Thus, similar to the Italian market, the lump sum settlement of claims is not projected correctly.

6.4.4. Pricing Procedure Results

The pricing procedure as described in Section 5.3.4 is applied to the Danish MTPL market. Hereby, the ultimate values per accident year of the copula-based SLD model are compared to the ones derived from the aggregated models. For this analysis, 116 simulation runs and splits between training and test data are performed based on 50 simulations per single claim. A layer of DKK 7 million xs DKK 3 million is used for this market. The related ultimate values per accident year are shown in Figure 6.19. The results of the copula-based SLD model are shown as boxplots and as line for the average ultimates.

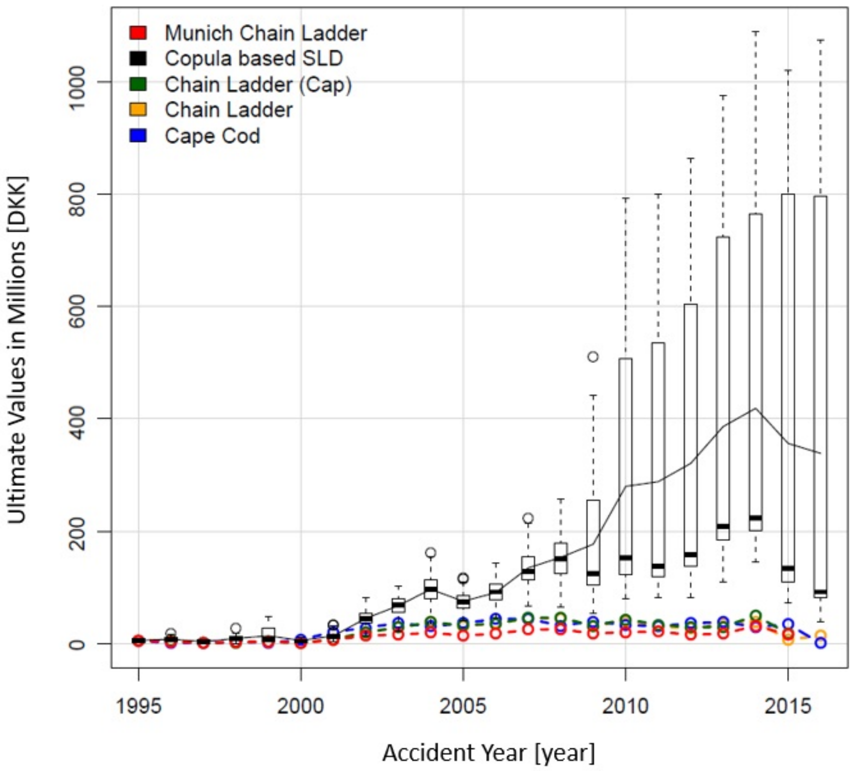


Figure 6.19.: Ultimates of Different Methods for the Danish MTPL Market.

It can be seen that the results of the aggregated methods are all close together and that only the Munich Chain Ladder method predicts slightly smaller values than the other aggregated models. The copula-based SLD model predicts similar values as well until the early 2000s after which the ultimate values start to increase for more recent accident years. This is related to the characteristics of the market. Since the claims are settled fast by law, this means that there are nearly no open claims left before 2000 which leaves less space for development of the claims. Afterwards, more claims are open, which results in a steady increase of the ultimate values towards more recent accident years. However, it can be seen that the interquartile distances are small for accident years before 2009 and that the means are close to the median values. Additionally, the whiskers are small compared to the later accident years. Here, the median of the predicted values is still small while the mean, the 75% quantile, and the upper whisker are increasing significantly. The distribution of the ultimate values in those years has a high right skew since an increasing number of simulations result in high ultimate values for those accident years. Hereby, more claims are open for those years which leaves more space for further predictions. Since the copula-based SLD model does not simulate positive run-off correctly, an overprediction of the ultimate values for single claims appears which is also observable for the backtest procedure. Consequently, this leads to the seen overestimation of the ultimate values for more recent accident years. This is also the case for the as-if frequency at the priority shown in Figure 6.20.

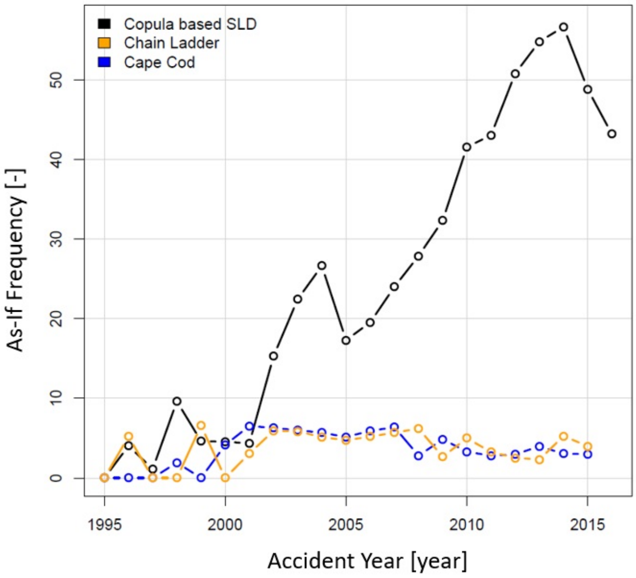


Figure 6.20.: As-If Frequency at DKK 3 Million for the Danish MTPL Market.

In respect of the frequencies, all as-if frequencies are close together until 2001. Then, the frequency of the copula-based SLD model starts to increase significantly for later accident years while the frequencies of the Cape Cod and Chain Ladder method are more steady and still close to each other. This behaviour of the frequencies is similar to the seen behaviour of the ultimate values. Hereby, a slight increase between 2001 and 2004 with a slight decrease in 2005 can be observed which occurs for the frequency as well. This shows again that the positive run-off of the claims is not handled correctly since this leads to a higher frequency at the priority and higher ultimate claim values per accident year.

In summary, the claims in Denmark are settled fast by law which is not simulated correctly by the copula-based SLD model. This leads to an overestimation of the frequency at the threshold and, consequently, to an overestimation of the ultimate values per accident year. This is more pronounced for more recent accident years since more claims are open at that stage leaving more space for the copula-based SLD model to predict the claims.

6.5. Discussion

To conclude this study, it has to be kept in mind that the analysis done so far is based on empirical data which does not allow a general statement. As seen in the market studies done in Chapter 5 and 6 for the backtest procedures, the distribution fits of the ultimate values per development year are better if the underlying data is less volatile and contains less jumps in the claim paths. However, this is related to the jump pattern, which pulls the simulated claims' distribution towards the starting distribution. This naturally allows a better performance of the copula-based SLD model with claims showing less volatility. In the case of markets with a short claim lifetime, e.g. Italy or Denmark, the copula-based SLD model does not work as intended. The short development period of the claims as well as the lump sum payments are not simulated correctly and the underlying pattern of the real data is not captured in the right way. As mentioned previously, this task should mainly be done by the jump pattern in combination with the general development scheme of the development copula. While the jump pattern shows a better assessment of the positive run-off for some markets, the ultimate distributions in general are not simulated correctly. This improves slightly if only the copula and/or the LDF surface is used for the simulations. Then, the ultimate distribution fits are better and only the tail developments are insufficient.

This also shows up when comparing the copula-based SLD model results with the results of commonly aggregated models. The positive run-off of the claims is not projected correctly for all markets, which results in a higher frequency at the priority and also higher ultimate values for the related accident years. Since a layer is used for the analysis and the claims are automatically capped for larger incurred values, this mainly shows the impact of a not correctly simulated positive run-off for the claims. This is also related to the volatility of the ultimate claim results which tend to be larger for markets with a longer lifetime of the claims. Since the proportion of the outstanding amounts is higher for those markets, it leaves more space for developments in the case of the copula-based SLD model resulting in a higher volatility.

Despite those findings, it has to be considered as well that all models are applied to the complete market data while an application for the pricing of a real XL reinsurance contract would only consider the client's data which is more sparse. Additionally, the results of the aggregated models are sometimes unusable since they are applied blindly while the Munich Chain Ladder model could not be applied in some cases. This will worsen if only the data of a single client is considered.

Putting these findings to a conclusion results in the fact that the development copula and the jump pattern have a similar task. While the development copula is capturing the development scheme of all claims in a cluster, the jump pattern is using similar data to predict the area of the next development step. However, the probabilities given in the development copula already provide the next development step. Since both requirements have to be met during the simulation, this overlaps leading to the seen effects of not being able to settle claims in time and to not have enough volatility in respect of the lower and upper tail developments. This is also driven by the fact that all closing LDFs are captured in the development copula but that the set of possible LDFs settling a claim is a null set. Thus, the short lifetime for claims in Italy and Denmark cannot be predicted correctly since a specific closing rate would be required for that task.

Furthermore, the next development step is drawn from the development copula under the assumption of gathering similar claims. However, the area of a cluster is too large here which has a stabilising effect on the claims' development, limiting the volatility and the impact on the lower and upper tail of the distribution.

6.6. Summary

In this section, the performance of the copula-based SLD model is discussed for four different markets. Starting with the different market characteristics, possible trend and influential effects on the data are analysed. Afterwards, the backtest and pricing procedure is applied. Hereby, it can be seen that the copula-based SLD model has a disadvantage with markets where claims have a short lifetime. A claim settlement in form of lump sums as it is done in Italy or Denmark is problematic insofar that the copula-based SLD model does not settle those claims in a timely manner but continues to develop the claim amounts. Consequently, the simulated ultimate distribution does not predict the actual distribution well resulting in higher ultimate values per accident year and an increased as-if frequency. In case of the claims' distributions also the lower and upper tail are not approximated correctly. This also accounts for the Swedish market where the long tail character in combination with larger outstanding amounts and a large number of IBNYR claims resulted in a large volatility and overestimation. For Malta, the results are acceptable since the claims show smaller developments and the number of claims is limited. The chapter concludes with a discussion of the model results and the finding that the jump pattern is not working as intended.

7. Discussion, Conclusion and Outlook

7.1. Advantages and Disadvantages of the SLD Model

Taking the results of the backtest and pricing procedure, as well as the theoretical considerations into account, several advantages and disadvantages of the newly developed copula-based SLD model can be formulated.

7.1.1. Advantages

Due to the character of a copula-based SLD model, it can be applied to any number of claims of a client as long as enough market data is available for the calibration of the model. While the model can only deal with monotonously increasing paid values, it can easily be extended for negative paid values as well. This also accounts for the inclusion of extreme LDFs which is explained further in the outlook in Section 7.4.

The main advantage of this copula-based SLD model is the setup of the model framework and the consideration of the point cloud consisting of the incurred value, the payment ratio, and the related LDFs. This basically means that similar claims for the development are not chosen according to the related development year but for the parameters in the point cloud. This setup in combination with the empirical copula for the development allows to simulate the claims until they are settled without assuming an additional pattern for a later split between paid and outstanding values of the incurred claims. The choice of the next development factors is based on a stochastic approach that allows to have new and unseen development factors for the claims' prediction.

Additionally, no tail factor is needed since it is already included in the claims' development. Even if this feature has to be considered carefully and the goodness of the tail estimation as well as the settlement of the claims have to be verified later on, this is a promising approach. Furthermore, the related memorylessness of the claims is required and assumed since the historical development of the claims is taken into consideration by applying the jump pattern.

Ultimately, the model itself can be explained easily to underwriters and other actuaries. Based on the point cloud, the clustering is done to find similar claims. The development is then based on a copula which is conditioned by the previous development of the claim and an assumption for the highest possible claim increase.

