

Optimal multi-fault tolerant control for PMSMs

Martin Ackermann

Vollständiger Abdruck der von der Fakultät für Elektrische Energiesysteme und Informationstechnik der Universität der Bundeswehr München zur Erlangung des akademischen Grades eines

Doktor-Ingenieurs

(Dr.-Ing.)

angenommenen Dissertation.

Gutachter 1: Prof. Dr. rer. nat. habil. Claus Hillermeier

Gutachter 2: Prof. Dr.-Ing. Dieter Gerling

Die Dissertation wurde am 03.06.2025 bei der Universität der Bundeswehr München eingereicht und durch die Fakultät für Elektrische Energiesysteme und Informationstechnik am 04.11.2025 angenommen. Die mündliche Prüfung fand am 25.11.2025 statt.

Abstract

This work presents an optimal multi-fault tolerant control strategy for Permanent Magnet Synchronous Machines (PMSMs). The foundation of the control system is the Subdomain Model, employing Maxwell equations for calculating the magnetic flux distribution with high accuracy. Based on this modeling, a feedforward and feedback control is developed, that enables precise tracking of high-dynamic mechanical trajectories.

The central focus is placed on the fault tolerance of the control. Combining the detailed physical description of the PMSM using Maxwell equations with an efficient mathematical formulation through constrained minimization problems, the developed control can be systematically adapted to motor-intrinsic electrical and magnetic faults requiring only minor adjustments. This generalized formulation not only allows the compensation of various individual faults, but also enables the simultaneous handling of multiple faults, resulting in a comprehensive multi-fault tolerant control.

Overall, the proposed approach is most suitable for safety-critical applications, such as electric aircraft propulsion systems, where operational continuity and fault resilience are paramount. The presented simulation results demonstrate the robustness of the control system across a wide range of fault scenarios, highlighting its potential for the deployment in demanding environments.

Kurzfassung

In dieser Arbeit ist die Entwicklung einer optimalen mehrfehlertoleranten Regelung für Permanentenerregte Synchronmaschinen (PMSM) dokumentiert. Die methodische Grundlage bildet das Subdomain Model, welches die Maxwell'schen Gleichungen löst und damit eine hochgenaue Berechnung des magnetischen Flusses ermöglicht. Auf Basis dieses Modells wird eine Vorsteuerung und eine Regelung entworfen, die eine präzise Einregelung vorgegebener mechanischer Trajektorien gewährleistet.

Der zentrale Fokus der Arbeit liegt auf der Fehlertoleranz des Regelungssystems. Durch die Kopplung der detaillierten elektromagnetischen Modellierung der PMSM mit einer effizienten mathematischen Formulierung in Form von Optimierungsproblemen unter Nebenbedingungen, lässt sich das Regelungssystem mit minimalem Anpassungsaufwand an verschiedene elektrische und magnetische Fehlerzustände adaptieren. Diese verallgemeinerte Struktur erlaubt nicht nur die Kompensation einzelner Fehlertypen, sondern auch die simultane Behandlung mehrerer Fehler, wodurch eine mehrfehlertolerante Regelung realisiert wird.

Der entwickelte Ansatz ist insbesondere für sicherheitskritische Anwendungen geeignet, beispielsweise in elektrischen Flugzeugantrieben, bei denen Betriebskontinuität und Fehlertoleranz von zentraler Bedeutung sind. Umfangreiche Simulationsergebnisse demonstrieren die Stabilität und Leistungsfähigkeit des Regelungssystems in einer Vielzahl relevanter Fehlerszenarien und bestätigen dessen Potenzial für den Einsatz in anspruchsvollen Betriebsumgebungen.

Acknowledgments

This thesis was composed during my work as a research assistant at the chair for Automation and Control at the University of the Bundeswehr Munich from 2021 to 2025. It was funded by the project ELAPSED of the Digital and Technology Research Center of the Bundeswehr (dtec.bw).

First of all, I would like to thank Prof. Dr. rer. nat. habil. Claus Hillermeier for the big degrees of freedom he granted me throughout this work, combined with his continuous support in all matters. Even during challenging phases he was always available with advice and encouragement. I have also greatly appreciated the in-depth discussions we had.

Equally, I would like to extend my thanks to Dr. rer. nat. José-Luis Marqués. Our intensive discussions not only had a significant impact on my professional development, but also further strengthened my passion for the natural sciences. In particular the great support during the initial stages laid the foundation of this work. I am also grateful for his guidance and assistance in many other aspects of this dissertation.

Furthermore, I would like to thank Prof. Dr.-Ing. Dieter Gerling for acting as the second reviewer and for evaluating this thesis.

Lastly I greatly wish to thank my family for their endless and unconditional support – in good as well as in challenging times. Without their encouragement, this work would not have been possible.

Note on linguistic editing:

ChatGPT was used as an extended dictionary for linguistic revision in selected sections of this work to improve clarity and readability. All methods and scientific contributions were developed independently by the author.

Contents

1	Introduction	1
1.1	Fundamental objective	1
1.2	Structure of the work	2
2	Fault tolerance for PMSMs	5
2.1	Fault classification	5
2.2	Fault tolerance from a hardware perspective	8
2.3	Fault tolerance from a control perspective	11
3	Approach and delimitation	15
3.1	Main idea & original contribution	16
3.2	Delimitation of the scope	16
3.3	Introduction of the control system	17
4	Subdomain Model	19
4.1	Basic approach	20
4.2	Important quantities	23
4.2.1	PM flux linkage & back-EMF	23
4.2.2	Inductance	24
4.2.3	Torque	25
4.3	Analysis	26
4.4	Usage as plant model	30
5	Optimal control for PMSMs	33
5.1	Desired trajectory	33
5.2	Optimal feedforward control	34
5.2.1	Optimal currents	34
5.2.2	Optimal field weakening	35
5.2.3	Evaluation	39
5.3	Optimal feedback control	43
5.3.1	Simplified motor dynamics	43
5.3.2	Full motor dynamics	45
5.3.3	Refinement of the optimization	47
5.4	Overall control system	50
5.4.1	Parametrization & initialization	50
5.4.2	Online control	50
5.4.3	Simulation environment	52
5.4.4	Evaluation	54

6	(Multi-) fault management	57
6.1	Open phase faults	58
6.2	Phase short circuits	64
6.3	Interturn short circuits	74
6.4	Demagnetization	84
6.5	High resistance connections	90
6.6	Multi-faults	95
6.7	Discussion	98
7	Fault adaptation and diagnosis	101
7.1	Adaptation to phase short circuits	101
7.1.1	Methodology	101
7.1.2	Evaluation	103
7.2	Diagnosis of partial demagnetization	106
7.2.1	Fitting of ellipses	109
7.2.2	Detection	110
7.2.3	Identification	111
7.2.4	Adaptation	112
7.2.5	Evaluation	113
7.3	Discussion	116
8	Analysis of the multi-fault tolerant control	117
8.1	Open phase and demagnetization	117
8.2	Interturn short circuit, high resistance connection and demagnetization . . .	121
8.3	Adaptation of phase short circuit and demagnetization	125
8.4	Concluding discussion	127
9	Conclusion and outlook	129
A	Detailed derivation of the Subdomain Model	131
A.1	Derivation of Poisson's equation	132
A.2	Subdomains	134
A.2.1	Subdomain I: slots	134
A.2.2	Subdomain II: air gap	138
A.2.3	Subdomain III: magnets	139
A.3	Continuity conditions	141
A.3.1	Subdomain II and III: airgap and magnets	142
A.3.2	Subdomain II and I: airgap and slots	143
A.4	Important Integrals	146
A.5	Resulting equation system	147
A.6	Further definitions	148
B	Considering flux barriers in the Subdomain Model	149
B.1	Subdomains	149
B.1.1	Subdomain IV: flux barriers	150
B.1.2	Subdomain V: surrounding air	152

B.2	Continuity conditions	153
B.2.1	Subdomain IV and V: flux barriers and surrounding air	153
B.2.2	Subdomain IV, II and I: flux barriers, air gap and slots	154
B.3	Resulting equation system	155
C	Considering short circuits in the Subdomain Model	159
C.1	Phase short circuits	159
C.1.1	Coefficients of the back-EMF	159
C.1.2	Coefficients of the torque capability	160
C.1.3	Faulty torque ripple	160
C.2	Interturn short circuits	162
C.2.1	Particular solution	162
C.2.2	Inductances	163
C.2.3	PM flux linkage & back-EMF	164
	Bibliography	165
	Publications	173
	List of Symbols	175

1 Introduction

Fault tolerant electrical drive systems are essential for safety-critical applications, as they must remain operational even in the event of critical failures. Their importance is growing steadily, in particular since emerging technologies such as electrified aircraft have become highly relevant in scientific research, driven by the global trend toward decarbonization [1].

From a hardware perspective, the reliability requirement can be addressed by selecting electrical machine topologies that inherently offer increased fault tolerance. Permanent Magnet Synchronous Machines (PMSMs) represent such a topology. In addition to their reliability, PMSMs are widely used due to their high efficiency, power density, and dynamic performance [2]. Especially in multi-phase configurations, i.e. machines with more than the conventional three phases, they provide additional degrees of freedom and redundancy. These can be exploited for fault compensation without requiring physical hardware reconfiguration [3, 4].

In addition to a resilient hardware design, an appropriate control strategy is essential to compensate faults during operation. Especially in highly safety-critical applications it is desirable to implement a control system that is as universally applicable to faults as possible to be capable of addressing even unlikely and complex fault scenarios. Despite substantial progress in the field, improving the robustness and extending the general applicability of fault tolerant control strategies remain key challenges in current research [2, 5, 6].

1.1 Fundamental objective

The fundamental objective of this work is to develop a control strategy for PMSMs, where the focus is placed on the fault tolerance. The aim is to create a control that is capable of ensuring highly dynamic and smooth operation even in the event of a fault. To extend the operational capabilities of the machine, an additional focus is placed on integrating a field weakening strategy that remains fully functional under fault conditions. This allows the system to operate across a wider speed and torque range without compromising safety or performance. In total the goal is the design of a control system suitable for highly safety-critical applications, such as electrified aircraft propulsion systems, where continuous and reliable drive operation is strictly required.

While the main emphasis lies in the development of the fault tolerant control itself, a secondary objective is the design of algorithms capable of detecting and identifying faults in real time and estimating the associated physical parameters required for an effective adaptation of the control.

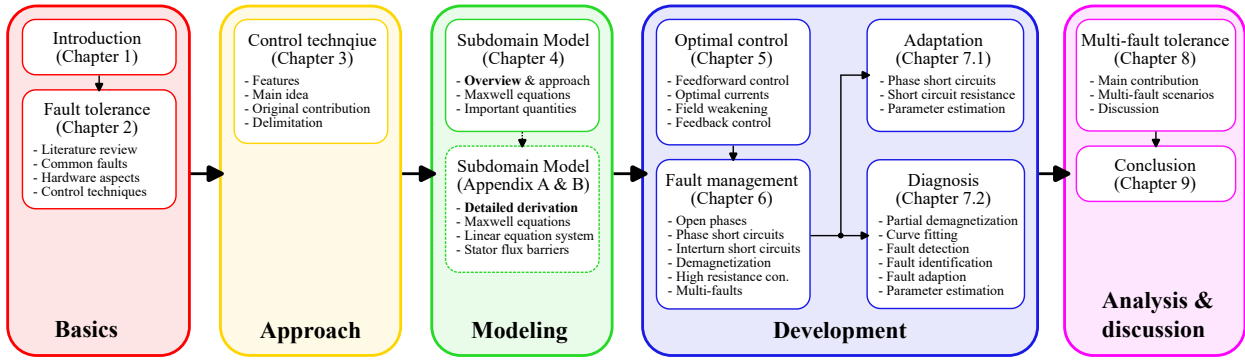


Figure 1.1: Structural overview of the work.

1.2 Structure of the work

The present work is structured into the following main sections: **Basics**, **Approach**, **Modeling**, **Development** and **Analysis & discussion**. These sections are illustrated in Fig. 1.1 and described in the following:

Basics: Building on this introduction, chapter 2 provides a literature overview of the physical and technical foundations relevant to the fault tolerance in PMSMs. This includes a classification and detailed discussion of typical motor-intrinsic fault types, covering electrical, mechanical, and magnetic faults. In addition, both relevant hardware-based fault tolerant design principles and control strategies are discussed.

Approach: Chapter 3 outlines the features, main idea and structure of the developed fault tolerant control system. Additionally, the scope of the investigation is narrowed down and the original contributions of the work are presented.

Modeling: Since the proposed control strategy relies on an analytical modeling of the PMSM, which is commonly referred as Subdomain Model in literature, chapter 4 provides a concise overview of the basic approach, derives the physical quantities relevant for the control, and evaluates the accuracy of the modeling. In addition, a complete derivation of the Subdomain Model is included in the appendices A and B. While these appendices are required for a comprehensive understanding of the control approach, they may be skipped by readers interested solely in a conceptual understanding.

Development: This part forms the core of the thesis, where the complete control system is developed. Chapter 5 describes the optimal control strategy derived from the Sub-domain Model equations under prefault conditions. Chapter 6 extends the control strategy to compensate motor-intrinsic electrical and magnetic faults. Each fault type is first treated individually, followed by a generalized formulation that enables the control under arbitrary multi-fault conditions. Chapter 7 then presents two diagnostic algorithms as part of this work, which allow fault detection and estimation of the parameters required for the control adaptation.

Analysis & discussion: Chapter 8 evaluates the performance of the proposed control system under a range of complex multi-fault scenarios. The findings are then discussed in terms of practical applicability, strengths, and limitations of the approach. Finally, chapter 9 provides the concluding remarks and an outlook on future research directions.

2 Fault tolerance for PMSMs

This chapter summarizes the physical and technical fundamentals relevant to this work. The most common fault types occurring in PMSMs are discussed in section 2.1. Furthermore, an overview of the fault tolerance from a hardware perspective is presented in section 2.2, addressing relevant design aspects to enhance the reliability. The chapter concludes with a discussion of existing fault tolerant control approaches (see section 2.3).

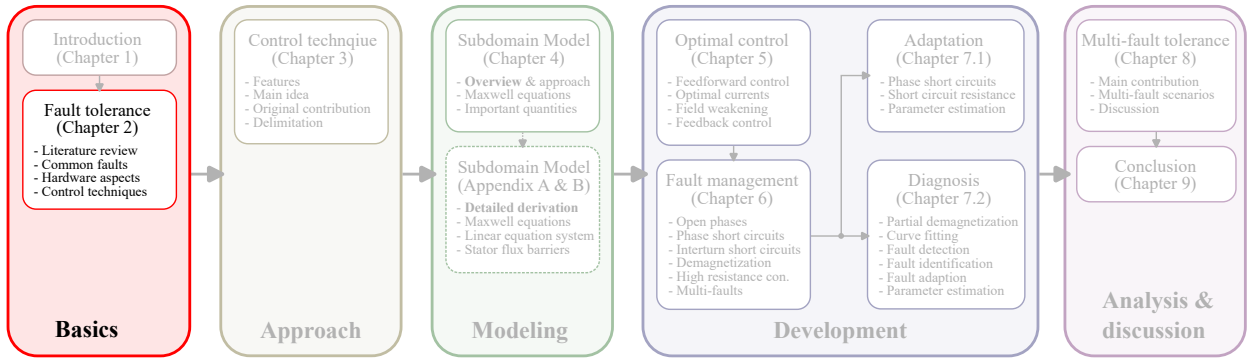


Figure 2.1: Structural overview of the work, with this chapter 2 highlighted.

2.1 Fault classification

Faults in PMSMs are typically classified into three main categories: **electrical**, **magnetic**, and **mechanical** faults (see Fig. 2.2). While the electrical and mechanical faults are common to most electric machine topologies, magnetic faults are specific for PMSMs (or other topologies with permanent magnets). The following provides an overview of typical fault types within each category.

Electrical faults

Electrical faults can be further divided into two subcategories: wiring faults and short circuits (see Fig. 2.2). The most common wiring fault is an open phase or open circuit, which may result from a triggered fuse, a loose phase connection, a cable break, or faults within the power electronics [5]. Without a fault tolerant control, an open phase can cause vibrations, large torque ripple, or even a complete loss of the controllability. This is particularly critical

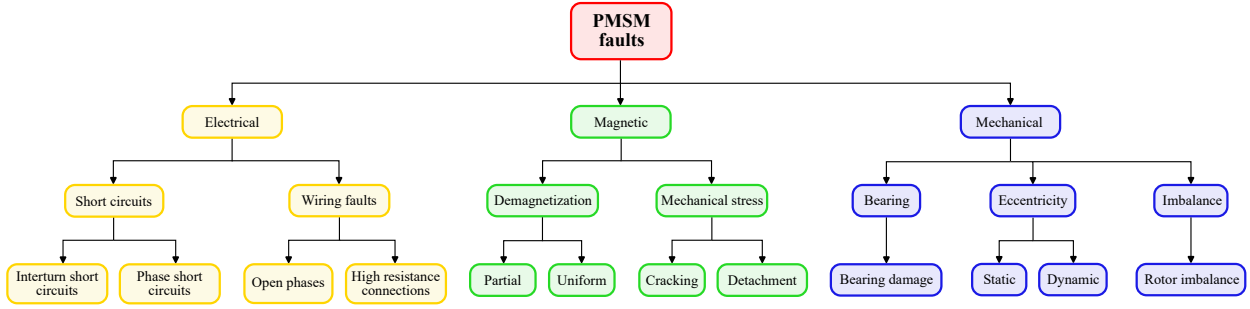


Figure 2.2: Overview of the most common PMSM faults [2, 7].

in conventional three-phase machines, where the absence of one phase makes it physically impossible to generate constant torque using the two remaining healthy phases [8].

Another relevant wiring fault is the high resistance connection, which manifests as an unintended increase in the effective phase resistance. It can be caused by poor workmanship, contact oxidation, or thermal cycling [9]. An imbalance in the phase currents and voltages, increased ohmic losses and local temperature rise are a result. This heating may further increase the resistance, which can cause in the worst case a melting of the conductor, resulting in a full open phase failure [10].

The relevant fault types within the second subcategory are phase-to-phase short circuits (commonly referred as phase short circuits) and interturn short circuits. Thereby interturn short circuits are considered to be one of the most frequent fault types in PMSMs [11]. As discussed in [12], the amplitude of the short circuit current during such a fault mostly depends on the effective short circuit impedance (expressed by the effective short circuit resistance and number of shorted coil turns). In cases where this impedance is low, the resulting current can reach critical levels which can cause an overheating and in the worst case a burning of the affected coil. The challenging feature of interturn short circuits is that both the diagnosis and the fault compensation are difficult.

In general, interturn and phase short circuits are commonly caused by insulation breakdown, which may result from mechanical vibrations, thermal stress, or insulation aging. The circulating short circuit currents can also induce torque oscillations, leading to increased vibrations and acoustic noise. Phase short circuits involve a direct short between one or multiple phases. This leads to the triggering of the corresponding fuse, resulting in an open phase condition. However, unlike a pure open phase fault, a phase short circuit also produces additional torque ripple due to residual currents flowing through the shorted winding [13].

Magnetic faults

Magnetic faults (see Fig. 2.2) are specific for machine topologies which use permanent magnets for the magnetic field excitation. Thereby the most common magnetic fault is demag-

netization, which can be classified as either uniform or partial.

Uniform demagnetization typically results from an overheating of the rotor magnets above the Curie temperature [14]. As a result, the torque capability is decreased whereby the currents for producing the same torque are increased. This, in turn, further raises the winding temperature and can exacerbate the demagnetization process, resulting in a self-reinforcing degradation loop that threatens the long-term reliability of the drive.

Partial demagnetization can arise from intense external magnetic fields, such as those occurring during short circuits [14]. In this case, the magnetization of individual magnets or sections of magnets is locally reduced, while other regions remain unaffected. The asymmetry in the magnetic field results in back-EMFs, which induce higher-order harmonic components in the phase currents. These harmonics can lead to increased torque ripple, mechanical vibrations, and acoustic noise, thereby affecting both performance and durability [15].

In addition, mechanical influences also pose a risk to the integrity of the magnets. Repeated mechanical stress, vibrations, or manufacturing defects can lead to cracking or chipping of the magnets. In severe cases, a magnet may even become detached from the rotor, for instance if the adhesive bond fails. This detachment is functionally similar to partial demagnetization, as the detached magnet no longer contributes to the magnetic flux and its relative permeability is close to that of air [16].