7.1.2. Disadvantages

While a basic explanation of the model can be done easily, getting a deeper understanding of the model is more difficult and requires a deeper mathematical understanding. Furthermore, the model can only be applied if market data is already available for the calibration. This means that an application to new markets cannot be done easily. While the claims' development is not based primarily on the time component any more, it includes the assumption that similar claims can be chosen by their incurred and payment ratio values. However, it is known for practitioners that the claims' development is also related to time which is so far only considered by the jump pattern. This implies that the historical structure of the claims will be used for the projection of the claims for the future. Since many factors play a role for the development of claims, this is underlying a large uncertainty and it is questionable if the historical projections are still true for the future projections. This is also related to a trend analysis of the market and to the fact that trends on single claims' basis cannot be eliminated so far.

While the model framework allows for a development of smaller claims in an also later stage of their development, it does not help for larger claims with a higher payment ratio since the claims database is still sparse for these kind of claims. Hence, the development of larger claims still contains a lot of uncertainty.

Modelling wise it has to be mentioned that the surface fit overestimates the actually seen highest development factors due to the smoothing process that is performed. While the impact of this should be minor since higher development factors have a smaller probability, this results in a tendency towards more conservative claims' values from an economic point of view.

Due to the complexity behind the model, it is currently not possible to derive the mean squared error analytically. Additionally, the runtime of the model is problematic considering all evaluations done in this thesis. However, for a later pricing, the Backtest procedure can be skipped, which improves the runtime significantly. Additionally, the number of claims, based on the clients data, is much smaller which will also have a positive impact on the runtime. Thus, the number of simulations could be increased. However, it should be possible to overcome this shortcoming for the pricing procedure in the future.

7.2. Conclusion on Requirements and Features

After evaluating the performance of the copula-based SLD model, it has to be discussed if the features stated in Section 3.1 and the requirements in Section 1.3.1 are met. With regard to the features, the following can be said:

- Handling sparse claims' data: This feature is fulfilled as shown for the Maltese market. Hereby, any number of single claims can be projected as long as market data for that country is available.
- New loss development factors: The LDFs are drawn from the fitted grid-type or Bernstein copula, leading to new and unseen LDFs in a first step. However, due to the usage of the respective quantile function, only values that have already been seen in the market can occur. This can be offset by considering a fit to the quantile function as a next step.
- Extreme loss development factors: The development itself does currently not incorporate the usage of extreme LDFs as such. However, such a feature could be added as stated briefly in Section 4.4.
- Claim settlement: The copula-based SLD model provides the possibility to develop each individual claim until it is settled. Hereby, not only the incurred values but also the payment ratio itself is developed.
- Claim history: Since the claims' development is guided by the jump pattern, the historically seen development patterns are incorporated. However, this part of the model is not working correctly as discussed in Section 6.5. Due to an overlapping effect with the copula, the jump pattern limits the run-off of claims during the simulation.
- Claim Sizes: Since the market data is separated into several clusters according to the incurred and payment ratio values, larger and smaller claims have a different development. Hereby, the claims are not only separated by claim sizes but also by payment ratio.
- Assumptions: In respect of the used assumptions, that are briefly summarized in Section 4.8, there are some assumptions like the homogeneity, time dependency, and the cluster similarity that are critical for the modelling. While all assumptions are still considered to be realistic, it appears that e.g. the cluster similarity assumption is too broad and does not go far enough.
- Simplicity: The model can be explained easily on a high level while a deeper understanding requires a mathematical background. Furthermore, the complexity of the model is similar to other SLD models, making an analysis of the impact of each model component challenging.

In summary, many key features are fulfilled by the copula-based SLD model and smaller adjustments can be done later to fulfil others as well. Following these features, the requirements stated in Section 1.3.1 towards an SLD model in general have to be discussed as well.

- Accuracy: In order to have accurate model results, the data quality is checked and possible trend effects can be offset partly by an indexation of the claims. Hence, an impact of poor data quality can be limited. In case of the modelling and similar to other SLD models, it is not possible to derive a mean squared error. However, a comparison with real data as well as with commonly used models is performed. Hence, the accuracy and validity of the model results can be evaluated and cross-checked.
- Competitiveness: Considering the real-time requirements, the runtime of the simulation is too long. Here, more work has to be invested into optimization and implementation. Additionally, the copula-based SLD model can only be applied if market data is already available limiting a possible application.
- Completeness: Most of the features stated in Section 3.1 are fulfilled as discussed above. Moreover, smaller adjustments can be done for the copula-based SLD model to fulfil others as well.
- Consistent and Reasonable: The model is considered to be contradiction-free and the used assumptions are reasonable as stated above.
- Correctness: The copula-based SLD model develops single claims up to their ultimate value resulting in an ultimate loss distribution. However, the jump pattern has to be reviewed and remodelled since it does not work correctly.
- Explainable: This is explained above in the simplicity feature.
- Flexibility: While a consideration of trend effects and expert opinion is not included yet, a possible implementation is stated in Section 4.3.3.
- Practicability: In terms of a usage in practice, the required data quality is given for the copula-based SLD model. However, further implementation and optimization efforts as well as an improvement of the jump pattern are required. Until then, the copula-based SLD model is not usable in practice.
- Realistic and plausible: It is possible to apply the copula-based SLD model on a sparse dataset to obtain an ultimate loss distribution. However, evaluations on several markets have shown that the results are not realistic since the run-off as well as the lifetime of the claims is not projected correctly.
- Robustness: The model results are not considered to be prone to outliers and extreme values in the underlying market dataset. However, every cedant gets its

own and individual ultimate loss distribution which is based on the underlying claims. Thus, extreme values in a sparse cedant dataset lead to a change of the ultimate loss distribution so that a fitting of the distribution might get challenging.

- Sensitivity: The model results are prone to changes in the input data if the simulated dataset has only sparse claims data. Hereby, slight changes of the incurred or payment ratio value have little impact while outliers lead to shifts in the ultimate loss distribution. Thus, small changes do not lead to a completely different ultimate loss distribution. This is also investigated exemplarily in Section 5.1 where the input parameters of a single claim are changed. Hereby, no unexpected behaviour of the simulated claim outputs can be observed.
- Simplification/Idealization: The point cloud in combination with the clustering and resulting copula fit cover the essential structure and are a promising approach. However, the impact of the LDF surface and the jump pattern is limited. Hereby, especially the jump pattern is not working as intended and leads to an overlapping effect with the copula. Hence, both conditions lead to an overengineering and overfitting of the main characteristics limiting the explainability and correctness of the model.
- Traceability: While the development of the model, the considered assumptions, and the logic behind are discussed in this thesis, a tracking of the model results over the simulations is time-consuming and cannot be easily done. Hence, the copula-based SLD model can be considered to be a black box model at its current stage. Hereby, especially the interferences of the different model components hinder a logical traceability of the model results.
- Uniqueness: With an increasing number of simulations and number of single claims, the ultimate loss distribution converges so that each simulated portfolio gets its own unique severity distribution.

To conclude this, the usage of the LDF surface and jump pattern are leading to several disadvantages so that the results of the copula-based SLD model cannot be considered in practice. Hence, some requirements are not fulfilled and further development has to be done.

7.3. Executive Summary and Conclusion

The main task of this thesis is to develop a market flexible individual loss model for the pricing of reinsurance XL contracts. Therefore, the model should provide an adequate estimation of the ultimate values, taking into account the most realistic claims' development pattern and settlement while avoiding unrealistic assumptions.

First of all, the data basis and classical reserving methods are analysed in Chapter 2. Hereby, the first research question stated in Section 1.4 focusing on the available data and possible influences is examined. This includes an assessment of the available data and information, their relations and the impact of external influences like inflation, repair costs or legal changes. This is done in order to answer the question whether the usage of market data is justified in respect of their homogeneity. On top of that, three traditional aggregated claims reserving methods that are used for a later comparison are stated.

The information gathered in Chapter 2 is then used to create a new and flexible model framework for the development of single claims in Chapter 3 answering the second research question. Therefore, possible parameters that can be used for a later modelling of single claims are identified. This leads to the main idea of using a point cloud based on all market claims consisting of the incurred values, the payment ratio, and the respective LDF as basis for the development. For the further claims' development the idea of considering similar claims is picked up and a clustering using the CLARA cluster algorithm is applied to the point cloud. Hereby, the goodness of the clustering with respect to the different input parameters and the usage of the reporting threshold is discussed.

Following the idea of similar claims, the copula-based SLD model is then derived in Chapter 4. Hereby, an adequate and realistic claims' development is targeted in line with the third research question. The development of a claim is based on similar claims contained in the same cluster for which a grid-type or Bernstein copula is fitted to the empirical copula of the respective cluster. While this can already be used for the development of a claim, a more realistic simulation of the claims is targeted. Therefore, the development factor derived from the copula is conditioned by the LDF surface for a restriction towards the maximal possible development factor and by the jump pattern to incorporate the previous development of the single claim. These two conditions aim at a more adequate and realistic claims' development within the stated framework. Afterwards, the chapter concludes with a short description of the IBNYR modelling as well as some key figures. The main steps for developing a single claim are described in Section 4.4 and are briefly summarized as follows:

1. Based on the latest incurred and payment ratio value, the related cluster is determined.
2. The cluster directly leads to the required copula. A further distinction can either be done according to the goodness of fit test for the copulas or by an initial choice of the actuary for the grid sizes and copula type.

3. The maximal possible LDF is determined from the LDF surface resulting in a limit for the maximal incurred value.

4. Based on the latest transition history of the claim, the related transition matrix and the transition probabilities are chosen.

5. The next development step is drawn from the copula [2.] under the restrictions given by the LDF surface [3.] and the development area drawn from the transition matrix [4.].

The evaluation of the copula-based SLD model is done in Chapter 5 and 6 focusing on the fourth research question. Hereby, the application and performance of the newly derived copula-based SLD model are the main focus of Chapter 5. Since the claims' development mainly depends on the latest incurred value, the payment ratio, and the history of the claim, the impact of those parameters and the stabilising effect of an increasing number of simulations is analysed. Hereby, technical difficulties in respect of the accuracy and runtime trade off are also discussed briefly. This is followed by describing two different test procedures and an evaluation of the German MTPL market. Therefore, the results of the copula-based SLD model are compared to actually occurred values in the backtest procedure. Additionally, a comparison with commonly used aggregated models is also done in the pricing procedure. The market analysis starts with the main characteristics of the market and an analysis of possible disturbing trend effects. Afterwards, the impact of the different model components is analysed. This is followed by evaluating four other markets in Chapter 6 to test the copula-based SLD model for different market characteristics.

After the evaluations done in Chapter 5 and 6, it has to be concluded that the model is not ready for a pricing of XL treaties yet. The main task of developing a flexible individual loss model for the pricing of reinsurance XL contracts is only fulfilled partly. While the main framework of the point cloud and the copula fit look promising, the jump pattern condition in particular is not working as intended. Thus, an adequate estimation of the ultimate values and a realistic claim settlement is not possible yet. While the mean squared error cannot be calculated as for other individual claims models, this study shows that a usage of an other main parameter than the time component for the development of single claims is an approach that is worth to be developed further. This is also shown in a more recent study by Alessio Carrato and Michele Visintin [40] using a framework based on the paid and outstanding values for their development and a machine learning algorithm to identify clusters of similar claim developments based on primary insurance data.

7.4. Outlook

As discussed partly in Section 6.5, the clustering should be removed and only points that are in a certain area around the current claims' position should be considered for the copula fit. This should improve the modelling of positive and negative run-off since this is currently based on all claims in one cluster smoothing the development and reducing the volatility in those cases.