Mechanical faults

Mechanical faults in electrical machines can be classified into three subcategories: eccentricity, imbalance, and bearing-related issues (see Fig. 2.2). Among these, bearing damage is the most prevalent type of mechanical fault [2]. It may arise due to factors such as mechanical overload, aging, or inadequate lubrication. As the bearings degrade, they can lead to mechanical vibrations and rotor eccentricity.

Rotor eccentricity is typically categorized as either static or dynamic, although both types can occur simultaneously in practice [7]. In all cases, the uniformity of the airgap is disrupted, resulting in an uneven magnetic flux distribution. This can significantly alter the back-EMF and the torque generation, often increasing torque ripple and acoustic noise. In severe cases, eccentricity may even lead to physical contact between the rotor and stator, potentially causing critical mechanical damage [2].

Rotor imbalance is another common mechanical issue, often originating from improper assembly or manufacturing tolerances. Additionally, the detachment of a permanent magnet from the rotor can cause an imbalance. Over time, such imbalances can accelerate wear on the bearings and lead to their failure.

2.2 Fault tolerance from a hardware perspective

As discussed in [5, 6], there exist various solutions to improve the fault tolerance of the overall drive system from a hardware perspective. In this section three hardware features are described, which improve the fault tolerance of PMSMs and which are of relevance for this work.

Multi-phase machines

Multi-phase machines are electrical machines that operate with more than the conventional three phases [3, 4]. Increasing the number of phases enhances the system's redundancy and provides additional degrees of freedom, which – in combination with a suitable fault tolerant control strategy – significantly improves the overall robustness of the drive against faults [5]. This built-in redundancy allows the machine to maintain full operation even in the presence of faults like open phases, making it especially suitable for safety-critical applications.

Beyond improved fault tolerance, multi-phase machines offer several other advantages compared to traditional three-phase machines. For the same power level, they require lower current per phase, which reduces thermal stress on the windings and eases the design requirements for the power electronics. Additionally, torque ripple is typically reduced, enabling smoother and more accurate operation [17].

In practical applications, the most commonly used multi-phase configurations are five-phase machines and machines with phase counts that are integer multiples of three, such as six- or nine-phase machines. These configurations allow a straightforward integration partly with existing hardware and control strategies while providing enhanced performance and fault resilience.

Concentrated windings

Electrical machines typically utilize one of two main types of windings: distributed or concentrated windings. The latter can be further classified into single layer and double layer concentrated windings¹. Each winding type offers specific advantages and disadvantages in terms of characteristics such as efficiency, torque ripple, and acoustic noise, depending on the application and machine topology [18–20]. However, when considering fault tolerance, concentrated windings offer distinct advantages over distributed windings. These benefits

¹Single layer concentrated windings are wrapped around alternating teeth and the double layers are wrapped around each teeth. In the latter case each slot contains two adjacent winding parts, whereas the slots in the single layer case contain only one winding part.

primarily arise from their inherent decoupling effects, which can be divided into three main categories [21]:

1. Magnetic decoupling
2. Electrical decoupling
3. Thermal decoupling

In general, the most important effect is the magnetic decoupling. Since concentrated windings do not overlap each other, their mutual coupling and thereby their mutual inductances are reduced compared to the distributed windings. This applies in particular to the single layer concentrated windings, since they have significantly lower mutual inductances compared to the double layers [22]. Without this decoupling, a fault like a short circuit in one phase could affect other phases by inducing high voltages. Consequently, the fault severity would be drastically increased. Due to the decreased contact area of concentrated windings, the electrical as well as thermal decoupling is improved as well, protecting adjacent phases from fault propagation. Again, this applies in particular to single layer concentrated windings, since they do not have a direct contact area at all.

Summarizing, from the perspective of the fault tolerance, single layer concentrated windings are preferable compared to the double layers due to the clearly enhanced decoupling in all of the above described three categories (magnetic, electrical, thermal).

Stator flux barriers

Stator flux barriers are additional gaps in the stator iron core, which have an influence on the magnetic flux distribution. By decreasing the amplitude of parasitic sub-harmonics and increasing the working harmonic of magnetic field in the airgap, the torque density is increased [23] and losses like eddy currents (especially within the magnets) are decreased [24]. Moreover, flux barriers offer some mass reduction advantages by enabling more material-efficient designs [25].

From a fault tolerance perspective, flux barriers enhance the decoupling effects already provided by concentrated windings (see subsection above). This improved decoupling contributes to better fault isolation and control flexibility under fault conditions. In addition, there are investigations of using the space provided by the flux barrier for integrated cooling solutions [26], which could further reduce the risk of thermal overload and thereby enhance the system's robustness against temperature-related faults.

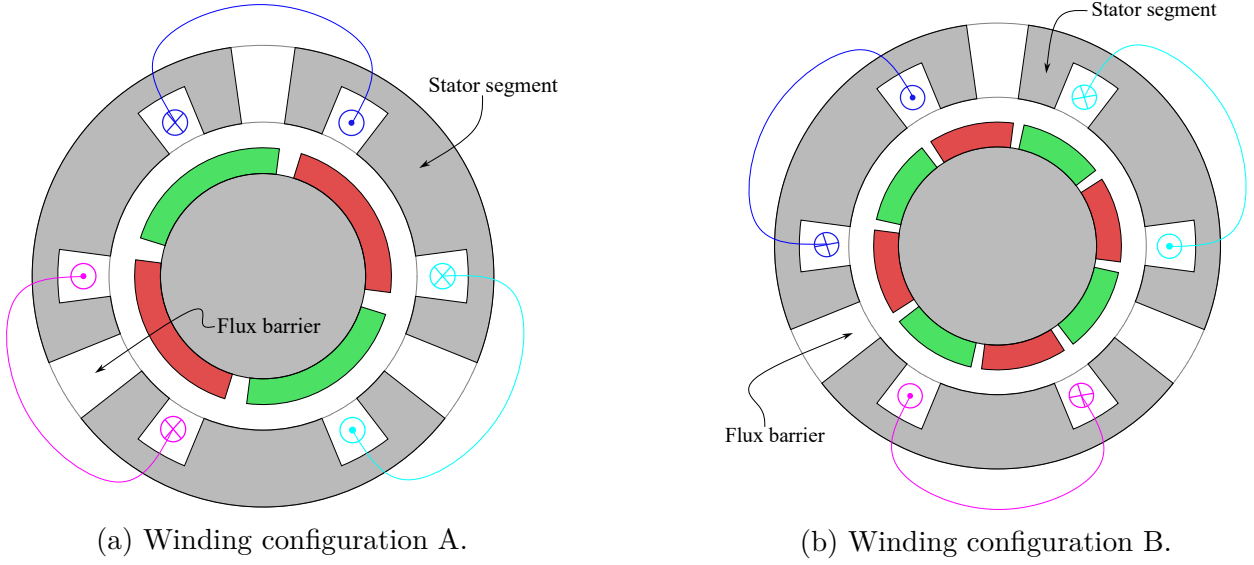


Figure 2.3: Qualitative structure of PMSMs with stator flux barriers in combination with two different configurations of single layer concentrated windings [27].

For a single layer concentrated winding with n_{SL} slots, two pole pair numbers p are generally feasible:

$$p = \frac{1}{2}(n_{SL} \pm 2). \quad (2.1)$$

When combined with stator flux barriers, two distinct winding configurations in turn can be realized [27], as illustrated in Fig. 2.3. In configuration A (Fig. 2.3a) each winding is placed *around* a single flux barrier, and in configuration B (Fig. 2.3b) each winding is placed *between* two flux barriers. Configuration A is generally better suited for machines with the lower number of pole pairs, while configuration B is more suitable for the higher pole pair variant. This is sketched as an example for a 6-slot design in Fig. 2.3a with $p = 2$ and Fig. 2.3b with $p = 4$. From a fault tolerance perspective, configuration B offers advantages. By placing each winding between two flux barriers, it provides physical isolation of the windings within their respective stator segments. This isolation improves fault containment, making the machine more robust against various fault conditions.

2.3 Fault tolerance from a control perspective

A wide range of fault tolerant control strategies for electrical machines has been proposed in the literature, as comprehensively reviewed in [5, 6]. The existing techniques differ significantly in terms of complexity, fault coverage, and required knowledge of fault parameters. Commonly addressed fault types include motor-intrinsic faults, sensor faults, and faults within the power electronics. In the following, an overview is provided of (motor-intrinsic) fault tolerant control strategies relevant to this work.

Compensation of open phase faults

Open phase faults are by far the most extensively investigated fault types in the literature. As a result, numerous control strategies have been developed to compensate this fault. These methods can generally be classified into categories such as vector space decomposition, direct torque control, and field oriented control approaches [6]. One of the most common approaches involves generating suitable current references by solving an optimization problem within a field oriented control framework. Several notable examples of this approach are outlined in the following. In [28], an open phase fault tolerant control strategy is proposed that determines optimal current references based on a cost function which minimizes losses. In contrast to conventional approaches that often primarily aim to minimize copper losses using a maximum torque per ampere strategy, the proposed method extends the optimization problem to account for iron core losses, thereby enabling the minimization of the total losses. In [29] additional constraints in the minimization problem for considering the voltage limit are incorporated, allowing field weakening with maximum permissible torque-speed limits in the presence of an open phase. To account for trapezoidal back-EMFs, which typically occur in machines with concentrated windings, an optimization problem incorporating the third-harmonic current component was introduced in [30]. A more generalized formulation of this optimization problem suitable for different winding configurations (star, pentagon, pentacle) was then presented in [31]. A solution for determining efficiently optimal reference currents under open phase conditions is proposed in [32], enabling reduced computational effort. Furthermore, a simplified approach that does not rely on solving an optimization problem is described in [33], where symmetrically distributed currents are directly calculated to enable the compensation of an open phase.

Overall, these studies demonstrate that constrained minimization problems for generating optimal current references are particularly effective in managing open phase faults. In most cases, fault compensation can be achieved efficiently by excluding the affected phase from the optimization process.

Compensation of phase short circuits

There are two categories of control strategies to compensate phase short circuits: those that reduce the short circuit current and others that compensate the faulty torque ripple [5]. The principle of the first category is to effectively reduce the PM flux linkage of the faulty coil by the healthy ones, minimizing the short circuit current. However, this can require huge phase currents since most fault tolerant drives have very low mutual inductances. As a result, only a limited number of publications address this approach. One example is presented in [34], where a magnet-flux nulling control method is proposed to reduce the short circuit current. It should be noted, however, that the control was developed for an induction machine with distributed windings and thus with high mutual inductances.

Control approaches in the second category are more popular. Thereby constrained minimization problems are frequently employed to compute optimal reference currents. By interpreting a phase short circuit as an open phase combined with an additional torque ripple, it can be efficiently incorporated into such an optimization framework. Examples of this approach are the references [35–38]. The above mentioned method presented in [33] for compensating an open phase fault is also applicable to phase short circuit faults, without the need to solve an optimization problem. An additional alternative to optimization-based approaches is proposed in [39], where an iterative learning control is introduced to successively adapt the control to the periodic torque ripple caused by the phase short circuit.

Compensation of interturn short circuits

Because of their complexity, fault tolerant control techniques which compensate interturn short circuits are relatively rare in literature [2]. Analogously to the phase short circuit, compensation methods for interturn short circuits can be divided into two categories, either reducing the short circuit current or compensating the resulting torque ripple. Compared to the phase short circuit, reducing the short circuit current is typically more effective, as the mutual inductance between the shorted and operational coil turns is comparatively high. For cases with low short circuit impedance this strategy is preferable, since the short circuit current can otherwise grow to critical values. Representative approaches to this include [40] and [41], where field weakening strategies are introduced to effectively reduce the back-EMF in the short circuited coil turns, thereby limiting the short circuit current. A related method is presented in [42], which aims to minimize the overall copper losses of the PMSM in presence of an interturn short circuit. Examples of compensation strategies from the second category include [43] and [44], where diagnostic algorithms are developed to estimate the relevant fault parameters of the interturn short circuit, along with fault tolerant control frameworks designed to compensate the resulting torque ripple.

Compensation of demagnetization faults

Fault tolerant control strategies addressing (partial) demagnetization in PMSMs are scarce in literature [5]. In [45] a current control scheme in the synchronous reference frame to handle demagnetization faults is derived. This is achieved by adapting the control references in response to observed changes in machine parameters, thus improving the robustness against local magnet degradation. A second approach can be found in [46], where a robust model predictive current control approach is proposed, incorporating a composite observer to estimate and compensate the flux reduction caused by partial demagnetization. The observer dynamically adjusts the internal model of the machine by estimating the magnet flux amplitude, thereby improving the accuracy of the predictive current control to achieve quick dynamic responses.

Since the author is not aware of any additional literature addressing the fault tolerant control for PMSMs in the presence of (partial) demagnetization faults, it is concluded that this (single) fault, compared to the others discussed in this section, requires the most extensive future research.

Compensation of high resistance connections

Some approaches have been proposed in the literature to compensate high resistance connections. In [45, 47], diagnostic strategies are combined with torque ripple compensation methods to mitigate the impact of such faults on machine performance. These approaches focus on estimating the increased resistance and adjusting the control signals accordingly to maintain balanced torque output. An advanced compensation method is presented in [9], where the controller actively minimizes copper losses by adapting the current references based on the estimated phase resistance. This approach not only enhances the efficiency, but also reduces the thermal stress on the affected winding. Another relevant contribution is found in [48], which introduces a robust current control scheme designed to handle high resistance asymmetries while ensuring low torque ripple.

Alternatively, if the resistance increases to a point where the compensation becomes ineffective or unsafe, the affected phase can be disconnected by opening the corresponding switches. This transforms the fault into an open phase condition, for which the established fault tolerant control strategies described above can be applied.

Multi-fault tolerant control

When it comes to compensate multiple faults simultaneously, only a very limited number of publications address multi-fault tolerant control strategies. Most existing approaches focus on specific two-fault scenarios. For instance, the previously mentioned reference [45] presents a control scheme capable of handling both high resistance connections and demagnetization faults. In [49], a combined diagnosis and control approach is developed considering short circuits as well as open phase faults. The most established dual-fault tolerant control approaches are those involving open phases and phase short circuits, due to the structural similarity of these faults. As described above, constrained minimization problems are commonly formulated to compute optimal current references in the presence of such faults [35–38]. A key benefit of these methods is that only minor adjustments in the optimization are required to account for each fault.

Summary

Based on the literature discussed in this section, it can be concluded that there already exist various single-fault tolerant control techniques. Also, there are some approaches addressing specific two fault scenarios. To the best knowledge of the author, no literature is known which explicitly addresses a generalized control approach capable of compensating arbitrary combinations of multiple faults. Nevertheless, it can be recognized that constrained minimization problems are a promising foundation for a multi-fault tolerant control, since the compensation can be achieved requiring only minor adjustments, and these adjustments are usually not contradictory.

3 Approach and delimitation

As discussed in section 2.3, there already exist various single-fault tolerant control techniques in the literature. Some approaches also address the simultaneous compensation of two faults. However, for highly safety-critical applications, it is desirable to have a fault tolerant control system that is as universally applicable as possible. The aim of this work is therefore to develop a control system with the following key features:

1. High fault tolerance with explicit focus on the multi-fault tolerant operation
2. Ripple-free torque generation even in the presence of faults
3. Efficient integration of stator flux barriers
4. Field weakening capability under fault conditions

The primary focus lies on the first feature, the development of a generalized fault tolerant control system for PMSMs that can handle a wide variety of faults, including all possible combinations. In addition, the control should ensure ripple-free torque generation to track accurately a desired trajectory of the rotor position, independently of the presence of faults. Since stator flux barriers have advantageous properties for improving the fault tolerance in

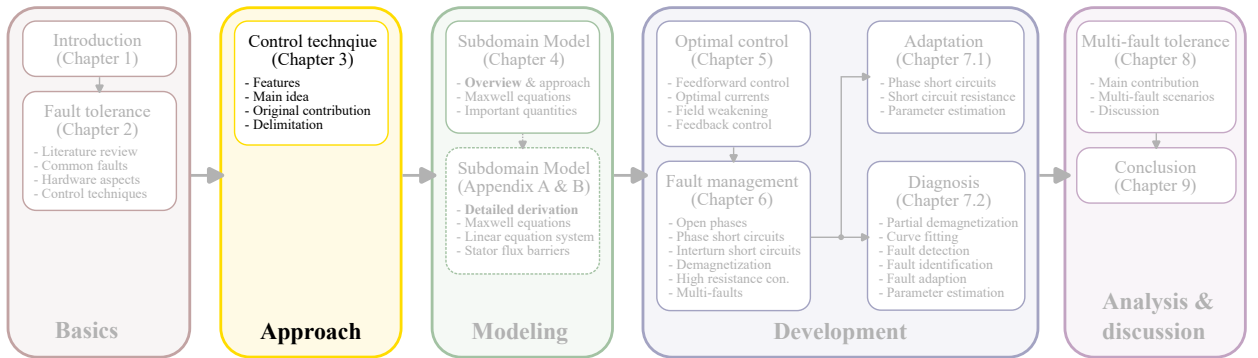


Figure 3.1: Structural overview of the work, with this chapter 3 highlighted.

PMSMs, another objective is their efficient integration into the control system. Nevertheless, the method should also remain fully applicable to conventional PMSMs without flux barriers. Finally, to ensure a wide operational range, the control system must include a field weakening strategy that remains fully functional across all considered fault conditions.

3.1 Main idea & original contribution

The main idea of this work is to use the Subdomain Model, a highly accurate modeling approach based on Maxwell equations [50–56], as basis of the control system. While the Subdomain Model has been extensively studied and applied for the detailed analytical modeling of electrical machines, it has not yet been employed for real-time control applications.

Solving the Maxwell equations of magnetostatics, the magnetic field distribution within the airgap can be precisely determined and all important physical quantities for describing the dynamics of a PMSM (as torque, PM-flux linkage, back-EMF and inductances) can be efficiently derived. Thereby the major advantage of the Subdomain Model is the high accuracy in combination with a good adaptability to faults. Furthermore, the mathematical formulation of the Subdomain Model can be optimized such that the resulting equations can be solved in real time. Based on these advantageous properties, the Subdomain Model is systematically integrated into a highly fault tolerant control system.

The original contributions of this work can be summarized as follows:

1. Control for PMSMs based on the highly accurate Subdomain Model which, by means of an efficient mathematical formulation, can be solved in real time.
2. High fault tolerance considering all relevant electrical and magnetic faults of PMSMs, including multi-fault operation (all possible combinations of faults).
3. Robust field weakening strategy that remains fully functional in the presence of faults.

In total, it is demonstrated that the Subdomain Model, traditionally used for the design and analysis of electrical machines, can be efficiently used as basis for the (multi-) fault tolerant control of PMSMs.

3.2 Delimitation of the scope

To enable an efficient application of the proposed method, the following delimitations and assumptions are made:

1. Only motor-intrinsic electrical and magnetic faults are considered.
2. The load torque is assumed to be extracted through a direct measurement of the machine's produced torque.
3. Magnetic saturation and other nonlinear magnetic effects are not considered.

Since the Subdomain Model is most efficient applicable to motor-intrinsic electrical and magnetic faults, only fault types of these categories are considered. Thereby mechanical faults, sensor faults or other faults external to the machine are excluded from the investigation. Regarding the load torque acting on the machine, it is assumed that it is extracted from a direct measurement of the generated torque. This is relevant for the determination of the desired torque. Due to the assumption of infinite permeable ferromagnetic iron in the Subdomain Model, the effect of magnetic saturation is not considered. Thereby the saturation is treated as an effective disturbance from the perspective of the control, that needs to be compensated in practice.

In general, it is assumed in this work that the relevant fault parameters are known. However, in order to illustrate the fundamental principles of the fault diagnosis and adaptation, two examples are provided: a diagnosis method for detecting and identifying partial demagnetization, and an adaptation algorithm for handling phase short circuits. Both methods are tailored on the properties of the developed control.

From a hardware perspective, the focus of this work is on multiphase PMSMs with single layer concentrated windings and surface magnets since a) they are one of the most relevant machine topologies for applications requiring high fault tolerance, b) the Subdomain Model as modeling basis is most efficiently applicable for this machine topology, and c) it simplifies the control due to the magnetic decoupling of the concentrated windings.