Additionally, the jump pattern has to be reworked. Its main task of catching the positive and negative run-off of already known claims is also done by the development copula. Thus, the jump pattern should only ensure that the settlement of claims is simulated according to market practice. In a market with many lump sums where the lifetime of claims is short, this should be projected and used for the claims' simulation which does not work currently. Thus, the jump pattern should be replaced by a closing rate which can also depend on the incurred and payment ratio values. Overall, the guidance of the claims' development should be done by the development copula while the jump pattern should focus on the settlement process.

If the modelling of the RBNS claims is working properly, the model has to be extended for negative paid values to allow a proper application for real world pricings. Additionally, an improvement of the runtime and an incorporation of extreme LDFs has to be targeted if necessary. Based on the RBNS model, a more sophisticated IBNYR model has to be build fitting into the used framework, modelling approach, and the basic idea behind the copula-based SLD model.

A. Tables and Graphs

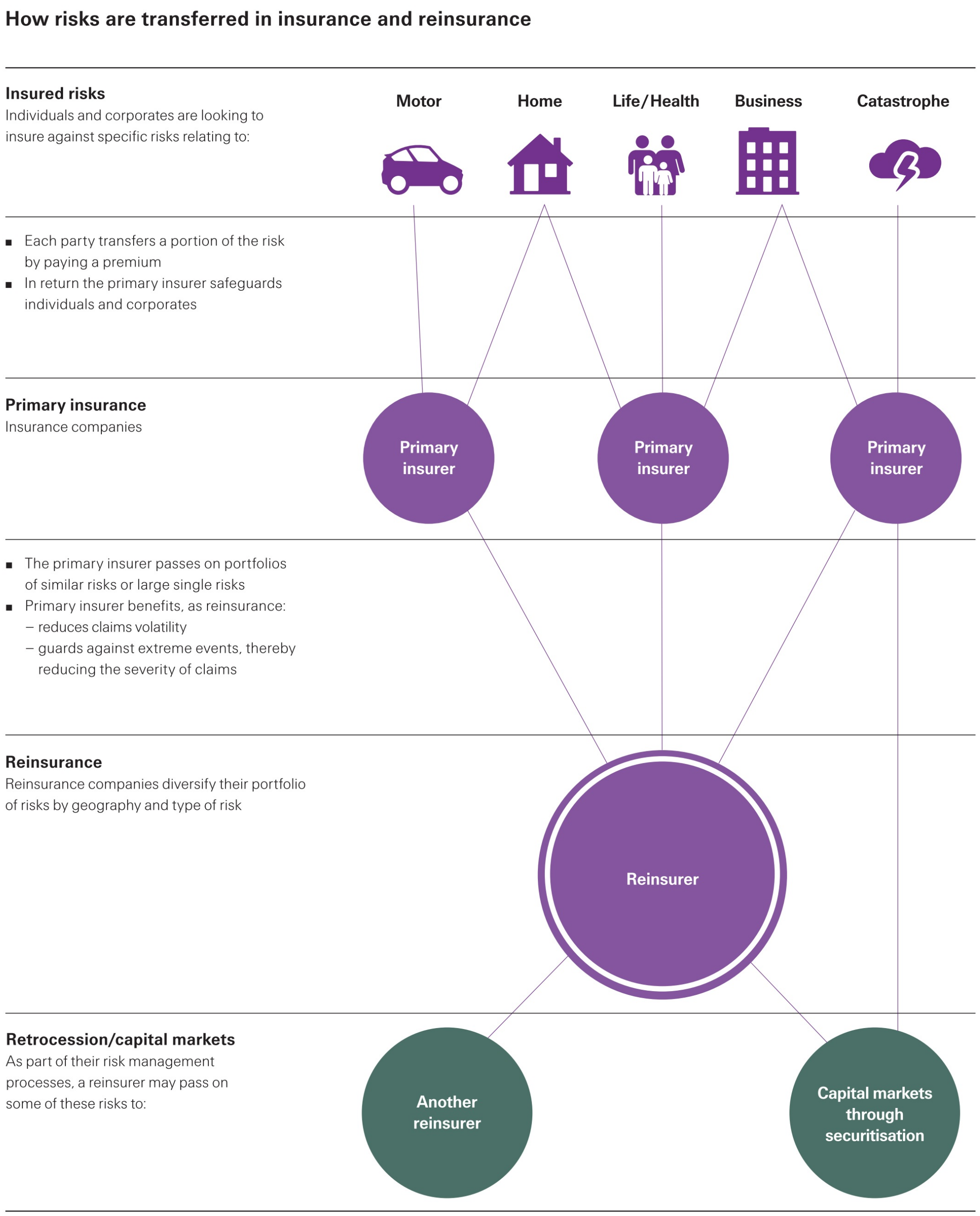


Figure A.1.: How Risks are Transferred in Insurance and Reinsurance based on Swiss Re [197].

A.1. Cluster Validity Indices

	Name of the index in **NbClust**	Optimal number of clusters
1.	`"ch"` (Calinski and Harabasz 1974)	Maximum value of the index
2.	`"duda"` (Duda and Hart 1973)	Smallest number of clusters such that index > criticalValue
3.	`"pseudot2"` (Duda and Hart 1973)	Smallest number of clusters such that index < criticalValue
4.	`"cindex"` (Hubert and Levin 1976)	Minimum value of the index
5.	`"gamma"` (Baker and Hubert 1975)	Maximum value of the index
6.	`"beale"` (Beale 1969)	Number of clusters such that critical value >= alpha
7.	`"ccc"` (Sarle 1983)	Maximum value of the index
8.	`"ptbiserial"` (Milligan 1980, 1981)	Maximum value of the index
9.	`"gplus"` (Rohlf 1974; Milligan 1981)	Minimum value of the index
10.	`"db"` (Davies and Bouldin 1979)	Minimum value of the index
11.	`"frey"` (Frey and Van Groenewoud 1972)	Cluster level before index value < 1.00
12.	`"hartigan"` (Hartigan 1975)	Maximum difference between hierarchy levels of the index
13.	`"tau"` (Rohlf 1974; Milligan 1981)	Maximum value of the index
14.	`"ratkowsky"` (Ratkowsky and Lance 1978)	Maximum value of the index
15.	`"scott"` (Scott and Symons 1971)	Maximum difference between hierarchy levels of the index
16.	`"marriot"` (Marriot 1971)	Max. value of second differences between levels of the index
17.	`"ball"` (Ball and Hall 1965)	Maximum difference between hierarchy levels of the index
18.	`"trcovw"` (Milligan and Cooper 1985)	Maximum difference between hierarchy levels of the index
19.	`"tracew"` (Milligan and Cooper 1985)	Max. value of second differences between levels
20.	`"friedman"` (Friedman and Rubin 1967)	Maximum difference between hierarchy levels of the index
21.	`"mcclain"` (McClain and Rao 1975)	Minimum value of the index
22.	`"rubin"` (Friedman and Rubin 1967)	Minimum value of second differences between levels
23.	`"kl"` (Krzanowski and Lai 1988)	Maximum value of the index
24.	`"silhouette"` (Rousseeuw 1987)	Maximum value of the index
25.	`"gap"` (Tibshirani *et al.* 2001)	Smallest number of clusters such that criticalValue >= 0
26.	`"dindex"` (Lebart *et al.* 2000)	Graphical method
27.	`"dunn"` (Dunn 1974)	Maximum value of the index
28.	`"hubert"` (Hubert and Arabie 1985)	Graphical method
29.	`"sdindex"` (Halkidi *et al.* 2000)	Minimum value of the index
30.	`"sdbw"` (Halkidi and Vazirgiannis 2001)	Minimum value of the index

Figure A.2.: Validity Indices in NbClust [44].

A.2. Result Tables

Incurred Values										
	Without Reporting Threshold					With Reporting Threshold				
Internal Validity:	mean	stdev	median	min	max	mean	stdev	median	min	max
Connectivity:	2.83	1.74	2.89	0.00	5.97	2.73	1.66	2.56	0.00	8.77
Dunn:	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Silhouette:	0.62	0.01	0.62	0.61	0.65	0.63	0.02	0.62	0.60	0.68
Stability Indices:										
APN:	0.02	0.02	0.02	0.00	0.08	0.03	0.02	0.04	0.00	0.07
AD:	1,023,100	23,774	1,015,981	985,865	1,084,999	1,256,286	34,120	1,249,867	1,200,495	1,337,470
ADM:	564,604	16,598	566,404	525,204	593,655	627,397	25,705	630,441	572,484	673,179
FOM:	827,450	11,850	825,713	801,364	858,264	965,207	9,975	964,893	942,919	988,377
Jaccard Similarity:	Cluster 1:	Cluster 2:				Cluster 1:	Cluster 2:			
bootstrap mean:	0.98	0.96				0.90	0.93			
noise mean:	0.95	0.92				0.89	0.87			
Number of Clusters:	1	2	3	4	5	1	2	3	4	5
# Simulations:	0	14	25	11	0	0	10	18	20	1

Table A.1.: Cluster Results Considering the Incurred Values and Loss Development Factors. In the Case of the Dataset with Applied Reporting Threshold, the Optimal Number of Clusters in One Simulation is Zero.

Payment Ratio										
	Without Reporting Threshold					With Reporting Threshold				
Internal Validity:	mean	stdev	median	min	max	mean	stdev	median	min	max
Connectivity:	14.95	31.86	4.42	2.57	117.06	91.07	7.68	89.80	69.80	106.64
Dunn:	3.36	5.87	0.05	0.00	19.39	0.00	0.00	0.00	0.00	0.00
Silhouette:	0.94	0.16	0.99	0.45	1.00	0.50	0.00	0.50	0.49	0.51
Stability Indices:										
APN:	0.15	0.09	0.20	0.01	0.25	0.06	0.02	0.05	0.01	0.09
AD:	0.95	0.41	0.74	0.57	1.78	0.27	0.00	0.27	0.26	0.28
ADM:	0.42	0.22	0.34	0.17	0.95	0.12	0.00	0.12	0.12	0.13
FOM:	6.57	7.24	2.63	1.32	18.76	0.25	0.01	0.25	0.22	0.27
Jaccard Similarity:	Cluster 1:	Cluster 2:				Cluster 1:	Cluster 2:			
bootstrap mean:	0.93	0.66				0.99	0.99			
noise mean:	0.91	0.30				0.99	0.99			
Number of Clusters:	1	2	3	4	5	1	2	3	4	5
# Simulations:	0	21	18	11	0	0	36	9	5	0

Table A.2.: Cluster Results Considering the Payment Ratio and Loss Development Factors.

Combination										
	Without Reporting Threshold					With Reporting Threshold				
Internal Validity:	mean	stdev	median	min	max	mean	stdev	median	min	max
Connectivity:	3.19	1.47	3.23	0.55	7.53	2.78	1.39	2.84	0.00	5.23
Dunn:	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Silhouette:	0.61	0.01	0.61	0.60	0.67	0.62	0.01	0.62	0.59	0.66
Stability Indices:										
APN:	0.07	0.08	0.01	0.00	0.17	0.17	0.01	0.16	0.14	0.21
AD:	846,659	32,200	837,863	812,618	969,951	1,032,945	31,683	1,025,481	984,620	1,105,914
ADM:	384,383	10,925	385,488	354,002	405,698	432,130	19,745	430,636	394,389	502,266
FOM:	561,039	12,629	558,544	537,789	588,666	646,129	13,083	645,038	621,493	671,427
Jaccard Similarity:	Cluster 1:	Cluster 2:	Cluster 3:			Cluster 1:	Cluster 2:	Cluster 3:		
bootstrap mean:	0.97	0.97	0.96			0.95	0.95	0.94		
noise mean:	0.95	0.94	0.90			0.92	0.91	0.87		
Number of Clusters:	1	2	3	4	5	1	2	3	4	5
# Simulations:	0	3	26	20	1	0	4	28	17	1

Table A.3.: Cluster Results Considering the Payment Ratio, Incurred Value, and Loss Development Factors.