3.3 Introduction of the control system

The developed control system, illustrated in Fig. 3.2, is primarily composed of a feedforward control and a feedback control. The task of the **feedforward control** is to accurately predict optimal currents and voltages for tracking a desired mechanical trajectory. This prediction is carried out by an analytical optimization based on the Subdomain Model, which

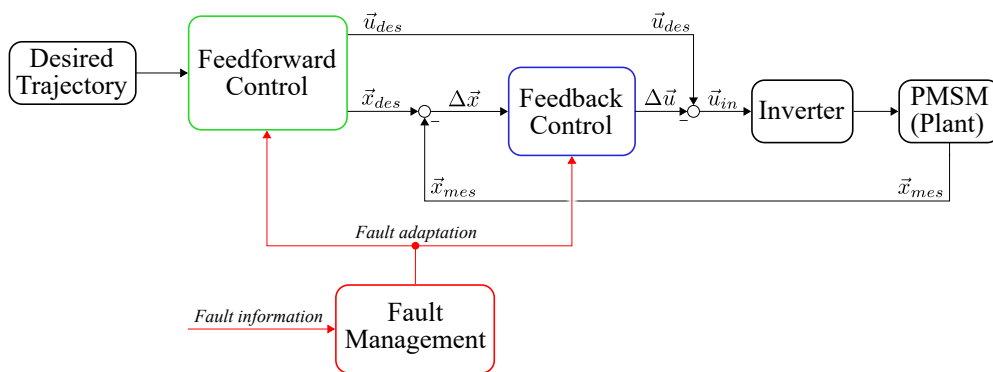


Figure 3.2: Schematic structure of the developed control system. All vectors $\vec{x}$ contain the relevant dynamic variables for the control.

is also capable of using the additional degrees of freedom available in multiphase machines. Since the relations between currents and the required voltages are known precisely, a field weakening strategy is implemented into the feedforward control incorporating the electrical limitations. All these computations are performed under the assumption of steady state conditions.

The **feedback control** is responsible for compensating deviations from the desired trajectory. Such deviations can arise from sudden disturbances, measurement delays, transients or other prediction errors of the feedforward control. To correct these deviations, optimally distributed voltage corrections are calculated and applied to the phases. In this way, the feedback control ensures the robustness and dynamic accuracy of the overall system.

To compensate faults, only minor modifications to the control system are required. Thereby the **fault management** represents the systematic adaptation of both the feedforward and feedback control based on the fault information, ensuring stability of the overall system despite the presence of electrical or magnetic failures. This enables the realization of a multi-fault tolerant control system.

4 Subdomain Model

The Subdomain Model is a modeling approach designed to calculate the magnetic flux distribution inside an electrical machine with high accuracy. The basic idea is to divide the machine into several subdomains and solve there the Maxwell equations of magnetostatics. The method was introduced in the 1990s, where some of the earliest references found is the publication series [50–53]. There, the open-circuit field was obtained by separating a slotless PMSM into the subdomains for the magnets and the air gap, solving the Maxwell equations [50]. The influence of the armature windings was incorporated by modeling the phase currents as current sheets placed on the stator surface [51]. To account for the effect of the stator slots, a relative permeance function was developed based on a conformal transformation [52]. Although the method demonstrated high accuracy [53], only the radial component of the magnetic field could be determined, making a precise computation of the resulting torque difficult.

Shortly thereafter, a version of the Subdomain Model including the stator slots as an additional subdomain under open-circuit conditions was presented [54]. This approach made it possible to describe the magnetic field of the rotor magnets only using Maxwell equations, whereby the consideration of a relative permeance function was not required anymore. As a result, both the radial and tangential components of the magnetic field could be calculated, allowing the cogging torque to be predicted precisely based on Maxwell’s stress tensor.

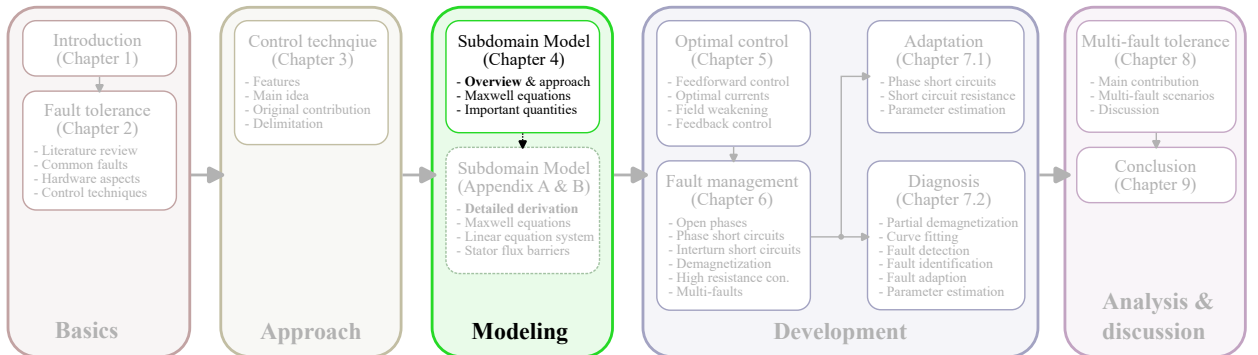


Figure 4.1: Structural overview of the work, with this chapter 4 highlighted.

The first reference that directly addressed the armature reaction field while considering the stator slots as a separate subdomain was presented in [55]. This approach was subsequently

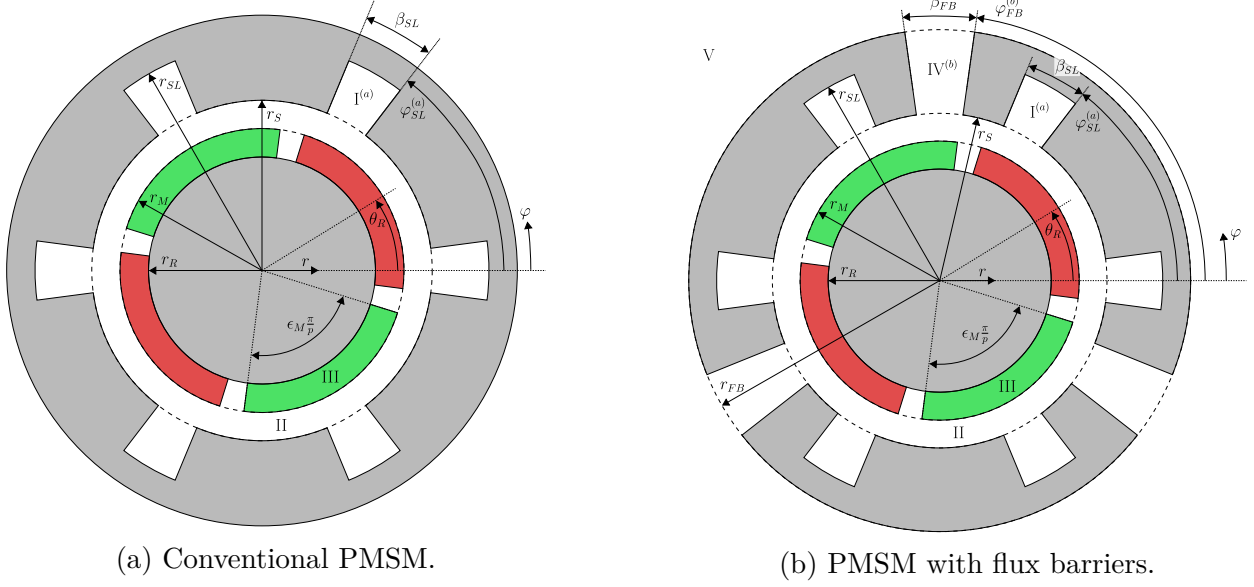


Figure 4.2: Considered geometry and relevant parameters in the Subdomain Model [60, 61].

employed to derive the Subdomain Model in its entirety, enabling the determination of both the open-circuit and the armature reaction fields to calculate the on-load torque, back-EMFs, and inductances [57].

While the Subdomain Model has been extended in various different ways, for example by considering stator tooth tips [56] or by incorporating magnetic saturation through virtual currents [58, 59], only the ‘raw model’ presented in [57] is used as the basis of this work. Thereby the Subdomain Model is derived for both a conventional PMSM and a PMSM with stator flux barriers. While the basic approach is briefly summarized in this chapter, detailed derivations for both machine types are provided in appendix A (conventional PMSM) and appendix B (PMSM with flux barriers).

4.1 Basic approach

The geometry of the conventional PMSM considered in the Subdomain Model is illustrated in Fig. 4.2a. The rotor, with radius r_R , has $2p$ magnets mounted on its surface, where p is the number of pole pairs. The magnets have a pole-arc to pole-pitch ratio of ϵ_M and are $r_M - r_R$ high. The stator, with inner radius r_S , contains n_{SL} slots, whereby each slot (a) is positioned at $\varphi_{SL}^{(a)}$, has a width of β_{SL} and a height of $r_{SL} - r_S$. Furthermore, each flux barrier (b) of the machine sketched in Fig. 4.2b is positioned at $\varphi_{FB}^{(b)}$, has a width of β_{FB} and a height of $r_{FB} - r_S$.

Based on this geometry, the following assumptions are made for the modeling:

1. Infinite permeable ferromagnetic material (neglecting magnetic saturation)
2. Infinite axial length of the electrical machine (side effects are neglected)
3. Rotor magnets are assumed to be radially magnetized with a relative recoil permeability equal to one
4. Each slot carries a homogeneously distributed current density oriented only in axial direction
5. Slot geometry is idealized to have only radial and tangential oriented surfaces
6. No time dependencies (Maxwell equations of magnetostatics)

All these assumptions made have in common that they drastically reduce the mathematical complexity and thereby the computational effort. By neglecting the side effects at the (axial) edges of the PMSM, the magnetic flux distribution only shows a radial and tangential dependency. This allows a two-dimensional modeling. Assuming an infinite permeability of the iron, the magnetic flux enters or leaves orthogonally such high permeable material. This, in combination with the assumed ideal slot geometry, allows an efficient formulation of the relevant boundary conditions. The solution of the Maxwell equations is achieved by using different Fourier expansions for each of three different subdomain types (see Fig. 4.2a): subdomain I^(a) for the stator slots with a given current density $J_z^{(a)}$ and (a) running from 1 to the total number of slots n_{SL} , subdomain II for the airgap and subdomain III for the rotor magnets with a given magnetization in radial direction. For the PMSM with the flux barriers (see Fig. 4.2b) two additional types of subdomains need to be considered: the subdomains IV^(b) for the flux barriers by themselves, with (b) running from 1 to the total number of flux barriers n_{FB} , and a subdomain V for the surrounding air.

The modeling procedure is carried out as illustrated in Fig. 4.3. Using the Maxwell equations of magnetostatics, a homogeneous solution can be obtained for each of the described subdomains. In addition, particular solutions are required for the slot subdomains and the magnet subdomain to account for the magnetic sources. In the next step, the boundary conditions at the interfaces to the high permeable iron are imposed. These are formulated such that the magnetic field enters or leaves the ferromagnetic material orthogonally. Subsequently continuity at each interface between two subdomains must be enforced. The resulting set of equations can be rearranged into a linear equation system for calculating the unknown Fourier coefficients. This equation system effectively depends on the two magnetic sources. The first source is the (radial) magnetization of the rotor magnets, whose spatial

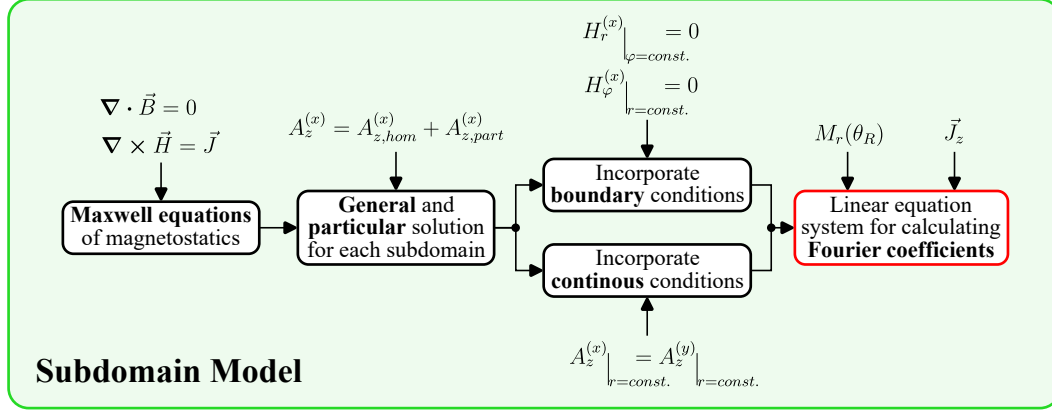


Figure 4.3: Modeling procedure of the Subdomain Model. (x) and (y) represent placeholders for the subdomain indices I-V.

distribution at actual rotor position θ_R is described by the following Fourier series

$$\mu_0 M_r(\theta_R) = \sum_{k \in \mathbb{N}} \frac{1}{r_R} \left[-a_k^{(M)}(\theta_R) \sin(k\varphi) + c_k^{(M)}(\theta_R) \cos(k\varphi) \right] \quad (4.1)$$

with φ the angular position and the Fourier coefficients

$$\begin{aligned} a_k^{(M)}(\theta_R) &= -r_R \mu_0 M_k \sin(k\theta_R), \\ c_k^{(M)}(\theta_R) &= +r_R \mu_0 M_k \cos(k\theta_R), \\ M_k &= \frac{B_{rem}}{\mu_0} \frac{4p}{k\pi} \sin\left(\frac{k\pi}{2p} \epsilon_M\right). \end{aligned} \quad (4.2)$$

The second source are the current densities flowing through each coil, which are summarized in the vector $\vec{J}_z$

$$\vec{J}_z = \left(J_z^{(c=1)} \quad J_z^{(c=2)} \quad \dots \quad J_z^{(c)} \quad \dots \quad J_z^{(c=n_{coil})} \right)^T. \quad (4.3)$$

Index z denotes the axial flow direction of the currents. By solving the linear equation system to determine the unknown Fourier coefficients, the magnetic flux distribution within the PMSM can be determined accurately with the Subdomain Model equations.

As described above, one basic assumption made is the infinite permeability of the iron. This assumption enables an efficient formulation of the boundary conditions at the iron interfaces. However, it also implies that the nonlinear magnetic saturation is neglected. In relation with this work the effect of magnetic saturation was incorporated into the Subdomain Model by virtual currents obtained from a magnetic equivalent circuit model [61]. While this represents an enhancement of the Subdomain Model, it is not compatible with the developed control. Therefore, this enhanced Subdomain Model is not further considered.

4.2 Important quantities

With the magnetic flux distribution determined by the Subdomain Model, the following important quantities for describing the dynamics of a PMSM can be obtained.

4.2.1 PM flux linkage & back-EMF

The PM flux linkage of slot (a) is calculated by a surface integral over the slot area S_{SL} of the vector potential¹ [57]

$$\Phi_{M,SL}^{(a)}(\theta_R) = \frac{\ell n_{turn}}{S_{SL}} \iint_{S_{SL}} A_z^{(I,a)} \Big|_{\vec{J}_z=\vec{0}} r dr d\varphi, \quad (4.4)$$

where ℓ denotes the active rotor length. For the case of no currents in all slots only the offset $a_0^{(I,a)}$ of the slot's vector potential contributes to the integral, which leads based on the assumed concentrated windings to the following equation for the PM flux linkage

$$\Phi_M^{(c)}(\theta_R) = \pm \left(\Phi_{M,SL}^{(c_l)}(\theta_R) - \Phi_{M,SL}^{(c_r)}(\theta_R) \right) = \pm \ell n_{turn} \left(a_0^{(I,c_l)} - a_0^{(I,c_r)} \right), \quad (4.5)$$

where c_l, c_r denote the left and right slot of coil (c). The sign in (4.5) indicates the relative winding orientation within the polyphase system. As described in section 4.1, all unknown Fourier coefficients can be calculated by a linear equation system, which is derived in appendix A.5 for the case of a conventional PMSM and in appendix B.3 for the case of a PMSM with flux barriers. By inserting the according part of the equation system² for calculating $a_0^{(I,a)}$, one obtains

$$\begin{aligned} \Phi_M^{(c)}(\theta_R) &= \pm \ell n_{turn} \left[\left(\left(\vec{Q}_{0A}^{(I,c_l)} \right)^T - \left(\vec{Q}_{0A}^{(I,c_r)} \right)^T \right) \vec{a}^{(M)}(\theta_R) + \left(\left(\vec{Q}_{0C}^{(I,c_l)} \right)^T - \left(\vec{Q}_{0C}^{(I,c_r)} \right)^T \right) \vec{c}^{(M)}(\theta_R) \right], \\ \vec{\Phi}_M(\theta_R) &= \left(\Phi_M^{(1)}(\theta_R) \quad \Phi_M^{(2)}(\theta_R) \quad \dots \quad \Phi_M^{(c)}(\theta_R) \quad \dots \quad \Phi_M^{(n_{coil})}(\theta_R) \right)^T. \end{aligned} \quad (4.6)$$

Thereby vector $\vec{Q}_{0x}^{(I,a)}$ contains effectively the geometric influence on the flux distribution caused by the magnetization within the slots. Taking the derivative of (4.6) with respect to the rotor angle and multiplying with the rotational speed ω_R one obtains the back-EMFs:

$$\begin{aligned} e^{(c)}(\theta_R) &= \pm \omega_R \ell n_{turn} \left[\left(\left(\vec{Q}_{0A}^{(I,c_l)} \right)^T - \left(\vec{Q}_{0A}^{(I,c_r)} \right)^T \right) \frac{\partial \vec{a}^{(M)}(\theta_R)}{\partial \theta_R} + \left(\left(\vec{Q}_{0C}^{(I,c_l)} \right)^T - \left(\vec{Q}_{0C}^{(I,c_r)} \right)^T \right) \frac{\partial \vec{c}^{(M)}(\theta_R)}{\partial \theta_R} \right], \\ \vec{e}(\theta_R) &= \left(e^{(1)}(\theta_R) \quad e^{(2)}(\theta_R) \quad \dots \quad e^{(c)}(\theta_R) \quad \dots \quad e^{(n_{coil})}(\theta_R) \right)^T. \end{aligned} \quad (4.7)$$

¹The vector potential $A_z^{(I,a)}$ of the slot (a) is defined by equation (A.25) in the appendix A.

²(A.73) for a conventional PMSM or (B.28) for a PMSM with flux barriers, in both cases with $\vec{J}_z = \vec{0}$.

Sine vectors $\vec{Q}_{0x}^{(I,a)}$ are constant over time, they can be calculated and stored in advance to have a real-time capable determination of the PM flux linkage and back-EMF.

4.2.2 Inductance

The self inductances can be calculated by the magnetic energy method [62], integrating over the slot area¹

$$L^{(c)} = \frac{n_{turn}\ell}{i^2} \sum_{a \in S^{(c)}} \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} \int_{r_S}^{r_{SL}} A_z^{(I,a)} \Big|_{|J_z^{(a)}| > 0} J_z^{(a)} r dr d\varphi \quad (4.8)$$

where $S^{(c)}$ is the set of all slots from coil (c). Thereby only the offset $a_0^{(I,a)}$ and the particular solution of the slot's vector potentials contribute to the integral, which leads to the following equation

$$L^{(c)} = + \ell n_{turn} \frac{a_0^{(I,c_l)} - a_0^{(I,c_r)}}{i} + \mu_0 \frac{\ell n_{turn}^2 \beta_{SL}}{S_{SL}^2} \left(\frac{r_{SL}^4}{4} (2 \ln(r_{SL}) - 1) - \frac{r_{SL}^2 r_S^2}{4} (2 \ln(r_S) - 1) - \frac{r_{SL}^4}{8} + \frac{r_S^4}{8} \right). \quad (4.9)$$

The sign of the current densities $J_z^{(a)} = \pm \frac{i}{S_{SL}}$ denote the relative current direction within the coil. Again, all unknown Fourier coefficients can be calculated by the linear equation system, which is derived in appendix A.5 for the case of a conventional PMSM and in appendix B.3 for the case of a PMSM with flux barriers. By inserting the according part of the equation system³ for calculating $a_0^{(I,a)}$, one obtains

$$L^{(c)} = + \ell n_{turn}^2 \frac{\pm P_0^{(I,c_l)}(c_l) \mp P_0^{(I,c_l)}(c_r) \mp P_0^{(I,c_r)}(c_l) \pm P_0^{(I,c_r)}(c_r)}{S_{SL}} + \mu_0 \frac{\ell n_{turn}^2 \beta_{SL}}{S_{SL}^2} \left(\frac{r_{SL}^4}{4} (2 \ln(r_{SL}) - 1) - \frac{r_{SL}^2 r_S^2}{4} (2 \ln(r_S) - 1) - \frac{r_{SL}^4}{8} + \frac{r_S^4}{8} \right),$$

$$\mathcal{L} = \text{diag} \left(L^{(1)} \ L^{(2)} \ \dots \ L^{(c)} \ \dots \ L^{(n_{coil})} \right). \quad (4.10)$$

Thereby $P_0^{(I,x)}(y)$ effectively contain the geometric influence (in part) on the flux distribution within the slots caused by the coils.