A.3. Copula Graphics

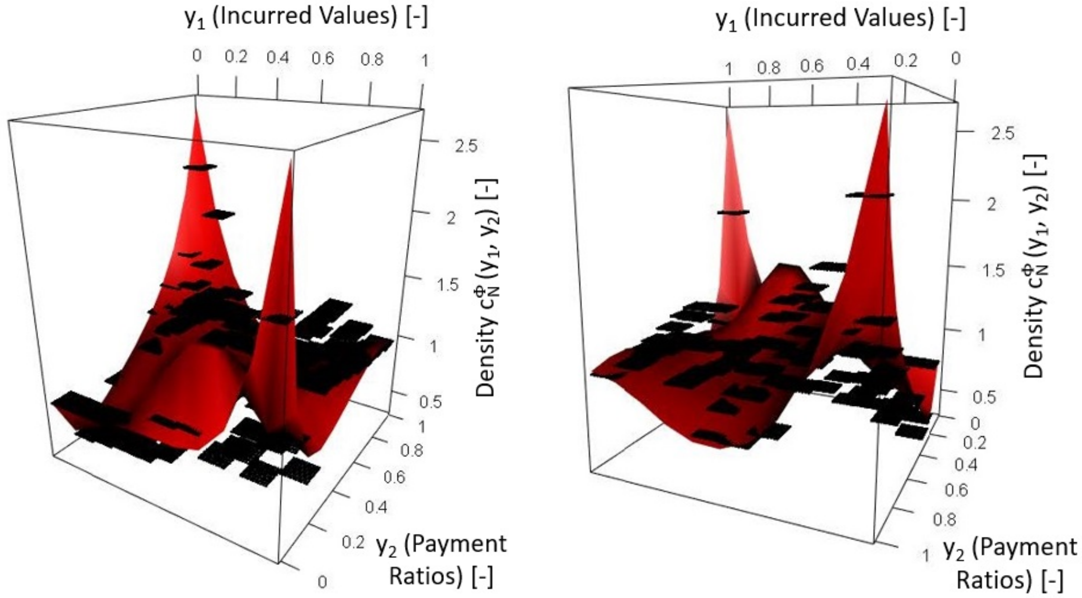


Figure A.3.: Grid-Type (Black) and Bernstein (Red) Copula Densities for the Additive Case with a Grid Size of $m = 10$ for Different View Points.

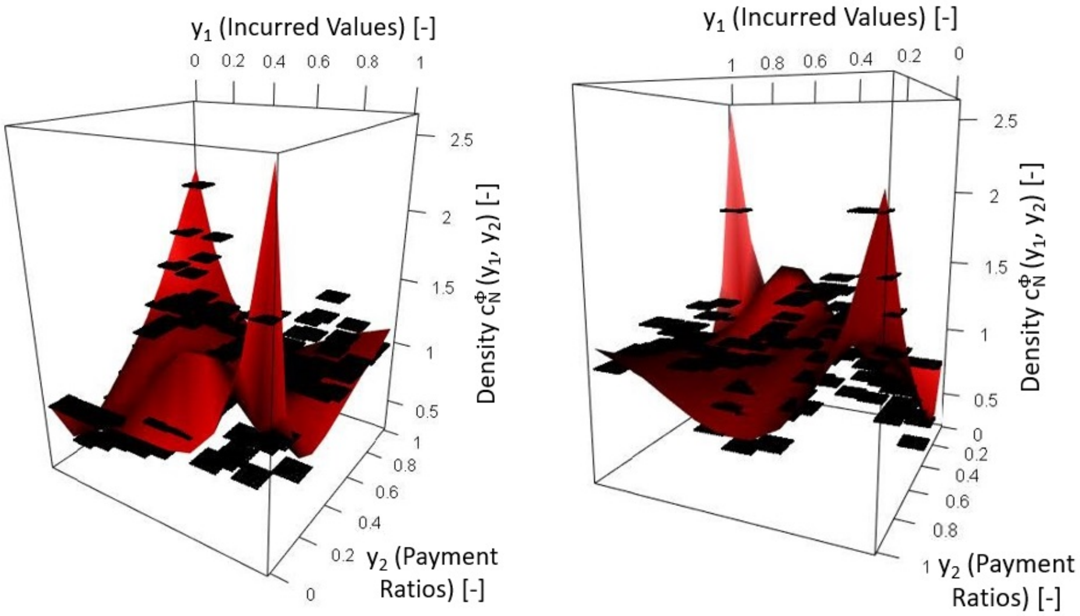


Figure A.4.: Grid-Type (Black) and Bernstein (Red) Copula Densities for the Multiplicative Case with a Grid Size of $m = 10$ for Different View Points.

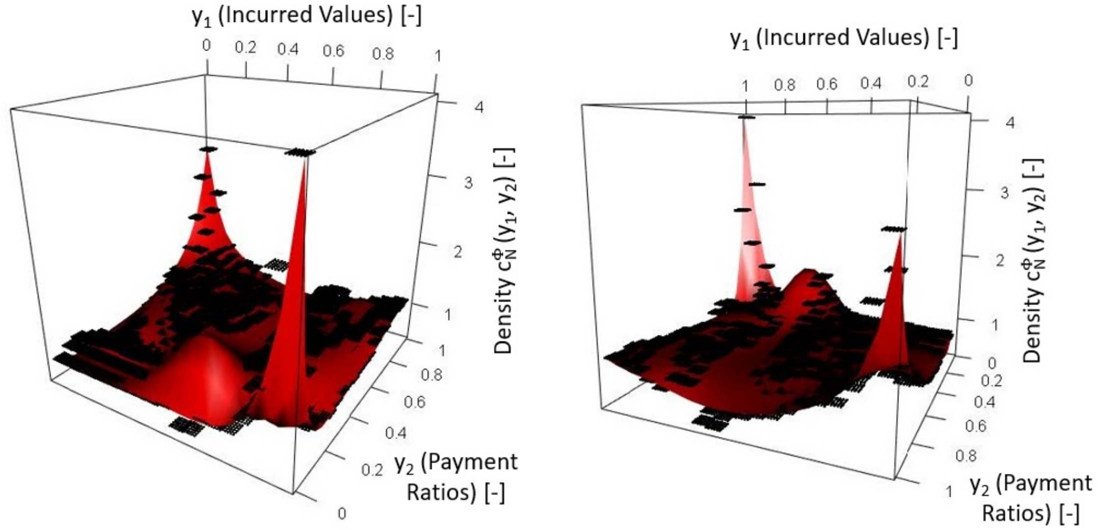


Figure A.5.: Grid-Type (Black) and Bernstein (Red) Copula Densities for the Additive Case with a Grid Size of $m = 20$ for Different View Points.

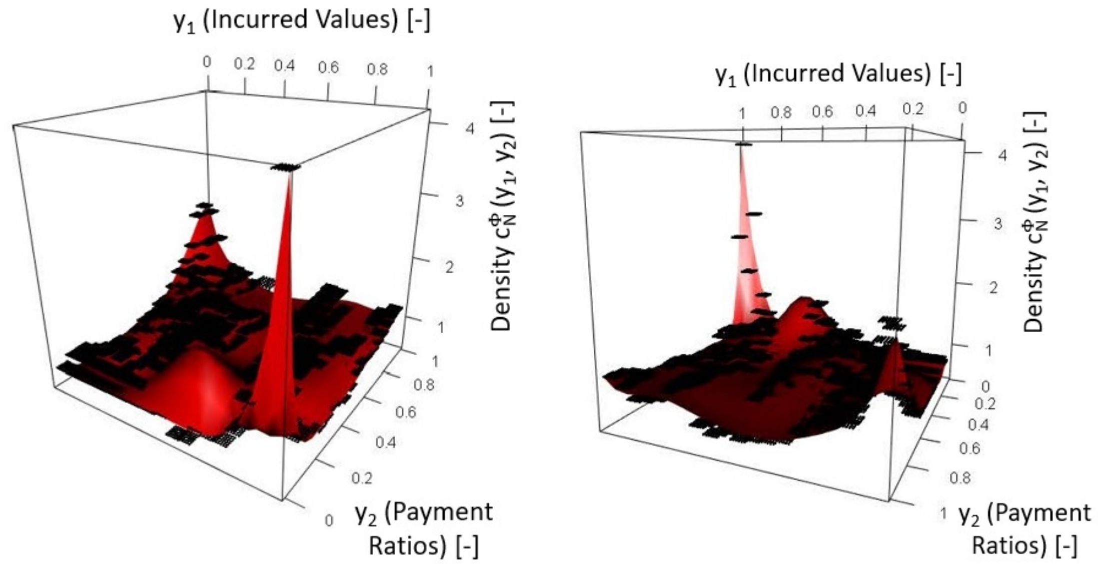


Figure A.6.: Grid-Type (Black) and Bernstein (Red) Copula Densities for the Multiplicative Case with a Grid Size of $m = 20$ for Different View Points.

A.4. LDF Surface Fits

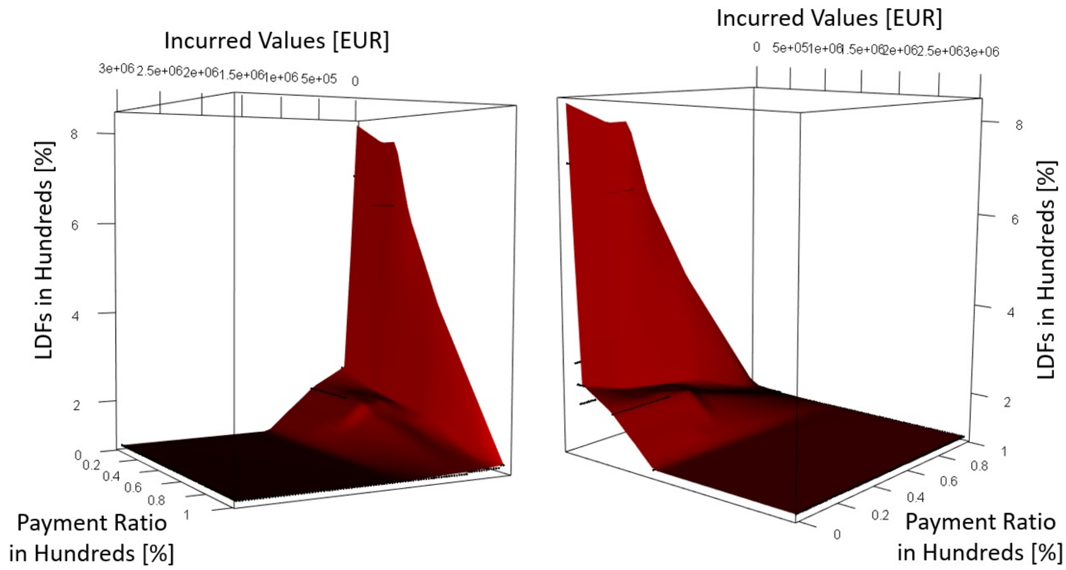


Figure A.7.: Smooth LDF Surface for Maltesian MTPL Market.