³(A.73) for a conventional PMSM or (B.28) for a PMSM with flux barriers, in both cases with $\vec{a}^{(M)} = \vec{c}^{(M)} = \vec{0}$.

4.2.3 Torque

With the magnetic field distribution within the air gap, the electromagnetic torque exerted on the rotor can be calculated from the Maxwell stress tensor⁴ [54]

$$T_{el} = \frac{\ell r_S^2}{\mu_0} \int_0^{2\pi} B_r^{(II)} B_\varphi^{(II)} d\varphi, \quad (4.11)$$

integrating over the interface between air gap and stator material. The solution of this integral is defined by:

$$T_{el} = \frac{\ell\pi}{\mu_0} \left(\vec{c}^{T(M)}(\theta_R) \text{diag}(\vec{G}) \vec{a}^{(III)} - \vec{a}^{T(M)}(\theta_R) \text{diag}(\vec{G}) \vec{c}^{(III)} \right), \quad (4.12)$$

whereby the geometry depending parts of this equation are summarized in vector $\vec{G}$; it is defined in the appendix A.6. Again, all unknown Fourier coefficients can be calculated by the linear equation system, which is derived in appendix A.5 for the case of a conventional PMSM and in appendix B.3 for the case of a PMSM with flux barriers. By inserting the according part of the equation system⁵ for calculating $\{\vec{a}^{(III)}, \vec{c}^{(III)}\}$, one obtains

$$\begin{aligned} T_{el} = & \frac{\ell\pi}{\mu_0} \underbrace{\left(\vec{c}^{T(M)}(\theta_R) \text{diag}(\vec{G}) \mathbf{Q}_{AA}^{(III)} \vec{a}^{(M)}(\theta_R) + \vec{c}^{T(M)}(\theta_R) \text{diag}(\vec{G}) \mathbf{Q}_{AC}^{(III)} \vec{c}^{(M)}(\theta_R) \right.} \\ & \left. - \vec{a}^{T(M)}(\theta_R) \text{diag}(\vec{G}) \mathbf{Q}_{CC}^{(III)} \vec{c}^{(M)}(\theta_R) - \vec{a}^{T(M)}(\theta_R) \text{diag}(\vec{G}) \mathbf{Q}_{CA}^{(III)} \vec{a}^{(M)}(\theta_R) \right)}_{Q(\theta_R)} \\ & + \frac{\ell\pi}{\mu_0} n_{turn} \underbrace{\left(\vec{c}^{T(M)}(\theta_R) \text{diag}(\vec{G}) \mathbf{P}_A^{(III)} - \vec{a}^{T(M)}(\theta_R) \text{diag}(\vec{G}) \mathbf{P}_C^{(III)} \right)}_{\vec{P}^T(\theta_R)} \vec{J}_z \end{aligned} \quad (4.13)$$

$\frac{\ell\pi}{\mu_0} Q(\theta_R)$ can be interpreted as the prediction of the cogging torque, and $\frac{\ell\pi}{\mu_0} \vec{P}(\theta_R)$ as the torque capability of each coil at current rotor angle. Thereby matrices $\text{diag}(\vec{G}) \mathbf{Q}_{xx}^{(III)}$ effectively describe the geometric influence on the cogging torque, which is quadratically depending on the Fourier coefficients of the magnetization $\{\vec{a}^{(M)}, \vec{c}^{(M)}\}$. And matrices $\text{diag}(\vec{G}) \mathbf{P}_x^{(III)}$ describe the geometric influence on the interaction of the magnetic field in the airgap caused by the coils ($\vec{J}_z$) with the one caused by the magnets $\{\vec{a}^{(M)}, \vec{c}^{(M)}\}$, whereby the electromagnetic torque produced by the PMSM is quantified. All matrices $\{\text{diag}(\vec{G}) \mathbf{Q}_{xx}^{(III)}, \text{diag}(\vec{G}) \mathbf{P}_x^{(III)}\}$ can be calculated and stored in advance to have a real-time capable determination of the produced torque. With $Q(\theta_R)$ and $\vec{P}(\theta_R)$ equation (4.13) can

⁴The radial ($B_r^{(II)}$) and tangential ($B_\varphi^{(II)}$) magnetic field in the airgap is defined by (A.49) and (A.50) in the appendix A.

⁵(A.72) for a conventional PMSM or (B.27) for a PMSM with flux barriers.

be formulated concisely:

$$T_{el} = \frac{\ell\pi}{\mu_0} \left(Q(\theta_R) + \vec{P}^T(\theta_R) \vec{J}_z \right). \quad (4.14)$$

In context with the developed control, the time derivatives of $\vec{P}$ and Q are of relevance. They are defined as follows:

$$\dot{\vec{P}} = \omega_R \frac{\partial \vec{P}}{\partial \theta_R} = \omega_R n_{turn} \left(\frac{\partial \vec{c}^{T(M)}}{\partial \theta_R} \text{diag}(\vec{G}) \mathbf{P}_A^{(III)} + \frac{\partial \vec{a}^{T(M)}}{\partial \theta_R} \text{diag}(\vec{G}) \mathbf{P}_C^{(III)} \right) \quad (4.15)$$

and

$$\begin{aligned} \dot{Q} = \omega_R \frac{\partial Q}{\partial \theta_R} = \omega_R & \left(\frac{\partial \vec{c}^{T(M)}}{\partial \theta_R} \text{diag}(\vec{G}) \mathbf{Q}_{AA}^{(III)} \vec{a}^{(M)} + \vec{c}^{T(M)} \text{diag}(\vec{G}) \mathbf{Q}_{AA}^{(III)} \frac{\partial \vec{a}^{(M)}}{\partial \theta_R} \right. \\ & + \frac{\partial \vec{c}^{T(M)}}{\partial \theta_R} \text{diag}(\vec{G}) \mathbf{Q}_{AC}^{(III)} \vec{c}^{(M)} + \vec{c}^{T(M)} \text{diag}(\vec{G}) \mathbf{Q}_{AC}^{(III)} \frac{\partial \vec{c}^{(M)}}{\partial \theta_R} \\ & - \frac{\partial \vec{a}^{T(M)}}{\partial \theta_R} \text{diag}(\vec{G}) \mathbf{Q}_{CC}^{(III)} \vec{c}^{(M)} - \vec{a}^{T(M)} \text{diag}(\vec{G}) \mathbf{Q}_{CC}^{(III)} \frac{\partial \vec{c}^{(M)}}{\partial \theta_R} \\ & \left. - \frac{\partial \vec{a}^{T(M)}}{\partial \theta_R} \text{diag}(\vec{G}) \mathbf{Q}_{CA}^{(III)} \vec{a}^{(M)} - \vec{a}^{T(M)} \text{diag}(\vec{G}) \mathbf{Q}_{CA}^{(III)} \frac{\partial \vec{a}^{(M)}}{\partial \theta_R} \right). \quad (4.16) \end{aligned}$$

4.3 Analysis

In literature, the Subdomain Model for conventional PMSMs is already intensely analyzed, showing a high agreement with Finite Element Analyses and experiments [57, 63–65]. Since the Subdomain Model for PMSMs with stator flux barriers has rarely been investigated yet, this is carried out in the following.

Therefore, the magnetic field, inductance, back-EMF and the cogging torque as well as on-load torque is determined by the Subdomain Model and compared with Finite Element Analyses. For this purpose a representative five-phase PMSM with single layer concentrated windings, stator flux barriers and surface magnets is investigated. Its most relevant parameters are listed in Tab. 4.1.

In Fig. 4.4 the on-load magnetic flux distribution at a constant radius inside the airgap is depicted (current amplitude $\hat{i} = 80\text{A}$). In the curve obtained from the Subdomain Model, some higher-frequent harmonics are visible. These harmonics arise from the finite number of Fourier coefficients used in the calculation. Apart from this, a high level of agreement between the Subdomain Model and the Finite Element Analysis can be observed for both the radial and tangential components of the magnetic field. A similar level of agreement can be recognized in Fig. 4.5, which shows the back-EMF for one electrical period at a rotational

speed of $n_R = 1500 \frac{1}{\text{min}}$.

Parameter	Value	Unit
Number of phases (m)	5	–
Number of pole pairs (p)	6	–
Number of coils (n_{coil})	5	–
Number of flux barriers (n_{FB})	5	–
Winding configuration ⁶	B	–
Coil inductance (L)	0.520	mH
Amplitude PM flux linkage ($\hat{\Phi}_M$)	82	mWb

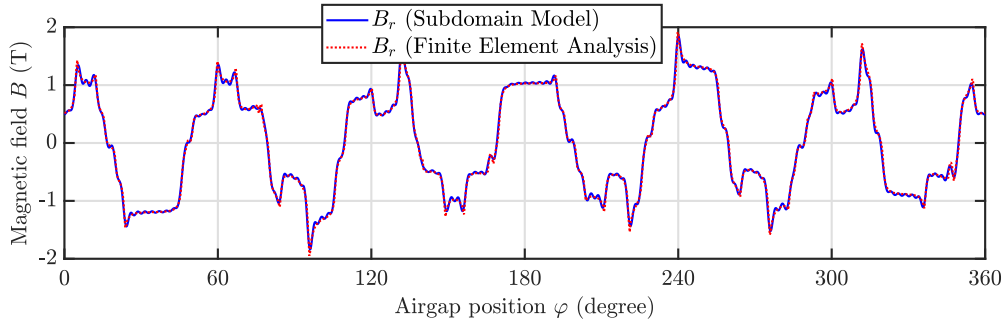
Table 4.1: Important parameters of the studied machine.

The cogging torque in dependence of the rotor position is visualized in Fig. 4.6. It stands out that it exhibits a relatively high frequency compared to the back-EMF. While the frequency of the back-EMF's fundamental wave over one mechanical period is equal to the number of pole pairs p , the frequency of the cogging torque corresponds to the least common multiple of the number of poles ($2p$) and the number of slots n_{SL} . For the investigated machine the least common multiple is 60 ($2p = 12$ and $n_{SL} = 10$) for one mechanical period, which results in a fundamental frequency of 10 over one electrical period. Overall, the Subdomain Model shows a high agreement with the Finite Element Analysis, confirming the model's accuracy in predicting this characteristic torque component.

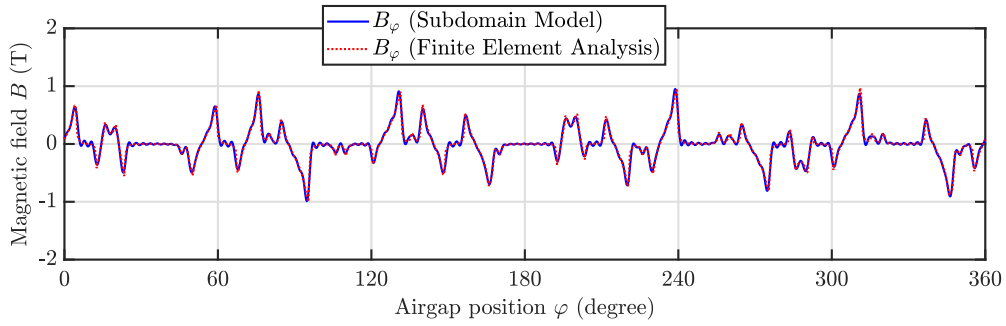
Fig. 4.7 shows the torque under load conditions, assuming sinusoidal currents with an amplitude of $\hat{i} = 80\text{A}$. Compared to the cogging torque, the agreement with the Finite Element Analysis is slightly lower, which may be attributed to differences in the calculation methods of the analytical Subdomain Model and the numerical Finite Element Analysis. This slight deviation is also noticeable in the self inductance values: according to the Subdomain Model each coil's self inductance of the investigated machine has a value of 0.520mH, whereas the Finite Element Analysis yields a value of 0.535mH. Nevertheless, a good agreement for both the on-load torque and the self-inductance between the two methods is still achieved.

Summarizing, it can be concluded that the Subdomain Model for a PMSM with stator flux barriers shows a high agreement with Finite Element Analysis for all relevant quantities. Thereby the Subdomain Model is running several orders of magnitudes faster than the Finite Element Analysis, making it suitable for control applications. However, it should be noted that in these simulations the magnetic saturation was not considered in both the Subdomain Model and the Finite Element Analysis. Depending on the current, the remanence of the magnets and the BH-curve of the ferromagnetic material, the accuracy of the Subdomain Model compared to the real PMSM may be reduced. From a control perspective, this deviation is treated as an effective disturbance that has to be compensated.

⁶Configuration of the concentrated windings in combination with stator flux barriers, see section 2.2.



(a) Radial magnetic field B_r at a constant radius within the airgap.



(b) Tangential magnetic field B_φ at a constant radius within the airgap.

Figure 4.4: Comparison of the on-load magnetic field determined by the **Subdomain Model** and by **Finite Element Analysis**.

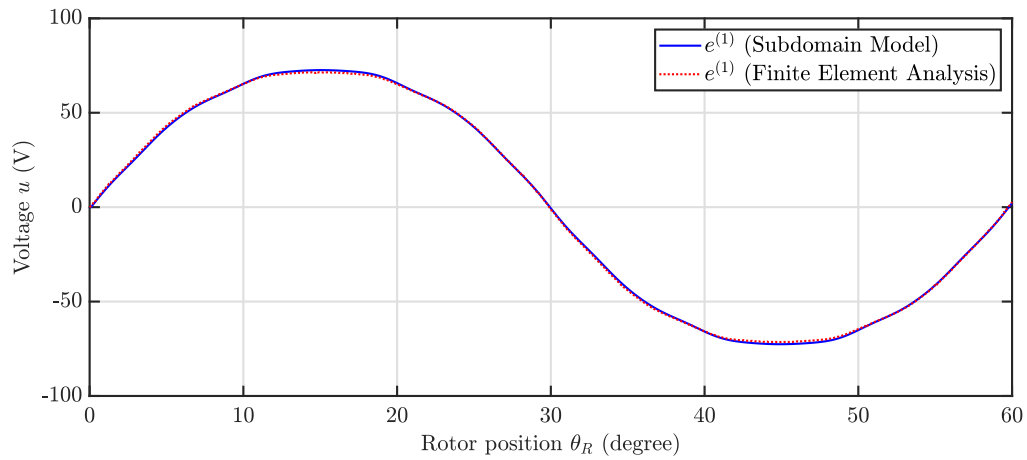


Figure 4.5: Calculated back-EMF $e^{(1)}$ of phase (1) over one electrical period determined by the **Subdomain Model** and by **Finite Element Analysis**.

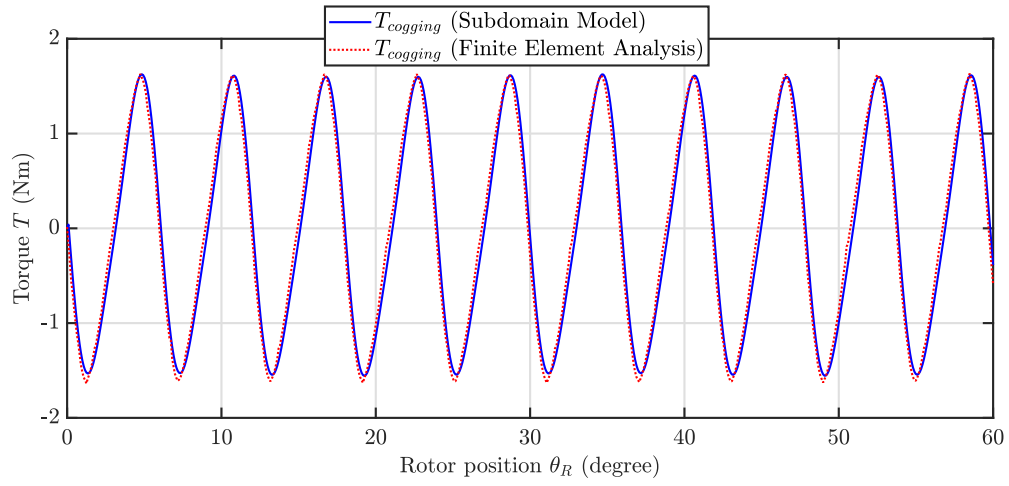


Figure 4.6: Calculated cogging torque $T_{cogging}$ over one electrical period determined by the **Subdomain Model** and by **Finite Element Analysis**.

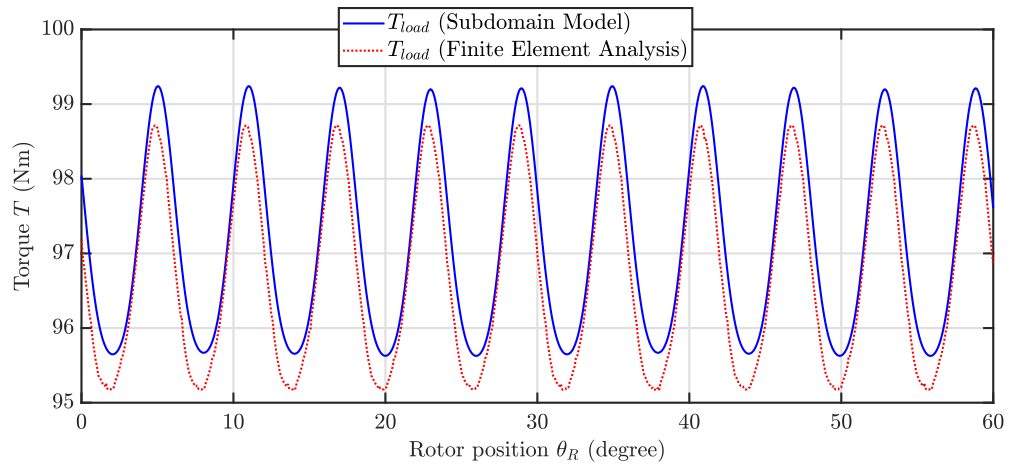


Figure 4.7: Calculated on-load torque T_{load} over one electrical period determined by the **Subdomain Model** and by **Finite Element Analysis**.

4.4 Usage as plant model

In the subsequent simulations for analyzing the control developed in this work, the Subdomain Model is used as plant model due to its high agreement with both Finite Element Analyses (see section 4.3) and experiments. However, since the Subdomain Model solely describes the relation between the currents flowing through the coils and the thereby produced torque, the dynamics of the currents (relation between input voltage and actual currents) and of the mechanics (relation between torque and rotor position as well as its derivatives) have to be considered for the full plant dynamics.

In addition, a model of an ideal inverter is used. It is assumed that the DC voltage source is constant and load-independent, and that the MOSFETs have an ideal (instantaneous) switching behavior. Considering the inverter, the continuous voltages $\vec{u}_{in}$ demanded by the control system are converted into discretized voltages $\vec{u}_{in}^*$, which are the actual inputs to the PMSM.

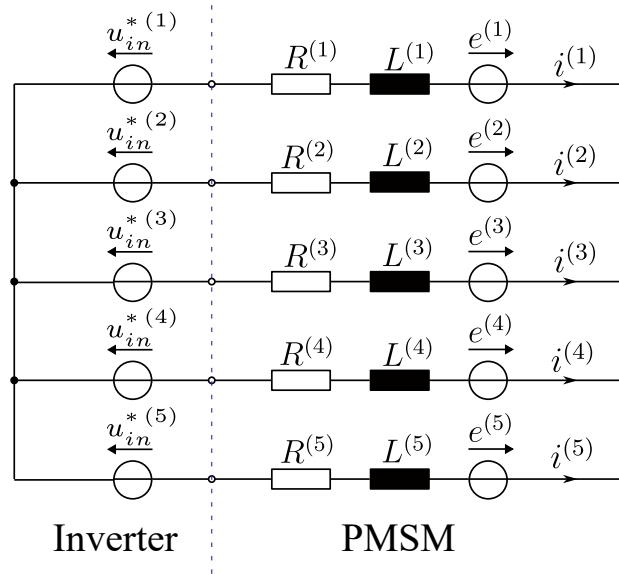


Figure 4.8: Electrical circuit considered of the drive with inverter and PMSM [66].

As discussed in section 2.2, the mutual inductances between the coils are very low and therefor negligible due to the magnetic decoupling of the concentrated windings. Considering the equivalent circuit shown in Fig. 4.8, the current dynamics can be described by the following system of differential equations:

$$\begin{pmatrix} u_{in}^{*(1)} - u_{in}^{*(2)} \\ u_{in}^{*(2)} - u_{in}^{*(3)} \\ u_{in}^{*(3)} - u_{in}^{*(4)} \\ u_{in}^{*(4)} - u_{in}^{*(5)} \\ 0 \end{pmatrix} = \begin{pmatrix} R^{(1)} & -R^{(2)} & 0 & 0 & 0 \\ 0 & R^{(2)} & -R^{(3)} & 0 & 0 \\ 0 & 0 & R^{(3)} & -R^{(4)} & 0 \\ 0 & 0 & 0 & R^{(4)} & -R^{(5)} \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \dot{i}_{act}^{(1)} \\ \dot{i}_{act}^{(2)} \\ \dot{i}_{act}^{(3)} \\ \dot{i}_{act}^{(4)} \\ \dot{i}_{act}^{(5)} \end{pmatrix} \\
 + \begin{pmatrix} L^{(1)} & -L^{(2)} & 0 & 0 & 0 \\ 0 & L^{(2)} & -L^{(3)} & 0 & 0 \\ 0 & 0 & L^{(3)} & -L^{(4)} & 0 \\ 0 & 0 & 0 & L^{(4)} & -L^{(5)} \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{di_{act}^{(1)}}{dt} \\ \frac{di_{act}^{(2)}}{dt} \\ \frac{di_{act}^{(3)}}{dt} \\ \frac{di_{act}^{(4)}}{dt} \\ \frac{di_{act}^{(5)}}{dt} \end{pmatrix} + \begin{pmatrix} e^{(1)} - e^{(2)} \\ e^{(2)} - e^{(3)} \\ e^{(3)} - e^{(4)} \\ e^{(4)} - e^{(5)} \\ 0 \end{pmatrix}. \quad (4.17)$$

Both the self-inductances and the back-EMFs can be determined precisely for each coil by (4.10) and (4.7). Solving (4.17), the actual currents $\vec{i}_{act}$ flowing through all coils are obtained, whereby the torque T_{el} produced by the PMSM can then be calculated by (4.14). This leads to the mechanical dynamics, which are defined as follows

$$\frac{d\theta_R}{dt} = \omega_R \quad (4.18)$$

$$\mathcal{J}_m \frac{d\omega_R}{dt} = T_{el} - T_m \quad (4.19)$$

with $\mathcal{J}_m$ the rotor's moment of inertia and T_m the load torque acting on the motor.

5 Optimal control for PMSMs

In this chapter, the complete control system based on the Subdomain Model is derived and described in detail. Thereby section 5.2 presents the feedforward control, which calculates optimal currents and voltages based on a desired mechanical trajectory. Additionally, a field weakening strategy is integrated into the feedforward control, considering the electrical limitations of the PMSM. In section 5.3 the feedback control is derived to compensate deviations from the desired trajectory caused by prediction errors of the feedforward control, measurement delays, or other unpredictable disturbances. The chapter concludes with section 5.4, where an overview of the control system and an initial evaluation is provided.

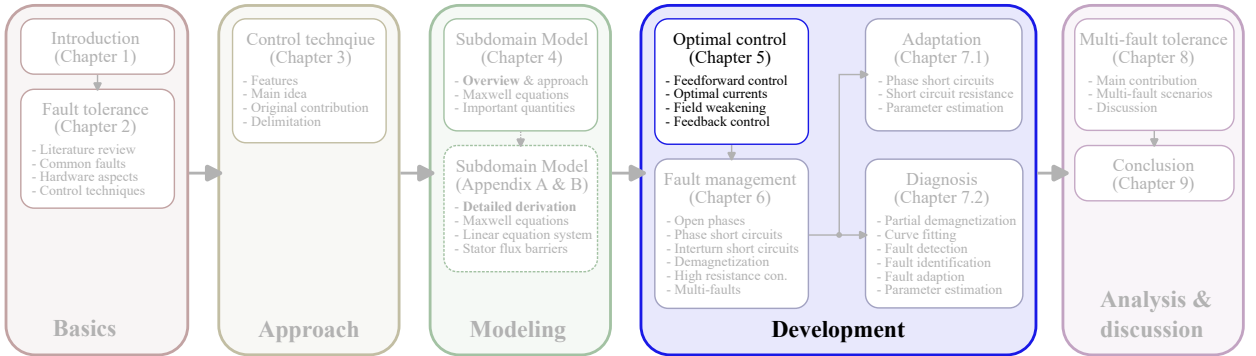


Figure 5.1: Structural overview of the work, with this chapter 5 highlighted.

It is important to note that this chapter focuses exclusively on prefault conditions. The necessary modifications and extensions of the control system to compensate motor-intrinsic electrical and magnetic faults as well as multi-fault conditions are intensely discussed in chapter 6.

5.1 Desired trajectory

With an arbitrary mechanical trajectory the desired variables $\vec{x}_{m,des}(t) = \{\theta_{R,des}(t), \omega_{R,des}(t)\}$ are defined at the actual time step t . Taking into account the load torque T_m , the desired torque required to track the mechanical trajectory can be determined by

$$T_{des} = T_m + J_m \frac{d\omega_{R,des}}{dt}, \quad (5.1)$$

This desired torque and the desired mechanical variables serve as inputs to the feedforward control. In practice, such trajectories would be generated by a higher-level control system, depending on the specific task of the drive. Since the design and implementation of such higher-level controllers is beyond the scope of this work, exemplary reference trajectories are utilized in the evaluations without direct reference to a specific application.

5.2 Optimal feedforward control

(The following section is based on [60].)

Using the analytical equations for torque (4.14), PM flux linkage (4.6), back-EMF (4.7) and inductance (4.10) derived in section 4.2, the knowledge of the Subdomain Model is extracted in a very efficient way. These equations are used for an analytical optimization of the desired currents.

5.2.1 Optimal currents

To reduce (copper) losses and avoid an overheating of the coils, the aim is to produce a desired torque with the least root mean square currents. In the sense of a maximum torque per ampere control strategy, an optimization problem is formulated whose cost function is composed of the square currents¹ [37, 38]:

$$\begin{aligned} \text{minimize } K(\vec{J}) &= \frac{1}{2} \vec{J}^T \vec{J} \\ \text{subject to } h_1(\vec{J}) &= \vec{1}^T \vec{J} = 0 \\ h_2(\vec{J}) &= \frac{\ell\pi}{\mu_0} \vec{P}^T(\theta_{R,des}) \vec{J} + \frac{\ell\pi}{\mu_0} Q(\theta_{R,des}) - T_{des} = 0 \end{aligned} \quad (5.2)$$

Equality constraint h_1 enforces an equilibrium of all currents due to the assumed star connection. Equality constraint h_2 demands for the currents to produce the desired torque T_{des} and to compensate the cogging torque $\frac{\ell\pi}{\mu_0} Q$. The solution of this constrained minimization problem must fulfill the condition [67]

$$\nabla_{\vec{J}} K + \lambda_1 \nabla_{\vec{J}} h_1 + \lambda_2 \nabla_{\vec{J}} h_2 = \vec{0}, \quad (5.3)$$

¹The index z for denoting the axial flowing direction of the current densities is omitted from here on to simplify the notation.

with Lagrangian multipliers λ_1 and λ_2 . This condition, together with the equality constraints can be efficiently written as a linear equation system in matrix notation:

$$\underbrace{\begin{pmatrix} \mathbf{I} & \vec{1} & \frac{\ell\pi}{\mu_0}\vec{P}|_{des} \\ \vec{1}^T & 0 & 0 \\ \frac{\ell\pi}{\mu_0}\vec{P}^T|_{des} & 0 & 0 \end{pmatrix}}_{\mathbf{M}} \underbrace{\begin{pmatrix} \vec{J} \\ \lambda_1 \\ \lambda_2 \end{pmatrix}}_{\vec{x}} = \underbrace{\begin{pmatrix} \vec{0} \\ 0 \\ T_{des} - \frac{\ell\pi}{\mu_0}Q|_{des} \end{pmatrix}}_{\vec{b}^*}, \quad (5.4)$$

where $\mathbf{I}$ is the identity matrix. By solving equation system (5.4), solution vector $\vec{x}$ can be determined, which then contains the optimal current densities $\vec{J}_{des}$.

5.2.2 Optimal field weakening

To consider field weakening, the minimization problem needs to be adjusted. This is done by adding a second term to the cost function, whose purpose is to adjust the magnetic field caused by the currents flowing through each of the coils to their PM flux linkages $\vec{\Phi}_M$:

$$\begin{aligned} \text{minimize } K(\vec{J}) &= \frac{1-\alpha}{2} \vec{J}^T \vec{J} + \frac{\alpha}{2} \left(\vec{J} + \frac{\mathcal{L}^{-1}\vec{\Phi}_M(\theta_{R,des})}{S_{SL}} \right)^T \left(\vec{J} + \frac{\mathcal{L}^{-1}\vec{\Phi}_M(\theta_{R,des})}{S_{SL}} \right) \\ \text{subject to } h_1(\vec{J}) &= \vec{1}^T \vec{J} = 0 \\ h_2(\vec{J}) &= \frac{\ell\pi}{\mu_0} \vec{P}^T(\theta_{R,des}) \vec{J} + \frac{\ell\pi}{\mu_0} Q(\theta_{R,des}) - T_{des} = 0 \end{aligned} \quad (5.5)$$

with α the field weakening coefficient having values from 0 to 1, depending on the need of field weakening at actual operation, and $\vec{\Phi}_M$ calculated by (4.6). The solution of this adjusted optimization problem only requires in equation system (5.4) a modification of vector $\vec{b}^*$:

$$\underbrace{\begin{pmatrix} \mathbf{I} & \vec{1} & \frac{\ell\pi}{\mu_0}\vec{P}|_{des} \\ \vec{1}^T & 0 & 0 \\ \frac{\ell\pi}{\mu_0}\vec{P}^T|_{des} & 0 & 0 \end{pmatrix}}_{\mathbf{M}} \underbrace{\begin{pmatrix} \vec{J} \\ \lambda_1 \\ \lambda_2 \end{pmatrix}}_{\vec{x}} = \underbrace{\begin{pmatrix} -\alpha \frac{\mathcal{L}^{-1}\vec{\Phi}_M}{S_{SL}}|_{des} \\ 0 \\ T_{des} - \frac{\ell\pi}{\mu_0}Q|_{des} \end{pmatrix}}_{\vec{b}}, \quad (5.6)$$

To evaluate whether or not field weakening is necessary at desired operation (meaning desired torque and rotational speed), the desired voltages need to be determined

$$\vec{u}_{des}(\alpha) = \mathcal{R}\vec{i}_{des}(\alpha) + \mathcal{L} \frac{d\vec{i}_{des}(\alpha)}{dt} + \vec{e}(\theta_{R,des}). \quad (5.7)$$

While $\vec{e}$ can be calculated by (4.7) and $\vec{i}_{des} = \vec{J}_{des} S_{SL}$ is based on the solution of the modified equation system (5.6), the time derivatives of the currents are not known yet. Since the only relevant time dependency of $\mathbf{M}$ and $\vec{b}$ comes from the rotor angle, the derivative of the currents based on the optimization can be easily calculated by differentiating the whole equation system, $\omega_R \frac{\partial}{\partial \theta_R} \mathbf{M} \vec{x} = \omega_R \frac{\partial}{\partial \theta_R} \vec{b}$. Combining the resulting equation system with the previous one (5.6), results in

$$\begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \dot{\mathbf{M}} & \mathbf{M} \end{pmatrix} \begin{pmatrix} \vec{x} \\ \dot{\vec{x}} \end{pmatrix} = \begin{pmatrix} \vec{b} \\ \dot{\vec{b}} \end{pmatrix}, \quad (5.8)$$

where

$$\dot{\mathbf{M}} = \begin{pmatrix} \mathbf{0} & \vec{0} & \frac{\ell\pi}{\mu_0} \vec{P} \Big|_{des} \\ \vec{0}^T & 0 & 0 \\ \frac{\ell\pi}{\mu_0} \vec{P}^T \Big|_{des} & 0 & 0 \end{pmatrix}, \quad \vec{x} = \begin{pmatrix} \vec{J} \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -\alpha \frac{\mathcal{L}^{-1} \vec{e}}{S_{SL}} \Big|_{des} \\ 0 \\ -\frac{\ell\pi}{\mu_0} \dot{Q} \Big|_{des} \end{pmatrix}. \quad (5.9)$$

The derivatives $\vec{P} = \omega_R \frac{\partial \vec{P}}{\partial \theta_R}$ and $\dot{Q} = \omega_R \frac{\partial Q}{\partial \theta_R}$ are determined by (4.15) and (4.16). Since in the feedforward control only steady states are considered, $\dot{T}_{des}$ and $\dot{\alpha}$ are neglected. Solving equation system (5.8) yields $\vec{J}_{des}$ and $\dot{\vec{J}}_{des}$, the first and forth component of $\begin{pmatrix} \vec{x} \\ \dot{\vec{x}} \end{pmatrix}$. Inserting these variables into (5.7) results in the desired voltages $\vec{u}_{des}$.

Once the desired voltages of each phase are determined, they can be adjusted to the complex plane

$$\underline{u}_{des}(\alpha) = \hat{u}_{des}(\alpha) e^{j\vartheta_{des}(\alpha)} = \sum_{c=1}^{n_{coil}} u_{des}^{(c)}(\alpha) e^{j\varphi_{phase}^{(c)}}, \quad (5.10)$$

where $\varphi_{phase}^{(c)}$ is the phase angle of each coil or phase, forming the desired voltage $\underline{u}_{des}$ with amplitude $\hat{u}_{des}$ and angle ϑ_{des} , both depending on α . This complex voltage can be compared to the limits of the power electronics as visualized in Fig. 5.2. In this example, the desired voltage **without field weakening** exceeds the limits of the power electronics – field weakening is necessary.

To calculate for a fixed operating point the optimal value of α , an iterative algorithm is used. This calculation is made offline, the result being stored in a lookup-table (LUT), see next subsection. The algorithm uses iteration index (i) and starts with ($i = 0$). Choosing an arbitrary rotor angle θ_R and setting initially $\alpha^{(i=0)} = 0$ the desired complex voltage vector is compared with the maximal available voltage from the power electronics. If the desired voltage $\underline{u}_{des}$ is smaller than the maximal voltage $\underline{u}_{pe,max}$ of the power electronics, α is equal

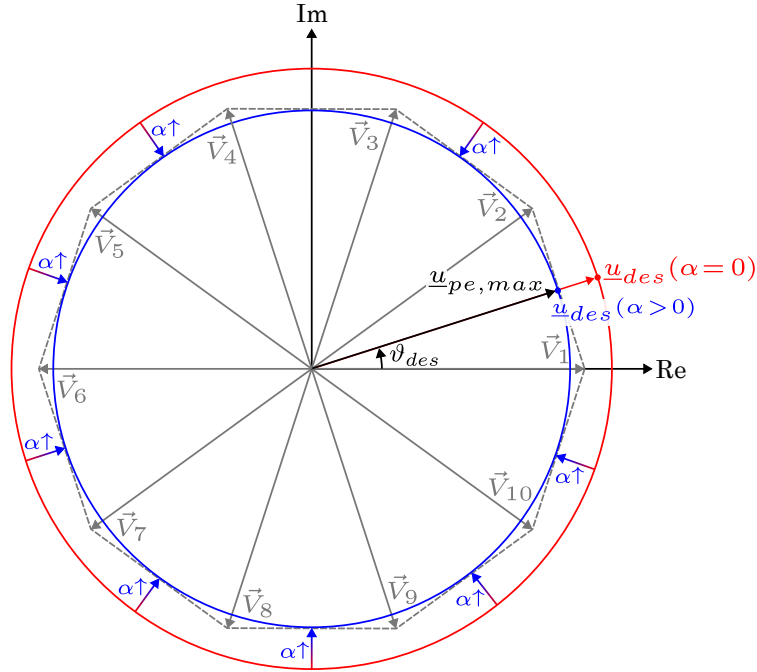


Figure 5.2: Space vector diagram for a conventional five-phase inverter (only the relevant, **outermost space vectors** are shown); exemplary trajectory of the desired voltage for one electrical period **without field weakening** and with **optimal field weakening** [60].

to zero for this operating point. Otherwise $i = i + 1$ and α is updated according to

$$\alpha^{(i)} = \alpha^{(i-1)} + \frac{\hat{u}_{des}(\alpha^{(i-1)}) - \hat{u}_{pe,max}}{\hat{u}_{pe,max}}. \quad (5.11)$$

Using this non zero $\alpha^{(i)}$ the desired complex voltage $\underline{u}_{des}(\alpha^{(i)})$ is recalculated by (5.8), (5.7), (5.10) and then compared to the maximal voltage of the power electronics. If it is within a specified tolerance band, the iterative procedure is finished, otherwise a further iteration $i = i + 1$ starts. If the solution for α is greater than 1, the desired operating point cannot be reached.

Due to the presence of faults the voltages can be non-sinusoidal and distributed unequally among the coils, such that the θ_R dependence in α has to be considered. For this reason the determination of α needs to be repeated over a whole electrical period, storing it as function of a sufficient number of discrete rotor angles within one electrical period. Choosing the maximum of the calculated values of α , **field weakening is optimally adjusted** to the defined operating point, as sketched in Fig. 5.2.

Overall field weakening strategy

In almost every application, voltage and current are limited. Hence the aim is to develop an overall field weakening strategy taking into account these electrical limitations. This is achieved by storing the values of α in dependency of the torque and rotational speed in a LUT to enable a feedforward control without need of any sensor input. Fig. 5.3 visualizes the qualitative structure of such a field weakening LUT.

In a first step, the maximal available torque T_{max} based on the specified maximal root mean square of the currents $I_{rms,max}$ needs to be determined. This can be done by calculating the desired currents for an arbitrarily chosen reference torque T_{ref} over a full electrical period at several discrete rotor angles using equation system (5.8) and then computing the RMS of each current $I_{rms,ref}$ at T_{ref} numerically. Since torque is linearly depending on the currents, T_{max} can be computed by

$$T_{max} = T_{ref} \frac{I_{rms,max}}{I_{rms,ref}}. \quad (5.12)$$

In the case of a fault, the currents can be distributed asymmetrically along the phases. For this reason, $I_{rms,ref}$ is determined as the largest RMS of all phases.

In the next step, for T_{max} the maximal rotational speed without field weakening is determined. This can be done in a similar way, calculating the desired complex voltage vectors at a reference rotational speed $\omega_{R,ref}$ for discrete rotor angles of an electrical period. Using the desired voltage $\underline{u}_{des,max}$ that exceeds the voltage limit most, and comparing it with the maximal available voltage of the power electronics $\underline{u}_{pe,max}$, the maximal rotational speed without need of field weakening based on the voltage limit can be determined by

$$\tilde{\omega}_{R,max} = \omega_{R,ref} \frac{|\underline{u}_{pe,max}|}{|\underline{u}_{des,max}|}. \quad (5.13)$$

Any operating point less than or equal to T_{max} and $\tilde{\omega}_{R,max}$ for torque and rotational speed is within the base speed range, where no field weakening is needed ($\alpha = 0$).

Now, the field weakening area has to be calculated ($\alpha \geq 0$). This is achieved by successively scanning the operating area at discrete values lower than T_{max} and greater than $\tilde{\omega}_{R,max}$ for torque and rotational speed. At each of these discrete operating points the field weakening coefficient α is determined by the algorithm explained in the section before.