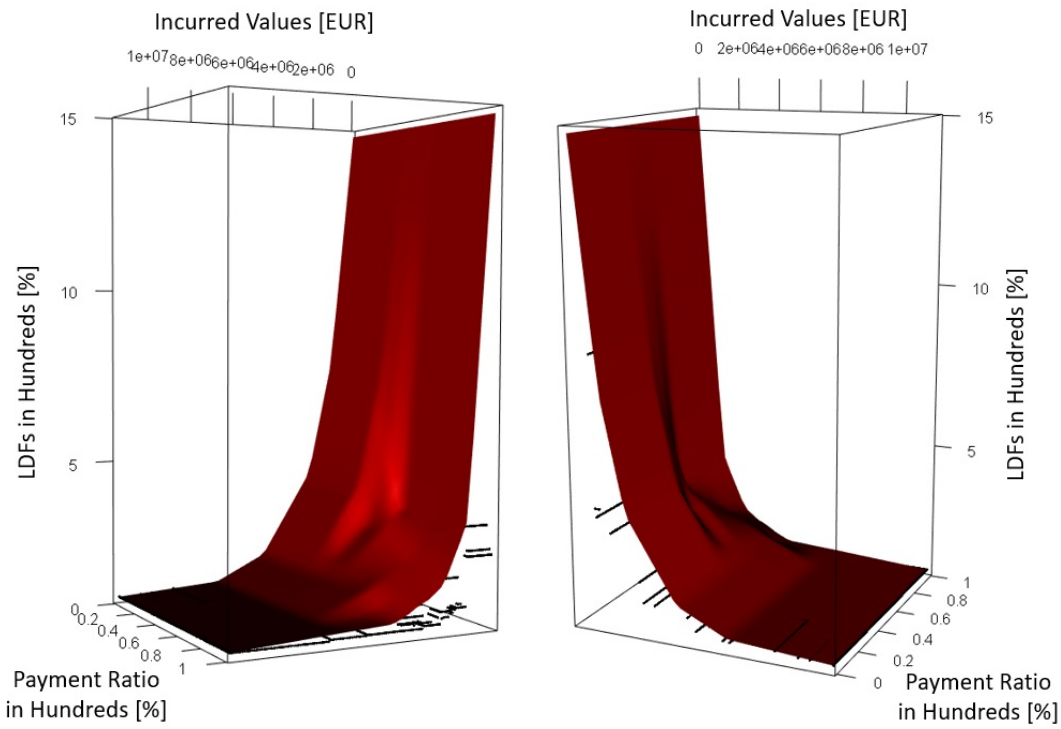


Figure A.8.: Smooth LDF Surface for Italian MTPL Market.

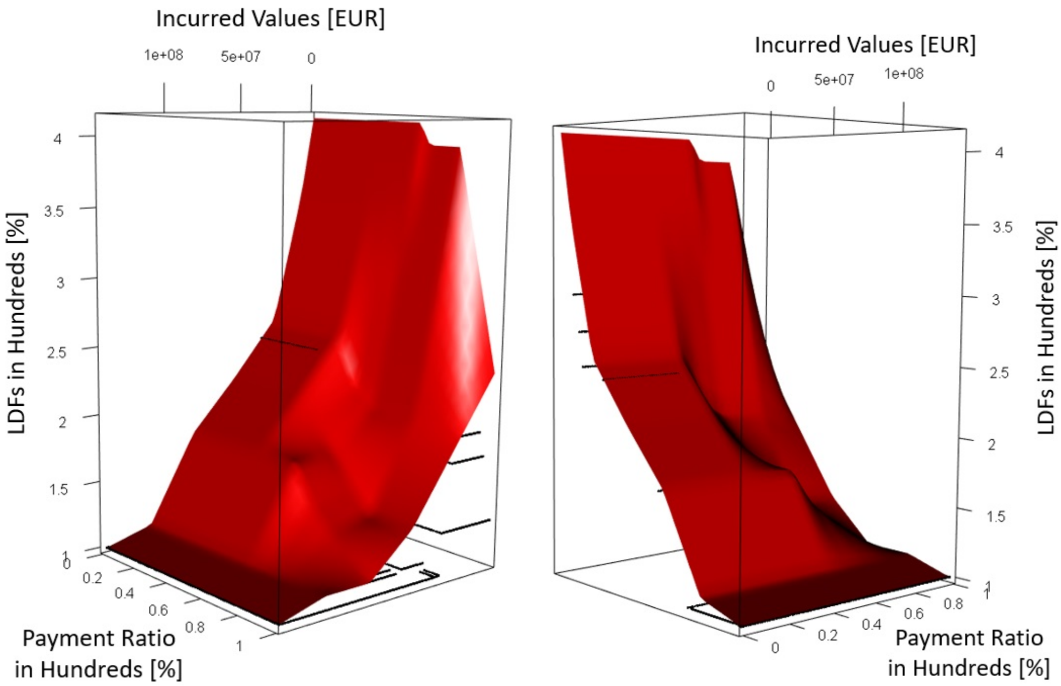


Figure A.9.: Smooth LDF Surface for Swedish MTPL Market.

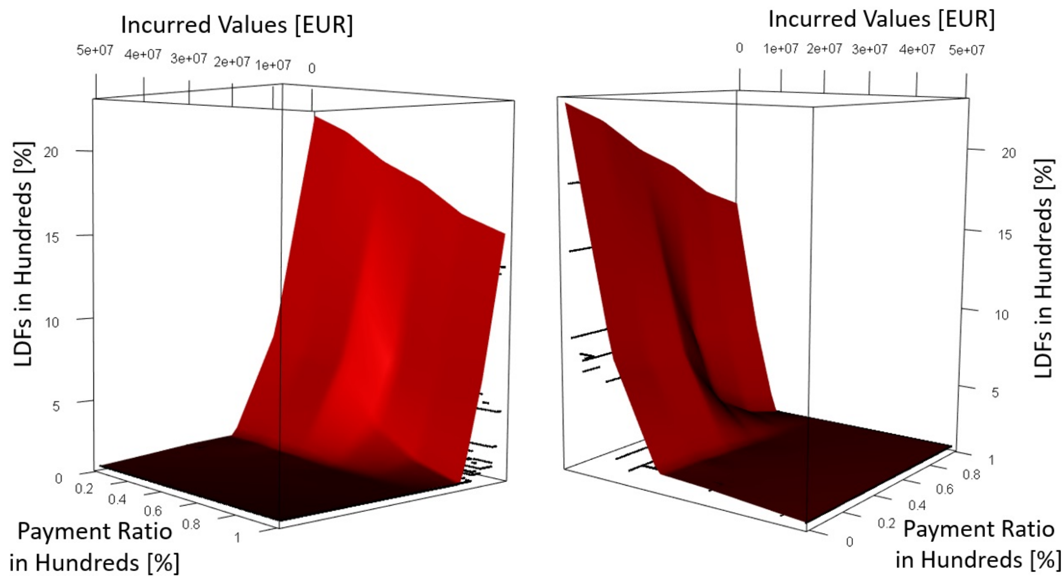


Figure A.10.: Smooth LDF Surface for Danish MTPL Market.

A.5. Residual Distances

	Number of Known Development Years																										
Development Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	0,10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	0,21	0,04	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	0,34	0,11	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	0,37	0,21	0,08	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	0,47	0,31	0,14	0,06	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	0,56	0,37	0,17	0,09	0,04	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	0,69	0,51	0,25	0,13	0,07	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	0,70	0,48	0,27	0,16	0,10	0,05	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	0,79	0,61	0,35	0,20	0,14	0,08	0,06	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	0,89	0,73	0,42	0,25	0,18	0,12	0,09	0,06	0,04	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
12	0,81	0,75	0,40	0,26	0,20	0,13	0,10	0,07	0,05	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
13	0,84	0,85	0,47	0,29	0,22	0,14	0,11	0,09	0,06	0,03	0,02	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	0,85	0,92	0,55	0,33	0,25	0,17	0,13	0,10	0,07	0,04	0,04	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	0,86	0,95	0,59	0,38	0,30	0,20	0,16	0,12	0,09	0,05	0,05	0,04	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-
16	0,79	0,79	0,54	0,38	0,35	0,28	0,25	0,24	0,18	0,14	0,12	0,11	0,12	0,09	0,06	-	-	-	-	-	-	-	-	-	-	-	-
17	0,96	0,90	0,63	0,43	0,41	0,32	0,29	0,28	0,24	0,20	0,18	0,17	0,17	0,14	0,09	0,02	-	-	-	-	-	-	-	-	-	-	-
18	1,32	1,29	0,81	0,57	0,50	0,38	0,35	0,31	0,29	0,23	0,21	0,22	0,25	0,22	0,16	0,06	0,03	-	-	-	-	-	-	-	-	-	-
19	1,50	1,19	0,78	0,58	0,58	0,47	0,48	0,47	0,38	0,32	0,27	0,29	0,37	0,31	0,25	0,11	0,10	0,03	-	-	-	-	-	-	-	-	-
20	2,23	1,27	0,95	0,73	1,02	0,86	0,87	0,85	0,71	0,71	0,63	0,67	0,79	0,70	0,64	0,36	0,33	0,20	0,08	-	-	-	-	-	-	-	-
21	2,45	1,41	1,14	0,97	1,50	1,16	1,12	1,18	0,96	1,01	1,03	1,19	1,38	1,15	1,08	0,67	0,54	0,38	0,25	0,09	-	-	-	-	-	-	-
22	3,01	1,33	1,22	1,02	1,68	1,40	1,17	1,25	0,93	0,92	0,98	1,16	1,43	1,29	1,26	0,92	0,83	0,66	0,42	0,21	0,09	-	-	-	-	-	-
23	2,31	2,00	1,58	1,44	2,26	1,91	1,53	1,47	1,15	1,04	1,09	1,36	1,74	1,51	1,49	1,24	1,20	0,97	0,73	0,42	0,25	0,15	-	-	-	-	-
24	7,85	3,21	2,85	1,75	2,58	2,26	1,90	2,15	1,84	1,68	1,46	1,68	2,15	2,04	2,03	1,72	1,71	1,43	1,25	1,01	0,68	0,56	0,25	-	-	-	-
25	7,75	4,41	3,39	2,50	3,30	2,76	2,25	2,65	2,56	2,41	2,05	2,11	3,03	2,81	2,69	2,34	2,26	1,97	1,84	1,42	0,96	0,83	0,52	0,31	-	-	-
26	21,11	26,65	14,45	6,34	5,09	3,98	2,98	2,73	2,90	2,76	2,57	3,22	3,10	3,02	2,80	2,41	2,25	2,06	2,45	2,17	1,83	1,66	1,25	1,24	1,06	-	-
27	19,04	25,50	14,18	10,40	10,34	3,82	3,83	3,61	3,52	2,87	2,80	4,14	3,50	3,09	3,32	2,95	2,93	2,69	2,85	3,04	2,66	2,38	2,11	2,32	2,29	1,82	-
28	18,98	20,43	16,73	14,28	15,56	15,32	12,81	10,58	10,99	8,56	7,69	3,81	4,84	4,82	4,77	4,84	8,29	8,43	8,83	5,13	7,89	7,99	8,53	10,45	10,87	8,77	7,27
28	-	-	-	5,82	6,02	6,26	6,37	6,22	6,56	3,85	3,63	4,04	3,35	3,31	2,08	2,25	2,80	2,97	2,24	-	-	-	-	-	-	-	-

Figure A.11.: Residual Distances Times 100, Full Model.