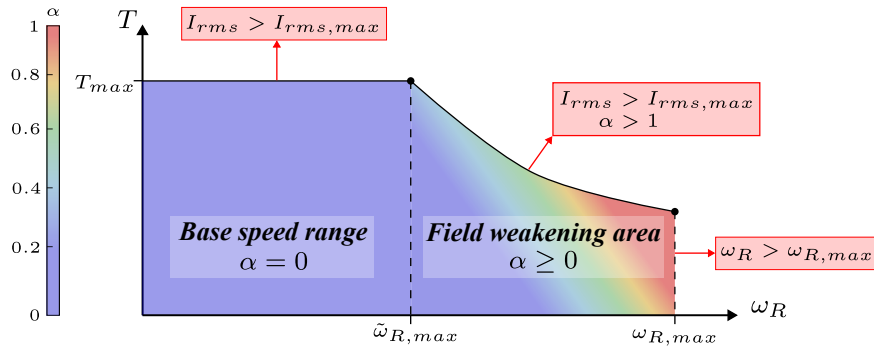


Figure 5.3: Qualitative structure of a field weakening LUT. **Blue** is the base speed range, where no field weakening is required ($\alpha = 0$). The field weakening area (**blue** to **red**) is the area that needs to be scanned, calculating the values of $\alpha \geq 0$ [60].

No solution is found, if one or more of the following cases are occurring:

1. Calculated field weakening coefficient exceeds a value of 1
2. The RMS of one or more phase currents exceeds the maximal allowed value $I_{rms,max}$
3. Rotational speed exceeds the maximal allowed value $\omega_{R,max}$ due to mechanical aspects

The calculated values of α in dependency of torque and rotational speed are stored in a LUT, which then contains the information about field weakening. This whole procedure can also be used under consideration of faults. Therefor, merely the feedforward control adjusted to the specified fault needs to be used for predicting the desired currents and voltages (see chapter 6).

For the online feedforward control, the desired currents based on the desired torque and rotational speed now can be calculated efficiently by interpolating within the stored LUT to extract the corresponding value of α , solving equation system (5.8) and determining the desired voltages by (5.7), requiring only low computational effort compared to the full Subdomain Model (for further explanations see also section 5.4).

5.2.3 Evaluation

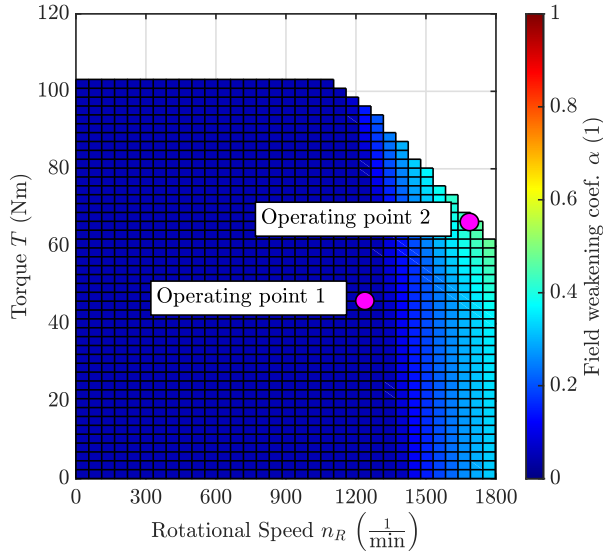
To evaluate the developed feedforward control it is used to predict the optimal desired currents and voltages at different operating points for the investigated five-phase PMSM, with its most relevant parameters listed in Tab. 4.1. In the following simulations, the desired voltage is transformed to the complex plane. Thereby a circle in the complex plane, which is equal to one electrical period, corresponds to voltages which are purely sinusoidal and evenly distributed along the phases. The greater the ripple, the higher the amplitudes of higher frequent harmonics. The more elliptical the voltages are, the higher is their asymmetrical

distribution (this is relevant for the fault tolerant operation discussed in chapter 6). To visualize compliance with the voltage limit of the power electronics, the resulting decagon of the space vectors is drawn. The desired voltage must not exceed this decagon.

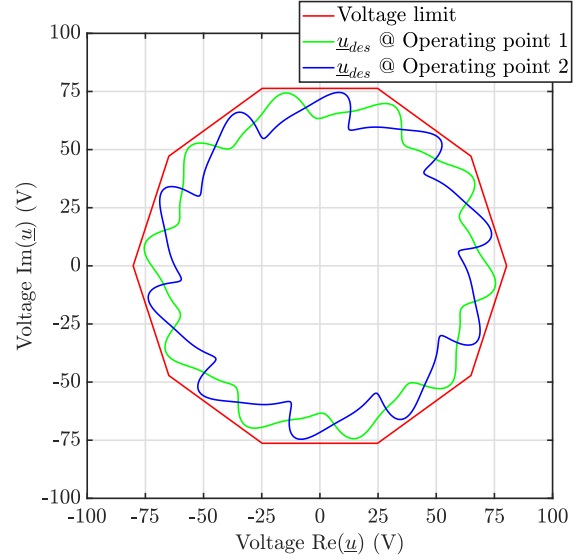
In Fig. 5.4a the resulting field weakening LUT is shown. It can be seen that the full field weakening potential is not utilized (α does not exceed values of 0.5 for any operating point). This is a result of the considered current limit. The desired voltages in the complex plane are shown for two exemplary operating points in Fig. 5.4b. It can be seen that the voltage amplitudes of operating point 1 and 2 are nearly the same. While operating point 1 is in an area where no field weakening is required, the voltage amplitudes at operating point 2 is reduced due to field weakening, since the voltages would otherwise exceed the voltage limit. For this reason, the current amplitudes relative to the produced torque of operating point 1 (Fig. 5.4c) are increased compared to those of operating point 2 (Fig. 5.4d)). Both the desired currents and voltages contain some harmonics, which are caused by the compensation of the relatively high frequent cogging torque (see Fig. 5.4e). Fig. 5.4f shows the torque capabilities of all coils. For operating point 1 (no field weakening) the desired currents of each coil, shown in Fig. 5.4c, tend to follow their torque capabilities to achieve maximum torque per ampere. In contrast, for operating point 2 requiring field weakening, the desired currents are phase shifted to their torque capabilities, whereby the relative voltage demand is decreased and the relative current demand is increased.

To recognize the influence of the flux barriers on the operating area as well as the desired currents and voltages Fig. 5.4 (PMSM *with* flux barriers) can be compared with Fig. 5.5 (same PMSM, but *without* flux barriers). Thereby the main difference can be seen in the field weakening LUTs: while the base speed range is increased for the PMSM *with* flux barriers compared to the one *without* flux barriers, the field weakening-area and -demand is decreased. This is caused by the flux concentration effect of the flux barriers, increasing effectively the torque capability (see Fig. 5.4f and Fig. 5.5f) but also in the same way the PM flux linkage and back-EMF. It is also evident in the desired currents: for operating point 1, where no field weakening is required, the desired currents for the PMSM *with* flux barriers are lower (see Fig. 5.4c and Fig. 5.5c), and for operating point 2, where field weakening is required, the desired currents are lower for the PMSM *without* flux barriers (see Fig. 5.4d and Fig. 5.5d).

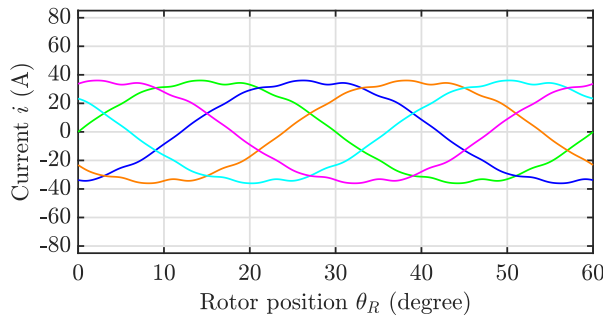
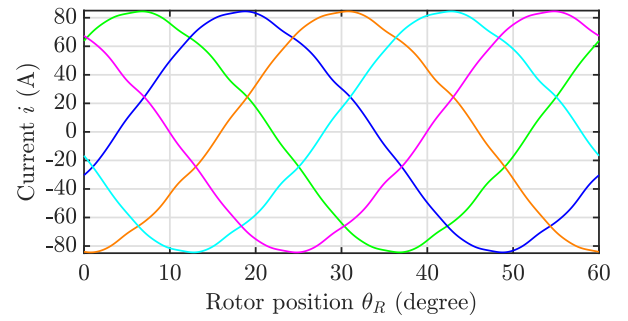
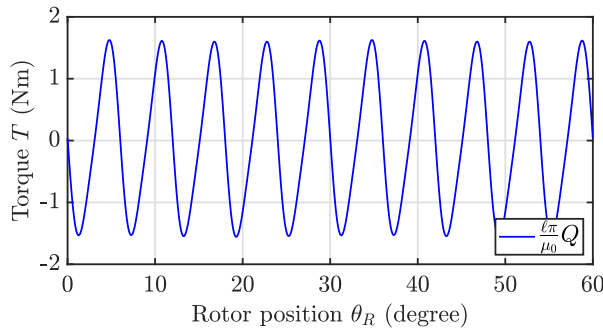
Because of the advantageous properties of the flux barriers for the fault tolerance (as discussed in section 2.2), only the PMSM *with* flux barriers is examined in all of following simulations from here on. However, the overall control system can be used in the same way for a conventional PMSM without flux barriers.



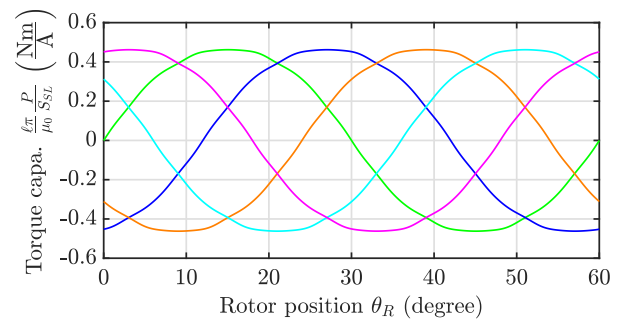
(a) Field weakening LUT.

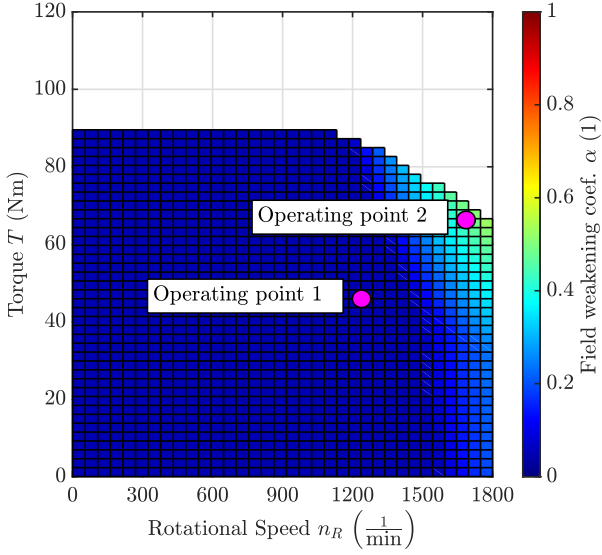


(b) Desired voltages in the complex plane.

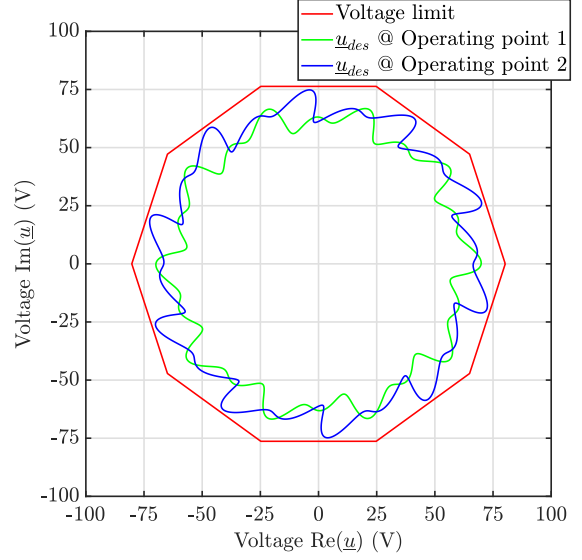
(c) Desired currents $\vec{i}_{des}$ at operating point 1.(d) Desired currents $\vec{i}_{des}$ at operating point 2.

(e) Predicted cogging torque.

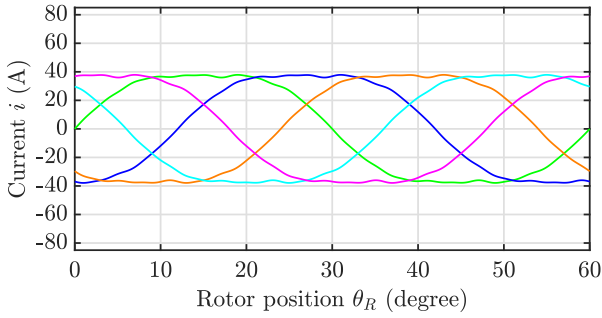
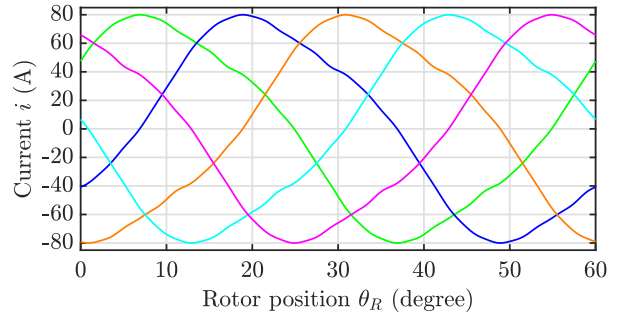
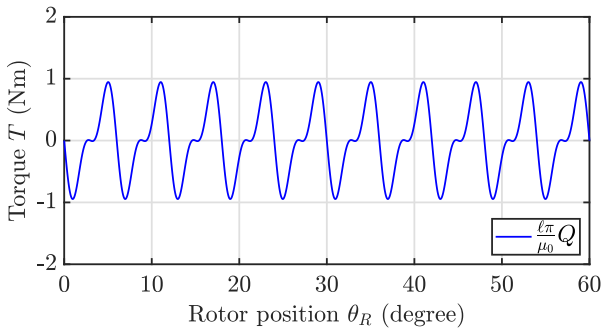
(f) Predicted torque capabilities $\frac{\ell_\pi}{\mu_0} \vec{P}$.Figure 5.4: Evaluation of the feedforward control for the PMSM *with* flux barriers.



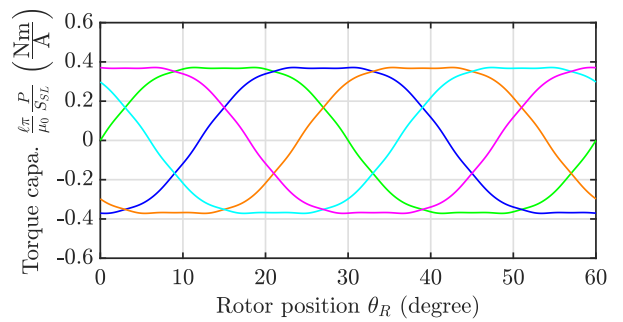
(a) Field weakening LUT.



(b) Desired voltages in the complex plane.

(c) Desired currents $\vec{i}_{des}$ at operating point 1.(d) Desired currents $\vec{i}_{des}$ at operating point 2.

(e) Predicted cogging torque.

(f) Predicted torque capabilities $\frac{\ell_\pi}{\mu_0} \vec{P}$.Figure 5.5: Evaluation of the feedforward control for the PMSM *without* flux barriers.

5.3 Optimal feedback control

(The following section is based on [66].)

In this section, the control system is completed by a feedback control to compensate deviations from the desired trajectory, caused by any kind of disturbances. Equally to the feedforward control, the analytical equations extracted from the Subdomain Model for calculating the torque (4.14), back-EMF (4.7) and inductance (4.10) are used as basis. To this end an efficient feedback control is introduced, compensating the deviations by small, optimally distributed corrections to the desired voltage determined by the feedforward control.

5.3.1 Simplified motor dynamics

As a first step, a feedback in the spirit of a current command control is developed. In this sense it is assumed that the current densities $\vec{J}$ can be directly supplied from an external control unit – in other words the currents are considered as effective inputs for the motor dynamics [68]. Thereby the equation of motion for the mechanical part of the motor is given by

$$\mathcal{J}_m \frac{d^2 \theta_R}{dt^2} = \frac{\pi \ell}{\mu_0} \left(Q(\theta_R) + \vec{P}^T(\theta_R) \vec{J} \right) - T_m. \quad (5.14)$$

Linearizing this equation at the desired rotor angle $\theta_{R,des}$ results in the following dynamics for a small deviation $\Delta \theta_R = \theta_{R,des} - \theta_{R,mes}$ between the desired and the measured rotor angle:

$$\frac{d^2 \Delta \theta_R}{dt^2} = \frac{\pi \ell}{\mu_0 \mathcal{J}_m} \left[\left(\frac{\partial Q}{\partial \theta_R} \Big|_{des} + \frac{\partial \vec{P}^T}{\partial \theta_R} \Big|_{des} \vec{J}_{des} \right) \Delta \theta_R + \vec{P}^T \Big|_{des} \Delta \vec{J} \right], \quad (5.15)$$

neglecting quadratic terms in the deviations. Note that the currents $\vec{J}_{des}$ determined by the feedforward control have to be corrected by $\Delta \vec{J}$ for obtaining the consistent deviation dynamics. By considering $\Delta \vec{J}$ as effective input variable, state feedback can be applied to impose an asymptotically stable behavior on the deviation dynamics

$$\frac{d^2 \Delta \theta_R}{dt^2} = -c_0 \Delta \theta_R - c_1 \frac{d \Delta \theta_R}{dt} \quad (5.16)$$

with constant coefficients $c_0, c_1 > 0$ according to the Routh-Hurwitz stability criterion. Equating the right-hand sides of (5.15) and (5.16) yields the following condition for the

required current corrections $\Delta \vec{J}$:

$$\vec{P}^T \Big|_{des} \Delta \vec{J} = \Delta \Xi. \quad (5.17)$$

$\Delta \Xi$ is the necessary scalar correction

$$\Delta \Xi = \Delta \Xi_{\Delta \theta_R} + \Delta \Xi_{\Delta \omega_R}, \quad (5.18)$$

consisting of the contribution

$$\Delta \Xi_{\Delta \theta_R} = - \left(\frac{\mu_0 \mathcal{J}_m}{\pi \ell} c_0 + \frac{\partial Q}{\partial \theta_R} \Big|_{des} + \frac{\partial \vec{P}^T}{\partial \theta_R} \Big|_{des} \vec{J}_{des} \right) \Delta \theta_R \quad (5.19)$$

and

$$\Delta \Xi_{\Delta \omega_R} = - \frac{\mu_0 \mathcal{J}_m}{\pi \ell} c_1 \Delta \omega_R \quad (5.20)$$

due to the deviation in rotor angle $\Delta \theta_R$ and in rotational speed $\frac{d\Delta \theta_R}{dt} = \Delta \omega_R$. The optimal distribution of this single scalar correction $\Delta \Xi$ among the coil currents can be found by solving the following minimization problem:

$$\begin{aligned} & \text{minimize } K(\Delta \vec{J}) = \frac{1}{2} \Delta \vec{J}^T \Delta \vec{J} \\ & \text{subject to } h(\Delta \vec{J}) = \vec{P}^T \Big|_{des} \Delta \vec{J} - \Delta \Xi = 0 \end{aligned} \quad (5.21)$$

Its solution reads

$$\Delta \vec{J} = \frac{\Delta \Xi}{\vec{P}^T \Big|_{des} \vec{P} \Big|_{des}} \vec{P} \Big|_{des}. \quad (5.22)$$

Summarizing, a deviation $\Delta \theta_R$ or $\Delta \omega_R$ from the desired trajectory can be compensated by correcting the desired currents generated from the feedforward control:

$$\vec{J}_{in} = \vec{J}_{des} - \Delta \vec{J}. \quad (5.23)$$

5.3.2 Full motor dynamics

The motor dynamics is actually driven by the voltages at each coil, and thus the internal current dynamics have to be incorporated in the model of the full system dynamics:

$$\mathcal{J}_m \frac{d^2 \theta_R}{dt^2} = \frac{\pi \ell}{\mu_0} \left(Q(\theta_R) + \vec{P}^T(\theta_R) \vec{J} \right) - T_m, \quad (5.24)$$

$$\mathcal{L} \frac{d\vec{J}}{dt} = -\mathcal{R}\vec{J} - \frac{\vec{e}(\theta_R)}{S_{SL}} + \frac{\vec{u}}{S_{SL}}. \quad (5.25)$$