		Number of Known Development Years																										
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
Development Year	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	2	0,08	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	3	0,14	0,03	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	4	0,19	0,07	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	5	0,22	0,14	0,07	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	6	0,26	0,19	0,11	0,06	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	7	0,32	0,23	0,13	0,08	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	8	0,36	0,32	0,17	0,10	0,04	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	9	0,35	0,33	0,19	0,12	0,06	0,04	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	10	0,42	0,45	0,25	0,15	0,08	0,05	0,03	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	11	0,48	0,50	0,28	0,18	0,09	0,07	0,05	0,04	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	12	0,42	0,51	0,27	0,20	0,10	0,07	0,05	0,04	0,03	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	13	0,47	0,60	0,33	0,24	0,12	0,08	0,07	0,06	0,05	0,04	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	14	0,49	0,66	0,40	0,28	0,14	0,10	0,08	0,07	0,06	0,05	0,05	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	15	0,55	0,76	0,47	0,30	0,14	0,11	0,09	0,08	0,07	0,06	0,06	0,05	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-
	16	0,50	0,62	0,36	0,27	0,16	0,12	0,12	0,11	0,10	0,09	0,08	0,06	0,05	0,05	0,03	-	-	-	-	-	-	-	-	-	-	-	-
	17	0,64	0,84	0,51	0,37	0,20	0,16	0,14	0,13	0,13	0,12	0,11	0,10	0,09	0,09	0,07	0,02	-	-	-	-	-	-	-	-	-	-	-
	18	0,97	1,15	0,66	0,52	0,24	0,17	0,16	0,16	0,16	0,14	0,13	0,12	0,12	0,13	0,10	0,05	0,03	-	-	-	-	-	-	-	-	-	-
	19	0,92	1,11	0,64	0,48	0,27	0,22	0,23	0,26	0,22	0,21	0,20	0,19	0,21	0,22	0,21	0,11	0,10	0,04	-	-	-	-	-	-	-	-	-
	20	1,15	0,85	0,50	0,41	0,48	0,40	0,41	0,43	0,38	0,37	0,33	0,35	0,41	0,41	0,39	0,21	0,20	0,11	0,04	-	-	-	-	-	-	-	-
	21	1,34	0,82	0,61	0,52	0,82	0,66	0,65	0,70	0,61	0,63	0,63	0,67	0,81	0,74	0,71	0,44	0,39	0,27	0,16	0,09	-	-	-	-	-	-	-
	22	2,18	1,07	0,72	0,63	1,06	0,86	0,77	0,79	0,62	0,61	0,60	0,65	0,85	0,78	0,73	0,48	0,46	0,35	0,26	0,14	0,09	-	-	-	-	-	-
	23	2,61	1,57	0,90	0,79	1,36	1,13	0,97	0,97	0,79	0,65	0,61	0,70	0,97	0,86	0,86	0,66	0,65	0,51	0,38	0,33	0,21	0,16	-	-	-	-	-
	24	4,35	2,17	1,45	1,24	1,70	1,34	1,24	1,35	1,13	0,92	0,73	0,84	1,16	1,09	1,09	0,96	1,03	0,82	0,74	0,65	0,43	0,42	0,19	-	-	-	-
	25	6,14	5,20	2,63	1,80	2,01	1,62	1,54	1,67	1,60	1,43	1,09	1,19	1,73	1,76	1,64	1,44	1,35	1,08	0,97	0,84	0,57	0,59	0,42	0,29	-	-	-
	26	24,81	21,69	10,50	5,20	3,37	2,24	1,87	2,06	2,03	1,85	1,80	2,13	2,30	2,14	1,80	1,62	1,55	1,34	1,27	1,40	1,23	1,32	1,24	1,24	1,01	-	-
	27	25,41	24,09	11,51	8,44	6,00	4,89	4,77	4,01	4,07	2,79	2,67	2,94	2,47	2,32	2,18	1,97	1,87	1,80	1,57	2,03	1,86	1,87	1,88	2,05	1,89	1,44	-
	28	20,99	22,24	24,43	20,00	20,31	15,47	11,91	14,73	15,12	7,92	7,38	5,10	4,63	6,31	3,93	4,18	4,68	4,92	5,23	7,29	7,32	7,17	7,91	8,67	8,56	8,16	7,65
	29	-	-	-	9,45	9,68	8,06	7,80	4,89	5,05	4,60	5,05	2,14	1,86	1,67	1,75	1,90	2,03	2,24	2,60	-	-	-	-	-	-	-	-

Figure A.12.: Residual Distances Times 100, Only Copula.

Development Year	Number of Known Development Years: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	0,08	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	0,14	0,04	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	0,19	0,11	0,03	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	0,26	0,23	0,12	0,03	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	0,33	0,33	0,19	0,09	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	0,42	0,43	0,26	0,14	0,05	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	0,52	0,60	0,36	0,22	0,11	0,05	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	0,46	0,60	0,40	0,24	0,12	0,06	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	0,57	0,85	0,55	0,35	0,17	0,09	0,04	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	0,67	0,99	0,65	0,43	0,22	0,13	0,06	0,04	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
12	0,62	1,02	0,63	0,45	0,23	0,13	0,08	0,05	0,04	0,02	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
13	0,74	1,13	0,75	0,53	0,26	0,16	0,11	0,08	0,06	0,04	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	0,77	1,25	0,86	0,59	0,31	0,20	0,14	0,11	0,09	0,07	0,07	0,04	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	0,94	1,41	0,99	0,63	0,33	0,21	0,14	0,11	0,09	0,08	0,09	0,06	0,04	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-
16	0,77	1,18	0,80	0,57	0,27	0,18	0,12	0,11	0,09	0,07	0,08	0,06	0,05	0,04	0,02	-	-	-	-	-	-	-	-	-	-	-	-
17	1,08	1,45	1,01	0,75	0,39	0,26	0,18	0,15	0,12	0,10	0,10	0,08	0,07	0,06	0,04	0,02	-	-	-	-	-	-	-	-	-	-	-
18	1,43	1,94	1,25	0,95	0,43	0,29	0,22	0,17	0,13	0,11	0,12	0,09	0,09	0,08	0,06	0,04	0,02	-	-	-	-	-	-	-	-	-	-
19	1,57	1,97	1,25	0,89	0,40	0,27	0,23	0,21	0,16	0,14	0,14	0,11	0,10	0,11	0,11	0,06	0,08	0,04	-	-	-	-	-	-	-	-	-
20	1,90	1,69	1,09	0,71	0,45	0,35	0,33	0,34	0,31	0,30	0,26	0,24	0,24	0,24	0,28	0,17	0,19	0,13	0,05	-	-	-	-	-	-	-	-
21	1,80	1,47	1,12	0,77	0,70	0,53	0,48	0,52	0,46	0,50	0,47	0,54	0,59	0,53	0,59	0,40	0,35	0,30	0,20	0,09	-	-	-	-	-	-	-
22	3,30	1,77	1,10	0,76	0,81	0,66	0,57	0,63	0,51	0,51	0,46	0,55	0,64	0,59	0,68	0,52	0,45	0,39	0,28	0,15	0,08	-	-	-	-	-	-
23	2,79	1,81	1,19	0,91	1,20	0,97	0,82	0,81	0,63	0,58	0,49	0,57	0,69	0,64	0,71	0,64	0,63	0,59	0,40	0,31	0,19	0,13	-	-	-	-	-
24	5,83	3,51	1,52	1,19	1,30	1,05	0,89	0,92	0,74	0,67	0,49	0,59	0,72	0,71	0,77	0,71	0,75	0,73	0,64	0,50	0,36	0,30	0,14	-	-	-	-
25	7,30	5,94	2,74	1,47	1,59	1,25	1,14	1,16	1,05	0,98	0,68	0,74	1,02	1,11	1,10	1,01	0,97	0,91	0,98	0,69	0,44	0,42	0,34	0,22	-	-	-
26	28,23	23,58	11,01	7,22	5,03	3,04	1,96	1,67	1,66	1,37	1,21	1,44	1,44	1,40	1,25	1,14	1,22	1,19	1,20	1,33	1,12	1,07	1,17	1,20	0,99	-	-
27	25,62	28,66	14,92	12,09	11,23	10,19	7,30	6,58	6,63	2,73	2,19	2,42	2,18	2,04	1,71	1,69	2,05	2,05	1,77	1,87	1,69	1,63	1,74	1,71	1,63	1,14	-
28	30,77	32,04	37,54	35,34	35,89	28,48	24,23	17,63	18,05	11,42	8,71	5,85	5,27	4,72	3,60	3,85	4,50	4,69	4,23	6,63	7,79	7,71	8,20	8,65	8,72	7,90	8,99
29	-	-	-	5,45	5,52	4,85	4,83	12,09	12,30	5,89	3,41	1,93	2,79	2,65	2,46	2,61	1,86	1,97	2,51	-	-	-	-	-	-	-	-

Figure A.13.: Residual Distances Times 100, Copula and LDF Surface.

	Number of Known Development Years																										
Development Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	0,12	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	0,21	0,03	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	0,30	0,09	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	0,34	0,17	0,07	0,03	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	0,42	0,25	0,12	0,07	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	0,50	0,29	0,15	0,09	0,04	0,02	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	0,58	0,38	0,19	0,12	0,08	0,04	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	0,59	0,38	0,22	0,15	0,10	0,07	0,04	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	0,64	0,47	0,28	0,19	0,13	0,10	0,07	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	0,69	0,51	0,30	0,21	0,17	0,14	0,10	0,06	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
12	0,61	0,48	0,27	0,21	0,19	0,16	0,13	0,09	0,06	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
13	0,66	0,54	0,31	0,24	0,22	0,18	0,15	0,12	0,07	0,04	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	0,62	0,53	0,34	0,26	0,22	0,18	0,15	0,14	0,10	0,07	0,06	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	0,61	0,56	0,34	0,26	0,23	0,20	0,18	0,17	0,13	0,08	0,08	0,04	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-
16	0,58	0,46	0,31	0,27	0,35	0,34	0,32	0,26	0,19	0,16	0,14	0,09	0,08	0,07	0,04	-	-	-	-	-	-	-	-	-	-	-	-
17	0,65	0,57	0,37	0,33	0,34	0,30	0,31	0,29	0,24	0,22	0,19	0,16	0,14	0,13	0,09	0,02	-	-	-	-	-	-	-	-	-	-	-
18	0,96	0,93	0,52	0,48	0,40	0,34	0,32	0,29	0,28	0,26	0,24	0,21	0,20	0,19	0,15	0,06	0,03	-	-	-	-	-	-	-	-	-	-
19	1,10	0,86	0,59	0,53	0,62	0,59	0,56	0,47	0,43	0,43	0,41	0,38	0,42	0,39	0,35	0,19	0,17	0,08	-	-	-	-	-	-	-	-	-
20	1,41	0,85	0,69	0,69	1,07	0,85	0,84	0,76	0,71	0,76	0,72	0,71	0,77	0,69	0,68	0,39	0,36	0,24	0,08	-	-	-	-	-	-	-	-
21	1,37	1,04	0,97	1,07	1,79	1,34	1,28	1,17	1,03	1,19	1,17	1,25	1,44	1,26	1,26	0,84	0,71	0,55	0,30	0,11	-	-	-	-	-	-	-
22	2,97	1,69	1,41	1,77	2,76	2,20	1,85	1,62	1,25	1,35	1,34	1,50	1,76	1,56	1,52	1,12	0,98	0,81	0,43	0,22	0,07	-	-	-	-	-	-
23	4,01	2,31	1,77	2,55	3,77	3,10	2,48	2,10	1,63	1,58	1,41	1,66	2,13	1,80	1,82	1,56	1,32	1,07	0,66	0,41	0,21	0,11	-	-	-	-	-
24	10,33	3,51	2,53	3,23	4,16	3,46	2,95	2,39	1,92	1,76	1,45	1,73	2,37	2,20	2,08	1,81	1,60	1,29	0,85	0,59	0,40	0,35	0,15	-	-	-	-
25	9,05	7,65	4,97	3,43	3,73	3,44	2,98	2,72	2,28	2,08	1,51	1,62	2,59	2,73	2,58	2,21	1,97	1,60	1,07	0,83	0,63	0,67	0,47	0,29	-	-	-
26	9,99	15,76	14,73	6,95	3,11	3,21	2,51	2,26	2,37	2,07	1,73	2,02	2,65	2,72	2,59	2,22	1,87	1,70	1,44	1,52	1,22	1,32	1,03	1,06	1,04	-	-
27	12,98	18,87	14,04	10,36	8,34	8,93	7,63	4,65	4,97	5,05	4,86	6,21	4,99	5,00	5,17	4,73	4,41	4,08	2,96	3,12	2,76	2,73	2,68	2,40	2,24	1,93	-
28	18,56	20,47	19,69	15,56	15,84	14,67	11,84	11,73	12,50	9,88	9,50	6,13	5,51	5,43	6,38	6,38	14,96	14,81	16,41	5,77	9,17	9,51	9,86	9,51	9,03	10,45	10,10
29	-	-	-	6,27	6,51	6,62	6,74	8,14	8,45	4,13	4,64	1,46	1,42	1,37	1,62	1,72	7,21	7,33	5,75	-	-	-	-	-	-	-	-