In principle, the currents calculated in (5.22), (5.23) could be used as target values for current control by appropriate input-voltages $\vec{u}_{in}$. Another way would be to form the input $\vec{u}_{in}$ by appropriate state feedback, with state variables $\{\theta_R, \omega_R, \vec{J}\}$. The approach for the feedback control in this work is different. It is based on a classification of the state variables: θ_R is supposed to track the desired trajectory $\theta_{R,des}(t)$ as close as possible (such that its deviation $\Delta\theta_R$ decays fast towards 0), whereas ω_R and $\vec{J}$ are only intermediate state variables, necessary to couple θ_R with the actual input variable $\vec{u}_{in}$. This coupling is achieved by forming the time derivative of (5.15) which, combined with (5.25), results in

$$\begin{aligned} \frac{d^3 \Delta\theta_R}{dt^3} = \frac{\pi \ell}{\mu_0 \mathcal{J}_m} & \left[\left(\frac{\partial^2 Q}{\partial \theta_R^2} \Big|_{des} + \frac{\partial^2 \vec{P}^T}{\partial \theta_R^2} \Big|_{des} \vec{J}_{des} \right) \omega_{R,des} \Delta\theta_R + \left(\frac{\partial Q}{\partial \theta_R} \Big|_{des} + \frac{\partial \vec{P}^T}{\partial \theta_R} \Big|_{des} \vec{J}_{des} \right) \Delta\omega_R \right. \\ & + \frac{\partial \vec{P}^T}{\partial \theta_R} \Big|_{des} \omega_{R,des} \Delta\vec{J} - \frac{\partial \vec{P}^T}{\partial \theta_R} \Big|_{des} \frac{\mathcal{L}^{-1}}{S_{SL}} (S_{SL} \mathcal{R} \vec{J}_{des} + \vec{e}_{des} - \vec{u}_{des}) \Delta\theta_R \\ & \left. - \vec{P}^T \Big|_{des} \frac{\mathcal{L}^{-1}}{S_{SL}} \left(S_{SL} \mathcal{R} \Delta\vec{J} + \frac{\partial \vec{e}}{\partial \theta_R} \Big|_{des} \Delta\theta_R + \frac{\vec{e}}{\omega_R} \Big|_{des} \Delta\omega_R - \Delta\vec{u} \right) \right], \end{aligned} \quad (5.26)$$

neglecting again quadratic deviation terms. $\Delta\vec{u}$ denotes the necessary deviation voltage from its feedforward value $\vec{u}_{des}$ and $\Delta\vec{J} = \vec{J}_{des} - \vec{J}_{mes}$. Again an asymptotically stable behavior is imposed on the mechanical deviation dynamics

$$\begin{aligned} \frac{d^3 \Delta\theta_R}{dt^3} &= -c_0 \Delta\theta_R - c_1 \frac{d\Delta\theta_R}{dt} - c_2 \frac{d^2 \Delta\theta_R}{dt^2} \\ &\stackrel{(5.15)}{=} - \left(c_0 + \frac{c_2 \pi \ell}{\mu_0 \mathcal{J}_m} \left(\frac{\partial Q}{\partial \theta_R} \Big|_{des} + \frac{\partial \vec{P}^T}{\partial \theta_R} \Big|_{des} \vec{J}_{des} \right) \right) \Delta\theta_R - c_1 \Delta\omega_R - \frac{c_2 \pi \ell}{\mu_0 \mathcal{J}_m} \vec{P}^T \Big|_{des} \Delta\vec{J}, \end{aligned} \quad (5.27)$$

with constant coefficients $c_0, c_1, c_2 > 0$ as well as $c_1 c_2 > c_0$ according to the Routh-Hurwitz stability criterion. Equating the right-hand sides of (5.26) and (5.27) results in the condition that has to be met by the voltage corrections $\Delta\vec{u}$ to the desired (feedforward) voltages $\vec{u}_{des}$.

This condition has the form of a single scalar equation:

$$\vec{P}^T|_{des} \frac{\mathcal{L}^{-1}}{S_{SL}} \Delta \vec{u} = \Delta \Xi' \quad (5.28)$$

$\Delta \Xi'$ is again the necessary scalar correction, but this time in consideration of the full motor dynamics. It is defined by

$$\Delta \Xi' = \Delta \Xi'_{\Delta \theta_R} + \Delta \Xi'_{\Delta \omega_R} + \Delta \Xi'_{\Delta J}, \quad (5.29)$$

with

$$\begin{aligned} \Delta \Xi'_{\Delta \theta_R} = & - \left[c_0 \frac{\mu_0 \mathcal{J}_m}{\pi \ell} + c_2 \left(\frac{\partial Q}{\partial \theta_R}|_{des} + \frac{\partial \vec{P}^T}{\partial \theta_R}|_{des} \vec{J}_{des} \right) + \left(\frac{\partial^2 Q}{\partial \theta_R^2}|_{des} + \frac{\partial^2 \vec{P}^T}{\partial \theta_R^2}|_{des} \vec{J}_{des} \right) \omega_{R,des} \right. \\ & \left. - \vec{P}^T|_{des} \frac{\mathcal{L}^{-1}}{S_{SL}} \frac{\partial \vec{e}}{\partial \theta_R}|_{des} - \frac{\partial \vec{P}^T}{\partial \theta_R}|_{des} \frac{\mathcal{L}^{-1}}{S_{SL}} (S_{SL} \mathcal{R} \vec{J}_{des} + \vec{e}|_{des} - \vec{u}_{des}) \right] \Delta \theta_R \end{aligned} \quad (5.30)$$

as the compensation term of an error in the rotor angle,

$$\Delta \Xi'_{\Delta \omega_R} = - \left[c_1 \frac{\mu_0 \mathcal{J}_m}{\pi \ell} + \left(\frac{\partial Q}{\partial \theta_R}|_{des} + \frac{\partial \vec{P}^T}{\partial \theta_R}|_{des} \vec{J}_{des} \right) - \vec{P}^T|_{des} \frac{\mathcal{L}^{-1}}{S_{SL}} \frac{\vec{e}}{\omega_R}|_{des} \right] \Delta \omega_R \quad (5.31)$$

as the compensation term of an error in the rotational speed,

$$\Delta \Xi'_{\Delta J} = - \left[c_2 \vec{P}^T|_{des} - \vec{P}^T|_{des} \mathcal{L}^{-1} \mathcal{R} + \frac{\partial \vec{P}^T}{\partial \theta_R}|_{des} \omega_{R,des} \right] \Delta \vec{J} \quad (5.32)$$

as the compensation term of an error in the currents.

Condition (5.28) leaves it open as a degree of freedom how this scalar $\Delta \Xi'$ is distributed among the coils. The aim of obtaining the smallest possible voltage corrections is achieved by solving the following optimization problem:

$$\text{minimize } K(\Delta \vec{u}) = \frac{1}{2} \Delta \vec{u}^T \Delta \vec{u} \quad (5.33)$$

$$\text{subject to } h(\Delta \vec{u}) = \vec{P}^T|_{des} \frac{\mathcal{L}^{-1}}{S_{SL}} \Delta \vec{u} - \Delta \Xi' = 0 \quad (5.34)$$

with solution equal to

$$\Delta \vec{u} = S_{SL} \frac{\Delta \Xi'}{\vec{P}^T|_{des} (\mathcal{L}^{-1})^2 \vec{P}|_{des}} \mathcal{L}^{-1} \vec{P}|_{des}. \quad (5.35)$$

Now, the optimal voltages $\vec{u}_{des}$ calculated by the feedforward control based on the desired (mechanical) trajectory are corrected by $\Delta\vec{u}$

$$\vec{u}_{in} = \vec{u}_{des} - \Delta\vec{u} \quad (5.36)$$

to produce the voltage input for the power electronics, compensating all kind of disturbances. If the correction $\Delta\vec{u}$ causes the input-voltage $\vec{u}_{in}$ to exceed the voltage limit of the power electronics, vector $\Delta\vec{u}$ is reduced by a factor so that $\vec{u}_{in}$ complies with that limit.

5.3.3 Refinement of the optimization

By means of cost function (5.33), the necessary scalar correction $\Delta\Xi'$ due to an error in the mechanical trajectory is compensated by the least square sum voltage corrections, minimizing $\Delta\vec{u}$. However, a deviation in the currents (which would contribute to the scalar $\Delta\Xi'_{\Delta J}$) is merely considered from a mechanical point of view due to constraint (5.34). Thereby only deviations of the torque-producing ('*quadrature*') component of the currents are compensated. The non torque-producing ('*direct*') component, which is incorporated by the field weakening, is not controlled yet. Thus, cost function (5.33) in the minimization problem needs to be modified:

$$\text{minimize } K(\Delta\vec{u}) = \frac{1}{2}(\Delta\vec{u} + \Delta\vec{v})^T(\Delta\vec{u} + \Delta\vec{v}) \quad (5.37)$$

$$\text{subject to } h(\Delta\vec{u}) = \vec{P}_{des}^T \left[\frac{\mathcal{L}^{-1}}{S_{SL}} \Delta\vec{u} - \Delta\Xi' \right] = 0 \quad (5.38)$$

This new cost function takes also into account an additional voltage deviation $\Delta\vec{v}$, which directly depends on the deviation of the currents

$$\Delta\vec{v} = \mathcal{L} \frac{\Delta\vec{J}}{\Delta t} S_{SL} = \mathcal{L} \frac{\vec{J}_{des} - \vec{J}_{mes}}{\Delta t} S_{SL} \quad (5.39)$$

where Δt is the discrete time step of the control. The deviation voltage is then compensated as closely as possible by the voltage correction of the feedback control, $\Delta\vec{u} \rightarrow -\Delta\vec{v}$.

For a better understanding, the relations between the vectors considered in the feedback control are shown in Fig. 5.6. Here, all voltage corrections $\Delta\vec{u}$ that point on the **red line** would fulfill equality constraint (5.38). In the context of the previous cost function (5.33), the optimal voltage correction $\Delta\vec{u}$ would be the smallest possible (**gray vector**). In Fig. 5.6 the current vectors $\vec{J}_{des}$ and $\vec{J}_{mes}$ (**light blue** and **blue vector**) are also drawn, and thus the resulting voltage deviation $\Delta\vec{v}$ (**dark green vector**). Due to the refined cost function (5.37), the optimal voltage correction compensates $\Delta\vec{v}$ as much as possible, while

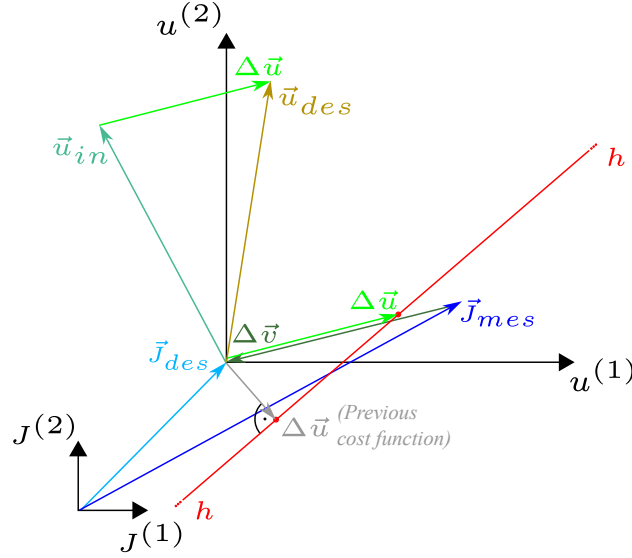


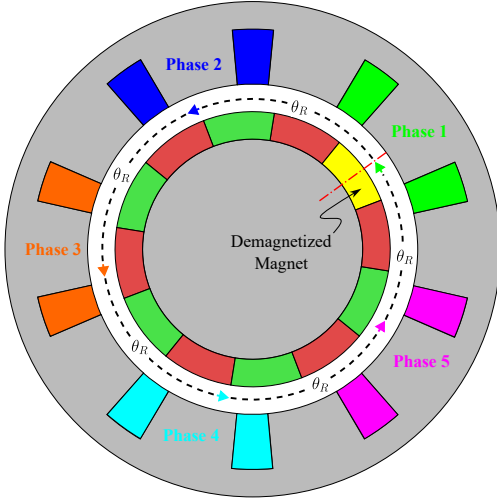
Figure 5.6: Geometric interpretation of the feedback control for a theoretical two phase machine. All voltage corrections $\Delta \vec{u}$ that point on the **red line** would fulfill equality constraint h in optimization problems (5.33) and (5.37).

still fulfilling equality constraint (5.38) (**green vector**). This allows the desired voltage $\vec{u}_{des}$ (**brown vector**) to be adjusted to form the crucial input voltage $\vec{u}_{in}$ (**turquoise vector**) for the power electronics, compensating simultaneously mechanical and electrical disturbances. By solving minimization problem (5.37), the following equation is obtained:

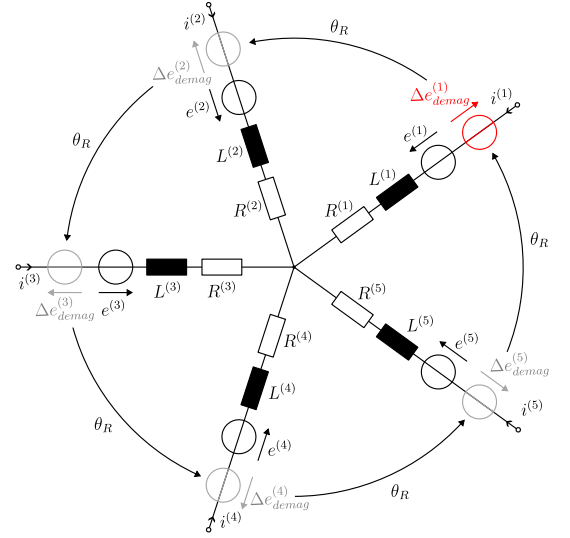
$$\Delta \vec{u} = S_{SL} \left(\frac{\Delta \Xi' + \vec{P}_{des}^T \frac{\Delta \vec{J}}{\Delta t}}{\vec{P}_{des}^T (\mathcal{L}^{-1})^2 \vec{P}_{des}} \mathcal{L}^{-1} \vec{P}_{des} - \mathcal{L} \frac{\Delta \vec{J}}{\Delta t} \right), \quad (5.40)$$

compensating $\Delta \Xi'$ and reducing $\Delta \vec{J}$ simultaneously.

Besides the control of this non-torque producing component of the currents, the refined cost function (5.37) has another advantageous property. In the 'healthy' case the sum of all voltages is usually equal (or close) to zero, resulting in a balanced polyphase system. If for example a partial demagnetization occurs and the control system is *not yet fault adapted*, this balance is disturbed. In Fig. 5.7b the equivalent circuit of a PMSM is drawn with partial demagnetization, which can be considered as an additional voltage source $\Delta e_{demag}^{(c)}$. This voltage source reduces effectively the 'healthy' back-EMF $e^{(c)}$ and is depending on the rotor position θ_R . Mainly if the **demagnetized magnet** passes directly a coil (as for example sketched in Fig. 5.7a for phase or coil $c = 1$), the according voltage source $\Delta e_{demag}^{(c)}$ is nonzero, while it is for the others (close to) zero due to the decoupling of the concentrated windings. With the first cost function (5.33) for the feedback control, this would result in an imbalance of the polyphase system and thereby in high deviations in all currents. Assuming



(a) A **demagnetized magnet** passes phase or coil $c = 1$.



(b) Equivalent circuit where the demagnetized magnet **passes coil $c = 1$** .

Figure 5.7: Partial demagnetization can be interpreted as superposition of the 'healthy' back-EMF $e^{(c)}$ by a 'counter-back-EMF' $\Delta e_{demag}^{(c)}$, both depending on rotor angle θ_R ; $\Delta e_{demag}^{(c)}$ is mainly nonzero when the demagnetized magnet passes directly coil (c) [69].

in a first step that $\Delta e_{demag}^{(c)}$ causes directly a change in the currents,

$$\Delta e_{demag}^{(c)} \frac{\Delta t}{S_{SL} L^{(c)}} \approx \Delta J^{(c)}, \quad (5.41)$$

the imbalance is (partially) corrected by the feedback control due to the refined cost function (5.37) in each time area where the demagnetized magnet passes coil (c)

$$\Delta u^{(c)} \rightarrow -\Delta v^{(c)} = -L^{(c)} \frac{\Delta J^{(c)}}{\Delta t} S_{SL}. \quad (5.42)$$

Thereby the polyphase system is rebalanced in part by the asymmetric distribution of the voltage correction. This property of the control is also used for the diagnosis and adaptation of partial demagnetization (see section 7.2).

5.4 Overall control system

This section provides an overview of the developed control system. While subsection 5.4.1 describes its parametrization and initialization, the subsequent operations during the online control are explained in subsection 5.4.2. An overview of the most important steps in both the offline and online phases are illustrated in Fig. 5.8. A simulation environment is described in subsection 5.4.3, and subsequently used in subsection 5.4.4 to evaluate the performance of the overall control system.

5.4.1 Parametrization & initialization

The parameters required for the parametrization of the control system are listed in Tab. 5.1. Thereby the geometric parameters are radii and angles describing effectively the rotor and stator size as well as the height of the magnets, the positions of the slots (and flux barrier) and their widths. They are also shown in Fig. 4.2. The magnetic parameters characterize the distribution of the rotor magnets, by the number of pole pairs, the pole-arc to pole-pitch ratio, and the magnetic remanence. Finally, the electrical parameters contain the effective resistance of each phase, the number of winding turns per coil, the DC-link voltage, and the maximum RMS value of the phase currents.

For the initialization, constant (geometry depending) matrices $\mathbf{P}, \mathbf{Q}$ of the linear equation system used to compute the Fourier coefficients must be pre-calculated², as highlighted in the **yellow area** in Fig. 5.8. Thereby the self inductances can be determined by equation (4.10). The field weakening LUTs are then generated based on the methodology described in subsection 5.2.2, both for the healthy condition and for each considered fault case.

5.4.2 Online control

The required computational steps for the online control at each time increment are illustrated in the **green area** of Fig. 5.8. Based on the desired trajectory $\{\theta_{R,des}, \omega_{R,des}\}$, Fourier coefficients $\{\vec{a}^{(M)}, \vec{c}^{(M)}\}$ and $\{\omega_R \frac{\partial \vec{a}^{(M)}}{\partial \theta_R}, \omega_R \frac{\partial \vec{c}^{(M)}}{\partial \theta_R}\}$ are firstly determined based on (4.2). From this, the quantities $\{\vec{P}, Q, \vec{\Phi}_M\}$ as well as their time derivatives $\{\dot{\vec{P}}, \dot{Q}, \dot{\vec{e}}\}$ are efficiently calculated by means of equations (4.14), (4.6), (4.7), (4.15), (4.16) and are then transferred to the feedforward and feedback control.

The feedforward control firstly extracts from the precalculated field weakening LUT the necessary field weakening coefficient α in dependence of the desired rotational speed $\omega_{R,des}$ and the desired torque T_{des} . Based on this, the optimal desired currents $\vec{i}_{des}$ and their derivatives $\frac{d\vec{i}_{des}}{dt}$ for each phase or coil are obtained by equation system (5.8). The corresponding desired

²see section A.5 for a conventional PMSM and section B.3 for a PMSM with stator flux barriers.