Figure A.14.: Residual Distances Times 100, Copula and Jump Pattern.

A.6. Results for the Maltese Market

Development Year	Number of Known Development Years															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	0,19	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	0,26	0,09	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	0,31	0,20	0,11	-	-	-	-	-	-	-	-	-	-	-	-	-
5	0,39	0,37	0,22	0,08	-	-	-	-	-	-	-	-	-	-	-	-
6	0,52	0,43	0,27	0,16	0,11	-	-	-	-	-	-	-	-	-	-	-
7	0,59	0,52	0,35	0,21	0,14	0,07	-	-	-	-	-	-	-	-	-	-
8	0,65	0,53	0,37	0,27	0,22	0,12	0,08	-	-	-	-	-	-	-	-	-
9	0,86	0,75	0,55	0,51	0,49	0,35	0,36	0,21	-	-	-	-	-	-	-	-
10	1,12	1,00	0,76	0,74	0,77	0,64	0,69	0,50	0,20	-	-	-	-	-	-	-
11	1,21	1,02	0,84	0,88	0,96	0,87	0,96	0,74	0,37	0,12	-	-	-	-	-	-
12	1,64	1,45	1,12	1,10	1,08	0,95	1,04	0,82	0,45	0,23	0,16	-	-	-	-	-
13	2,11	1,87	1,53	1,56	1,59	1,35	1,49	1,26	0,83	0,55	0,43	0,32	-	-	-	-
14	2,85	2,07	1,83	2,10	2,23	1,98	2,00	1,73	1,27	1,00	0,79	0,71	0,33	-	-	-
15	5,12	3,35	2,77	2,66	2,80	2,64	2,63	2,44	2,12	1,94	1,76	1,68	1,19	0,64	-	-
16	4,12	3,88	4,28	4,92	5,13	5,12	5,19	5,06	4,84	5,04	4,73	4,55	3,46	2,46	0,65	-
17	3,40	2,95	3,05	3,61	3,68	3,62	3,62	3,54	3,45	3,42	3,19	3,04	2,34	1,90	0,64	-
18	3,50	3,02	3,06	3,60	3,66	3,61	3,61	3,53	3,44	3,38	3,17	3,02	2,22	1,68	0,65	-
19	3,56	2,89	2,18	2,47	2,27	2,20	2,15	2,11	2,13	1,81	1,85	1,68	1,37	1,12	0,90	-

Figure A.15.: Residual Distances Times 100, Maltese MTPL Market.

A.7. Results for the Italian Market

	Number of Known Development Years																	
Development Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	0,05	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	0,22	0,05	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	0,53	0,16	0,03	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	0,79	0,29	0,08	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	1,06	0,41	0,14	0,04	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-
7	1,33	0,52	0,17	0,05	0,01	0,00	-	-	-	-	-	-	-	-	-	-	-	-
8	1,69	0,72	0,28	0,09	0,03	0,01	0,00	-	-	-	-	-	-	-	-	-	-	-
9	2,10	0,97	0,40	0,15	0,06	0,02	0,01	0,00	-	-	-	-	-	-	-	-	-	-
10	2,42	1,15	0,49	0,20	0,09	0,04	0,02	0,01	0,00	-	-	-	-	-	-	-	-	-
11	2,68	1,34	0,57	0,25	0,12	0,06	0,04	0,02	0,01	0,00	-	-	-	-	-	-	-	-
12	3,29	1,75	0,76	0,38	0,19	0,08	0,05	0,03	0,01	0,01	0,00	-	-	-	-	-	-	-
13	4,15	2,20	1,01	0,60	0,34	0,18	0,11	0,07	0,04	0,03	0,03	0,01	-	-	-	-	-	-
14	6,03	3,21	1,62	1,01	0,65	0,34	0,20	0,13	0,09	0,08	0,08	0,04	0,02	-	-	-	-	-
15	5,24	2,45	1,05	0,57	0,41	0,43	0,45	0,33	0,25	0,22	0,21	0,15	0,09	0,05	-	-	-	-
16	10,61	5,80	3,35	2,06	1,22	0,54	0,34	0,24	0,22	0,21	0,27	0,20	0,18	0,18	0,19	-	-	-
17	10,71	5,63	3,06	2,10	1,46	1,07	1,04	0,83	0,72	0,68	0,67	0,54	0,42	0,31	0,36	0,02	-	-
18	16,47	6,86	3,89	2,59	1,56	1,22	1,14	1,00	0,91	0,91	0,91	0,75	0,66	0,50	0,62	0,10	0,19	-
19	29,31	17,36	12,62	10,01	5,86	2,87	1,76	1,34	1,19	1,19	1,41	1,22	1,28	1,07	1,58	0,35	1,19	0,87
20	-	-	-	-	1,00	3,72	4,73	4,59	3,94	3,45	2,82	2,50	1,90	1,38	1,17	1,47	0,45	0,33

Figure A.16.: Residual Distances Times 100, Italian MTPL Market.

A.8. Results for the Swedish Market

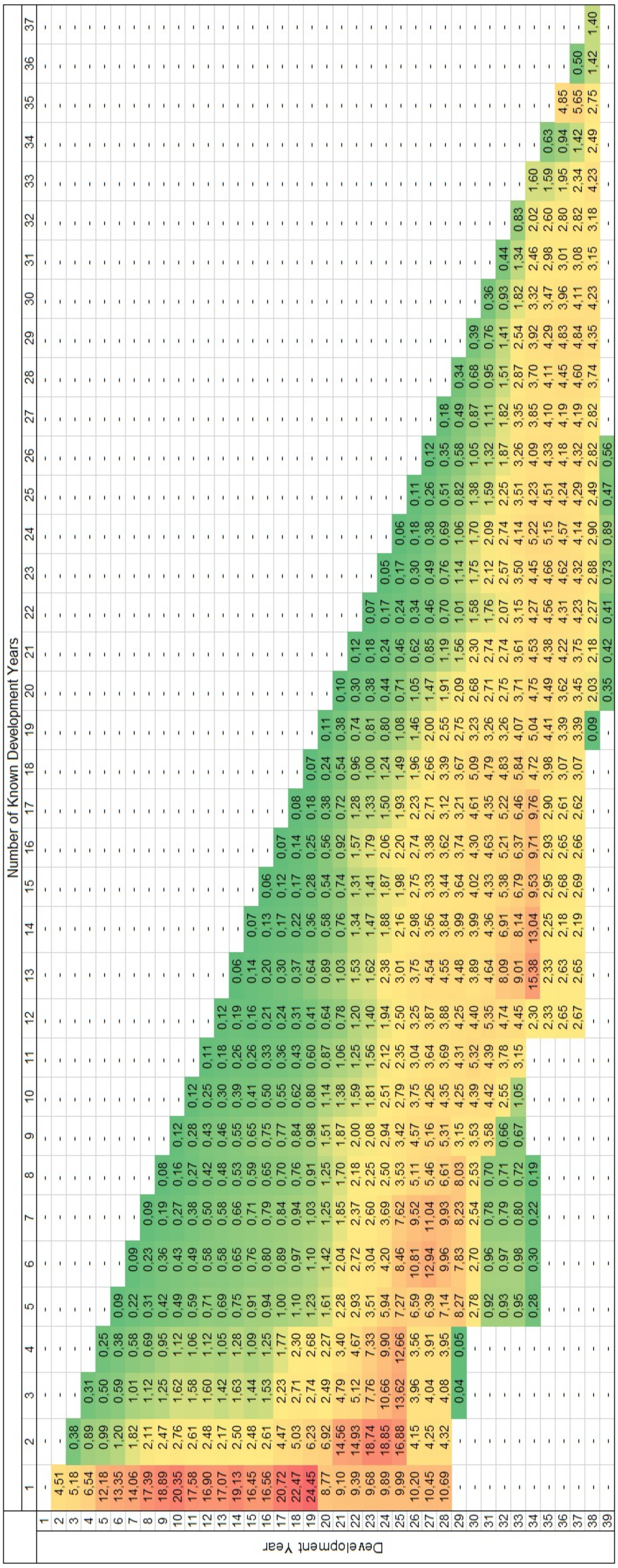


Figure A.17.: Residual Distances Times 100, Swedish MTPL Market.

A.9. Results for the Danish Market

Development Years	Number of Known Development Years: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	0,09	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	0,63	0,18	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	1,49	0,75	0,21	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	3,12	2,11	1,04	0,18	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	4,67	3,59	1,96	0,55	0,04	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	6,11	5,17	3,08	1,07	0,18	0,03	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	7,15	6,40	3,98	1,60	0,36	0,10	0,01	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	7,93	7,34	4,66	2,01	0,53	0,18	0,04	0,00	-	-	-	-	-	-	-	-	-	-	-	-	-
10	8,92	8,44	5,60	2,47	0,74	0,30	0,10	0,03	0,00	-	-	-	-	-	-	-	-	-	-	-	-
11	10,09	9,42	6,47	3,05	1,04	0,49	0,19	0,09	0,03	0,01	-	-	-	-	-	-	-	-	-	-	-
12	9,89	10,18	6,99	3,50	1,28	0,66	0,29	0,16	0,09	0,04	0,01	-	-	-	-	-	-	-	-	-	-
13	10,44	10,71	7,68	3,85	1,44	0,77	0,38	0,23	0,15	0,09	0,04	0,01	-	-	-	-	-	-	-	-	-
14	11,49	11,65	8,63	4,48	1,69	0,91	0,51	0,33	0,22	0,14	0,08	0,07	0,06	-	-	-	-	-	-	-	-
15	12,04	11,85	9,30	5,22	1,97	1,14	0,72	0,51	0,34	0,28	0,19	0,18	0,21	0,10	-	-	-	-	-	-	-
16	15,02	11,09	9,18	5,89	2,70	1,58	1,09	0,83	0,66	0,61	0,50	0,46	0,65	0,50	0,43	-	-	-	-	-	-
17	17,73	11,18	8,93	5,65	3,06	1,92	1,50	1,22	1,01	0,97	0,90	0,91	1,15	1,01	1,06	0,52	-	-	-	-	-
18	23,55	11,74	9,14	5,83	3,14	2,28	1,82	1,63	1,55	1,49	1,41	1,47	1,48	1,25	1,16	0,60	0,21	-	-	-	-
19	30,21	13,76	10,49	6,68	3,81	2,55	2,14	1,91	1,68	1,70	1,68	1,94	2,20	1,98	1,99	1,31	0,74	0,46	-	-	-
20	22,80	18,77	19,29	12,13	6,17	5,50	4,62	4,06	2,60	4,44	4,24	4,46	7,39	5,82	6,65	5,13	4,44	3,97	1,74	-	-
21	22,19	22,72	22,61	17,76	12,51	9,69	9,13	8,76	8,53	8,80	8,34	9,10	11,14	10,07	9,96	7,48	6,15	6,04	2,19	1,10	-
22	28,93	29,00	28,25	23,74	20,63	22,30	18,58	17,49	12,39	15,64	19,33	15,13	20,76	20,83	24,06	20,35	19,22	19,41	15,24	10,66	8,91
23	-	-	-	11,12	13,18	12,19	16,28	19,14	27,33	21,53	17,88	17,57	7,95	8,06	-	-	-	-	-	-	-

Figure A.18.: Residual Distances Times 100, Danish MTPL Market.

B. R Packages

Besides the standard packages included in the R version 3.5.0, the following packages in Table B.1 are used additionally.

Package Name	Package Version	Description
akima	0.6.2	The function ,interp'is used to interpolate the data of the LDF surface in order to get a smoother fit.
caret	6.0.80	A data splitting function is used to split the market data into test and training data.
ChainLadder	0.2.6	This package includes an implementation for the Chain Ladder and Munich Chain Ladder method.
cluster	2.0.7.1	This package provides an implementation of the CLARA and PAM clustering algorithm.
clValid	0.6.6	Several validation measures for clustering results and the corresponding number of clusters are provided.
cobs	1.3.3	This package provides the computation of constrained B-Splines.
data.table	1.11.4	Helpful function for data wrangling.
doParallel	1.0.11	Needed for a parallel implementation of the Monte Carlo simulations and to set up the cores.
doSNOW	1.0.16	This package provides a function to register a parallel backend with the foreach (doParallel) package.
dplyr	0.7.5	Helpful function for data wrangling.
fields	9.6	Used to create an image plot for the data visualization.
fpc	2.1.11	The function ,clusterboot' is used for a clusterwise cluster stability assessment by resampling.
kdecopula	0.9.2	Allows for the estimation of the bivariate kernel copula density of a Bernstein copula.
lmomco	2.3.1	With this package, several distribution fits by using the L-Moments can be computed.
mgcv	1.8.23	Helpful function for data wrangling.
NbClust	3.0	This package provides 30 indexes for determining the optimal number of clusters in a dataset.
parallel	3.5.0	Used to detect the number of cores for a parallel setup of the simulations.
rgl	0.99.16	Used to create 3d interactive plots, e.g. for the visualization of the point cloud.
sp	1.3.1	This package provides helpful functions to work on multivariate grids.
tcltk	3.5.0	Used to track the progress within parallel simulations.
tictoc	1.0	Helpful function to determine the runtime of code segments.
VGAM	1.0.5	Provides the density and distribution function of the Pareto(I) distribution.

Table B.1.: Used R Packages, Package Versions, and Short Description.

C. Program Structure and Pseudo-Code

In this section, the structure of the program is stated briefly and the main limitation which currently prevents a faster runtime is explained and shown in a pseudo code. The program is structured in three parts shown in Figure C.1.

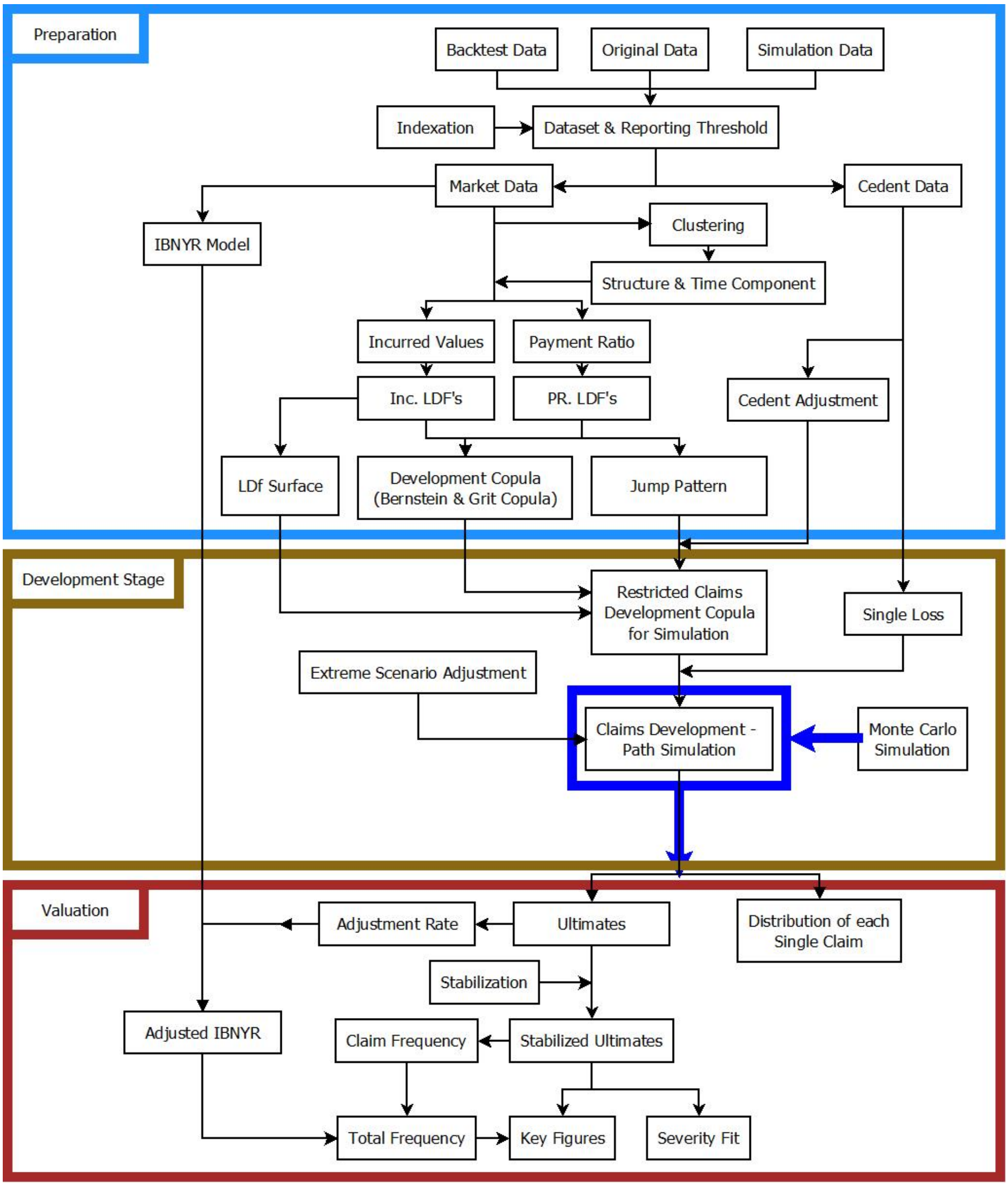


Figure C.1.: Program Structure of the Copula-Based SLD Model.

The program itself can work with any sort of data triangles, split into paid and outstanding values as stated in Section 2.3 in Table 2.1. The preparation phase has two main tasks. First, it is used to calibrate the copula-based SLD model. The market data is clustered and the copulas are fitted. Then, both conditions, the LDF surface and the jump patterns, are calibrated. Additionally, the IBNYR model is calibrated based on the market data. The whole calibration can be done separately and before the actual pricing of a reinsurance XL treaty. The second function of this phase is to prepare the dataset for the pricing by applying the indexation.

For a later application, the claims triangle of the cedant is developed. Therefore, each single loss is developed by applying the copula and both restrictions. For the pricing of cedant data, an adjustment of the jump pattern is also possible to take cedant specific idiosyncrasies into account. Additionally, extreme scenario adjustments can be applied here. Afterwards, the single claims are developed multiple times during the Monte Carlo simulation which is programmed in a parallel version to use multiple cores.

The last stage is the valuation of the model results. This includes an adjustment of the IBNYR claims as well as the stabilization of the claims. The layering of a contract can be applied on single claim basis and the total frequency at the priority as well as other key figures can be estimated.

The main shortcoming in the way the copula-based SLD model is programmed right now is the development stage. While the runtime of the preparation stage is not that important since it can be done before the renewal period, the development of the cedants data is under time pressure. Therefore, the simulation part that is responsible for the long runtime is shown exemplarily as pseudo code in Algorithm 1.

As input data, the latest incurred value and the related payment ratio of the claim as well as the claims' history are required. Based on this information, the responsible cluster and the related probabilities of the jump pattern can be chosen. Additionally, the maximum possible LDF can be determined from this information. Starting with line 10, a suitable LDF is searched for. Therefore, one value is drawn from the copula, depending on the respective cluster, which is then used to determine the new incurred value and payment ratio. If a copula value exists, the first condition is checked in line 15. If the newly derived development step fulfils the LDF surface requirement, the second condition is checked in line 18. Hereby, the respective jump area for the claim is drawn from the jump probabilities. Only if the newly derived claim value is within the jump area, this is a suitable next loss development step. However, it often appears in practice that the newly derived development step is not within the jump area. In those cases, the whole procedure is repeated and a new value is drawn for the copula and jump area based on the related probabilities. Thus, the estimation of a suitable LDF lying in the intersection of both conditions is currently based on coincidence.

This could be overcome if a conditioned drawing from the copula would be possible. Then, the jump area could be estimated and based on that, a LDF could be drawn from the specific region of the copula. However, according to the authors knowledge, no possibility to draw values from a copula under several conditions exists so far. Apart

Algorithm 1 Development Step - Choosing the Next Development Factor

```
Inc ← the latest incurred value of the claim
PR ← the latest payment ratio of the claim
ClaimsPath ← current historical claim path
Input: Inc, PR, ClaimsPath

Cluster ← ClusterMembership(Inc, PR)
JumpProbability ← JumpPattern
maxLDF ← LDFSurface(Inc, PR)

while no suitable LDF do
    CopulaValue ← DrawFromCopula(# = 1, Cluster)
    IncNew ← Inc + △Inc(CopulaValue)
    PRNew ← PR + △PR(CopulaValue)
    if ∃ CopulaValue then
        Cond_1 ← LDFSurfaceAcceptance(IncNew, PRNew)
        if Cond_1 == TRUE then
            JumpArea ← JumpProbability(ClaimsPath)
            Cond_2 ← JumpFilterAcceptance(IncNew, PRNew, JumpArea)
            if Cond_2 == TRUE then
                return ← (IncNew, PRnew)
            end if
        end if
    end if
end while

Output: next development step in form of (IncNew, PRNew)
```

from this, it is also possible to draw the jump area first and to simulate more copula outputs afterwards. However, this has not shown a significant impact on the overall runtime. Since this procedure has to be done many times for one claim and also for all claims in the portfolio, this limitation currently drives the overall runtime.

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Author’s Declaration

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of any other degree or diploma of the university or other institute of higher learning, except where due acknowledgment has been made in the text.