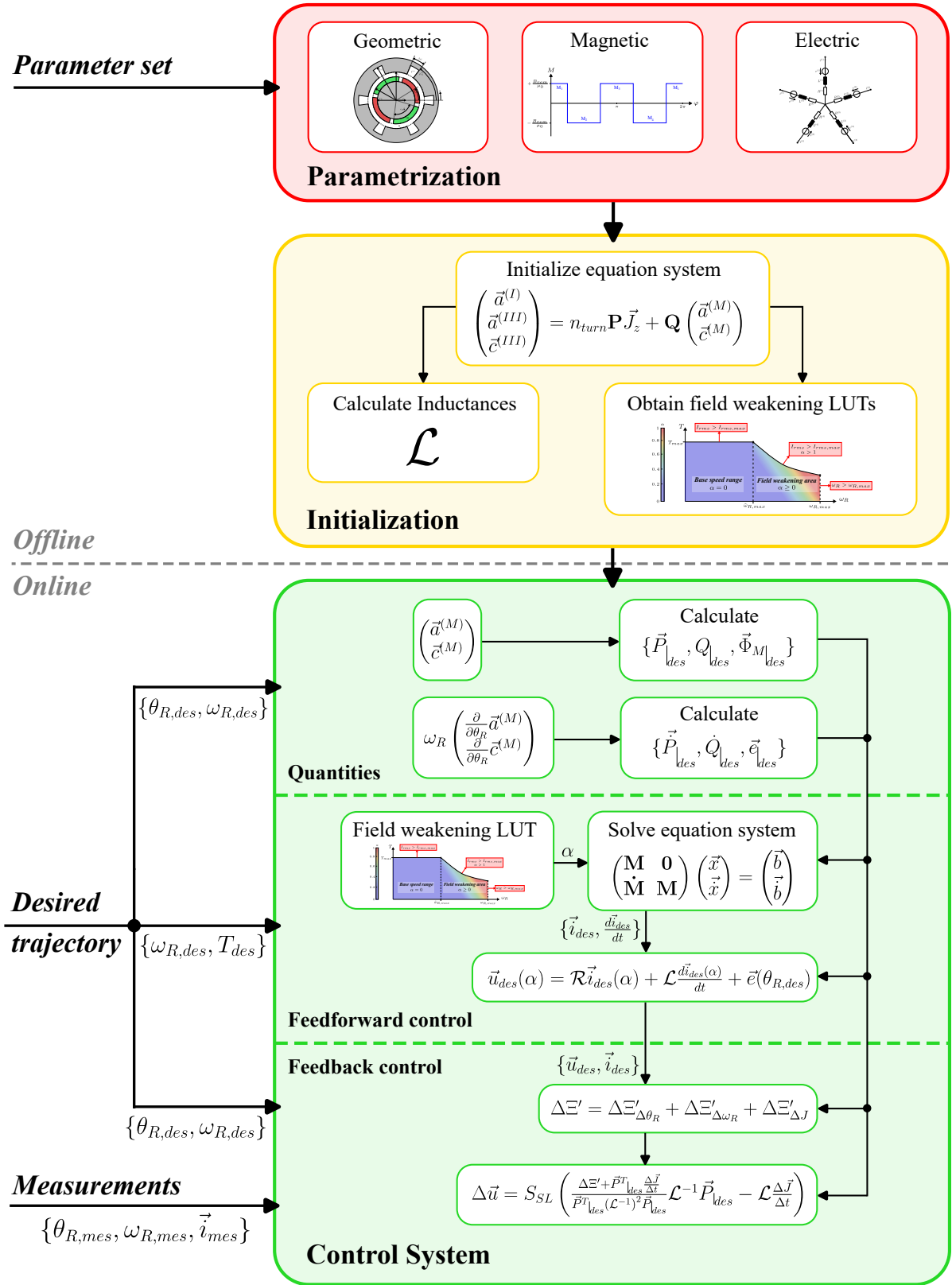


Figure 5.8: Qualitative overview over the **required parameters** (listed in Tab. 5.1), the most relevant offline operations during **initialization** and the most relevant computational steps that need to be done in the **online control system** for each time increment.

Category	Parameter	Description	Unit
Geometric	r_R	Rotor radius	m
	r_M	Magnet radius	m
	r_S	Stator radius	m
	r_{SL}	Slot radius	m
	β_{SL}	Slot width	rad
	$\vec{\varphi}_{SL}$	Slot positions	rad
	r_{FB}	Flux barrier radius	m
	β_{FB}	Flux barrier width	rad
	$\vec{\varphi}_{FB}$	Flux barrier positions	rad
	ℓ	Active rotor length	m
Magnetic	p	Number of pole pairs	–
	ϵ_M	Pole-arc to pole-pitch ratio	–
	B_{rem}	Magnetic remanence	T
Electric	$\mathcal{R}$	Coil resistances	Ω
	n_{turn}	Number of winding turns	–
	$I_{rms,max}$	Maximal RMS of phase currents	A
	U_{DC}	DC voltage	V
Mechanical	$n_{R,max}$	Maximal rotational speed	$\frac{1}{\min}$
	$\mathcal{J}_m$	Rotor's moment of inertia	kg m ²

Table 5.1: **Parameter set** required to parameterize the control system. The parameters marked blue are specific for **PMSMs with flux barriers**. Vectors $\vec{\varphi}_{SL}$ and $\vec{\varphi}_{FB}$ contain all angular slot and flux barrier positions and diagonal matrix $\mathcal{R}$ contains the effective resistance of each phase or coil.

voltages $\vec{u}_{des}$ are subsequently determined using equation (5.7).

The feedback control then summarizes by equation (5.29) deviations $\{\Delta\theta_R, \Delta\omega_R, \Delta\vec{J}\}$ from the desired values in the required (mechanical) correction $\Delta\Xi'$. To compensate $\Delta\Xi'$ and reduce the current deviations $\Delta\vec{J}$, the feedback control computes the optimally distributed voltage corrections $\Delta\vec{u}$ using equation (5.40). Correcting the desired voltages $\vec{u}_{des}$ with $\Delta\vec{u}$, the input for the power electronics is obtained (5.36). This ensures an accurate and robust tracking of the desired trajectory.

5.4.3 Simulation environment

To evaluate the developed control system it is implemented in a Matlab/Simulink environment. The Subdomain Model described in section 4.4 is used as plant model. The control system is realized based on the structure shown in Fig. 5.8. The time step of the control system is set to $100\mu s$ ($f_{control} = 10\text{kHz}$) and the time step of the plant simulation to $10\mu s$ ($f_{plant} = 100\text{kHz}$) to improve the numerical accuracy of the current dynamics. Thereby the input voltages $\vec{u}_{in}$ for the power electronics are kept constant during the 10 time steps of

the plant dynamics.

In order to have a realistic simulation, the Subdomain Model acting as the plant and the control system itself are parameterized with parameter sets which have a discrepancy between each other. Thereby *real* parameters are meant to be the ones of the plant model, *estimated* the ones of the control system. The discrepancies between real and estimated parameters effectively result in an error in the resistance, inductance and PM flux linkage, which needs to be compensated by the feedback control. Since the field weakening LUTs are calculated assuming steady-states, they are determined based on an assumed DC input voltage of the inverter which is 5% lower than the real one to have a voltage tolerance for transients. For all measurements (rotor position, rotational speed, phase currents and torque) Gaussian white noise is taken into account. These signals in the analyzes are denoted with the index *mes* and they are filtered using a moving average before being processed by the control system. For the (ideal) power electronics, a five phase three level inverter is taken into account [70]. The chosen modulation strategy is a carrier-based PWM with min-max injection [71]. The resulting structure of the overall simulation environment is sketched in Fig. 5.9.

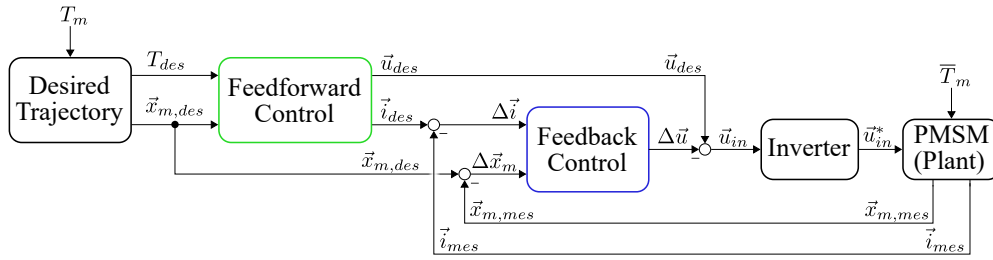


Figure 5.9: Structure of the overall control system. Only the most relevant signals and blocks are shown. $\vec{x}_m = (\theta_R \ \omega_R)^T$ are the relevant mechanical variables [60, 66].

5.4.4 Evaluation

In the following, the performance of the developed control system is analyzed based on the simulation environment described above. The parameter set for the parameterization of the control system is listed in Tab. 5.2. The desired rotational speed as well as the assumed load torque during simulation is shown in Fig. 5.10. The simulated discrepancy between the *estimated* and the *real* parameter set results in deviations, which can be summarized in the four most relevant quantities: the coil resistance, coil inductance, PM flux linkage and DC voltage. They are listed in Tab. 5.3.

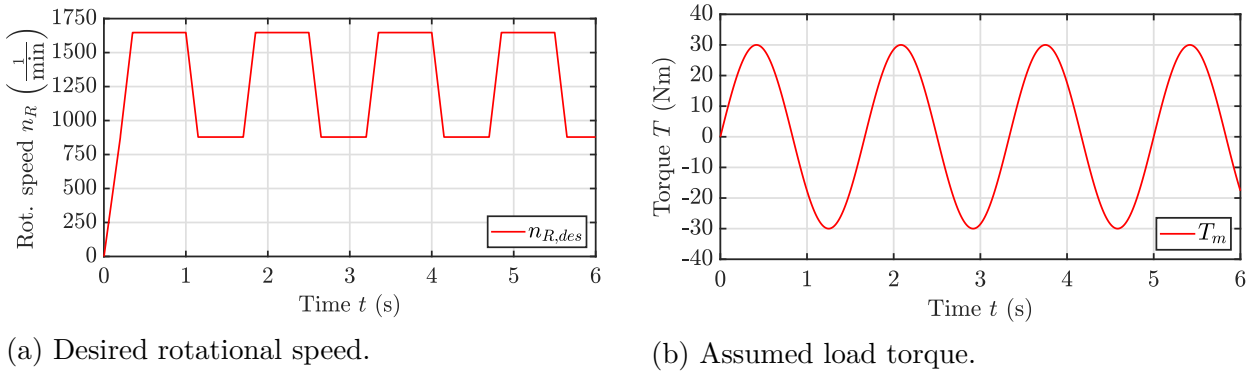
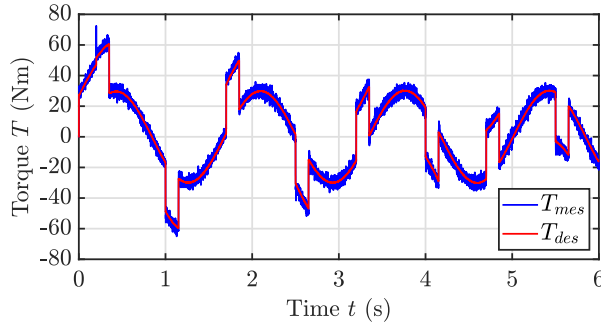
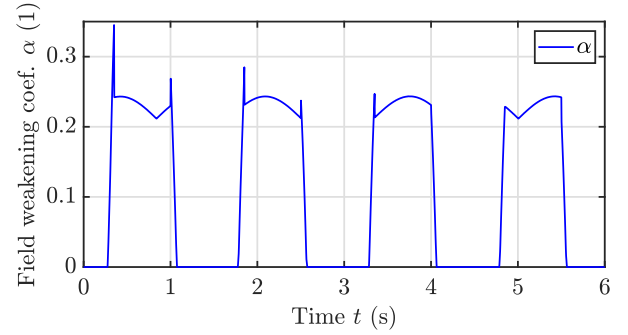


Figure 5.10: Desired rotational speed and assumed load torque during the simulation.

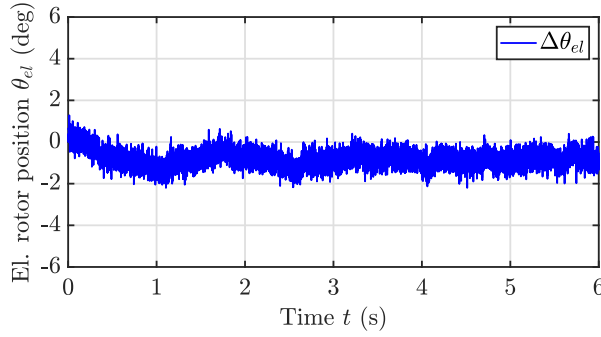
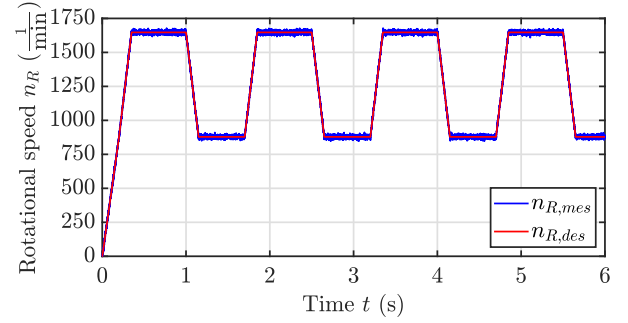
The simulation results are displayed in Fig. 5.11. Here, Fig. 5.11a shows the actual torque in comparison to the desired one. Thereby a ripple in the produced torque of the simulated machine can be observed. This is caused in combination by the prediction error of the feed-forward control due to the parameter discrepancies, the simulated noise of the measurements and the inverter injecting high frequent harmonics in the currents. However, the deviation of the rotor angle from the desired trajectory in electrical degrees, $\Delta\theta_{el}$, remains small over the whole simulation period (see Fig. 5.11c). In the time periods with increased desired rotational speed, field weakening becomes necessary (see also field weakening LUT in 5.4a), resulting in nonzero values for field weakening coefficient α (see Fig. 5.11b). Fig. 5.11e and Fig. 5.11f show the actual currents at time periods, where there is a step in the rotational speed and thereby in the desired torque. A comparison with the desired current waveforms (Fig. 5.11g and Fig. 5.11h) reveals that the feedback control enables fast transitions in the currents to avoid any relevant deviation from the desired mechanical trajectory.



(a) Actual and desired torque.



(b) Field weakening coefficient.

(c) Deviation of the rotor angle ($\Delta\theta_{el} = p\Delta\theta_R$).

(d) Actual and desired rotational speed.

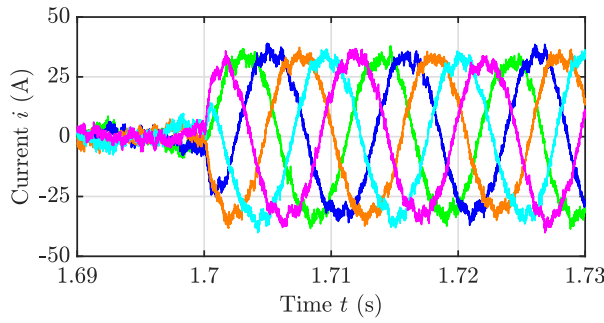
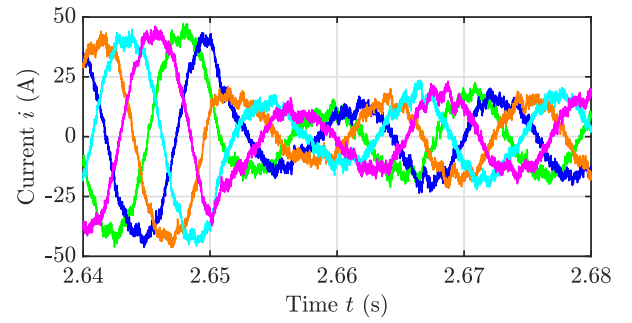
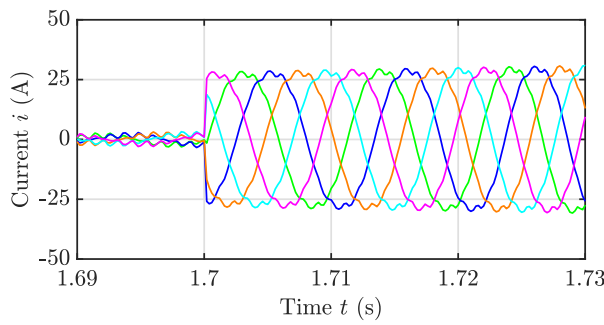
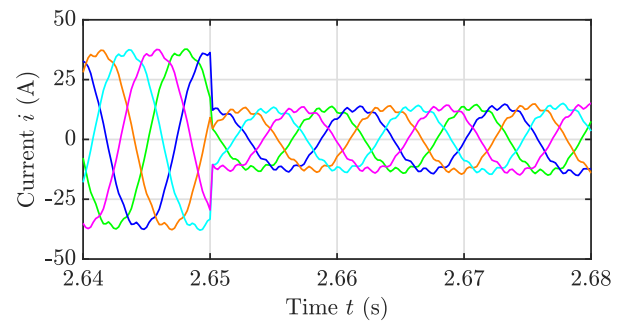
(e) Actual currents $\vec{i}_{mes}$ at time period 1.(f) Actual currents $\vec{i}_{mes}$ at time period 2.(g) Desired currents $\vec{i}_{des}$ at time period 1.(h) Desired currents $\vec{i}_{des}$ at time period 2.

Figure 5.11: Evaluation of the overall control system under dynamic operation.

Category	Parameter	Value	Unit
Geometric	r_R	53.15×10^{-3}	m
	r_M	61.0×10^{-3}	m
	r_S	61.75×10^{-3}	m
	r_{SL}	85.0×10^{-3}	m
	β_{SL}	11.0	degree
	$\vec{\varphi}_{SL}$	$(12 \ 48 \ 84 \ 120 \ 156 \ 192 \ 228 \ 264 \ 300 \ 336)^T$	degree
	r_{FB}	95.0×10^{-3}	m
	β_{FB}	9.3	degree
	$\vec{\varphi}_{FB}$	$(0 \ 72 \ 144 \ 216 \ 288)^T$	degree
	ℓ	85.0×10^{-3}	m
Magnetic	p	6	–
	ϵ_M	0.86	–
	B_{rem}	1.2	T
Electric	$\mathcal{R}$	diag(0.1 0.1 0.1 0.1 0.1)	Ω
	n_{turn}	35	–
	$I_{rms,max}$	60	A
	U_{DC}	125	V
Mechanical	$n_{R,max}$	1800	$\frac{1}{\text{min}}$
	$\mathcal{J}_m$	0.06	kg m ²

Table 5.2: (*Estimated*) parameter set for the control system.

Parameter	Real	Estimate	Unit
DC Voltage (U_{DC})	130	124	U
Coil resistance (R)	0.115	0.100	Ω
Coil inductance (L)	0.487	0.520	mH
Amplitude PM flux linkage ($\hat{\Phi}_M$)	74	82	mWb

Table 5.3: Discrepancy between the estimated and real parameter set, summarized in the four most relevant quantities.

6 (Multi-) fault management

While chapter 5 focused solely on prefault conditions, this chapter presents the required modifications in the plant model, feedforward control, and feedback control to account for motor-intrinsic electrical and magnetic faults (sections 6.1 to 6.5). Section 6.6 introduces a generalized formulation of both the feedforward and feedback control that enables fault compensation not only for single faults but also for their arbitrary combination to ensure multi-fault tolerance. For a better overview, all fault handling strategies developed within this work are summarized in Tab. 6.1, with reference to their corresponding sections.

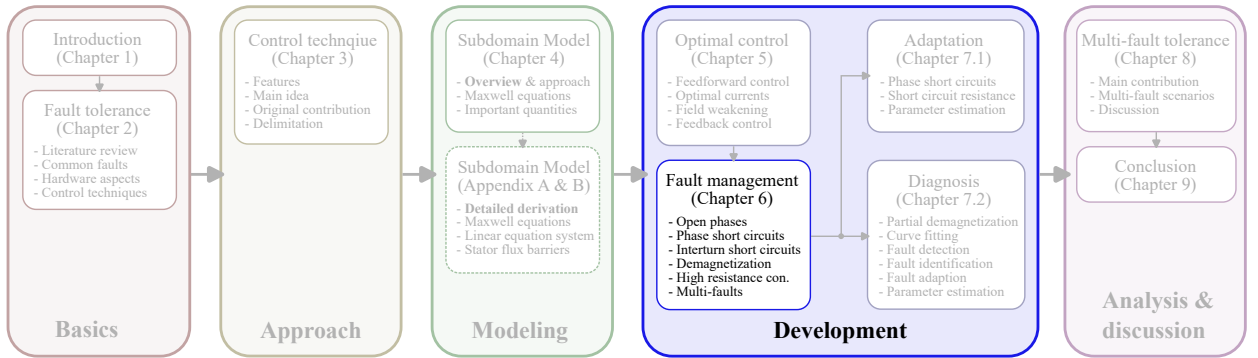


Figure 6.1: Structural overview of the work, with this chapter 6 highlighted.

To evaluate the performance of the developed control system under fault tolerant operation, each fault type is analyzed using the simulation environment described in section 5.4.3. The parameter set applied corresponds to the values listed in Tab. 5.1. Analogously to the previous evaluation, a discrepancy between the *estimated* parameter set (the one for the control system) and the *real* parameter set (the one for the plant model) is considered. This leads to deviations in the relevant physical quantities, which are summarized in Tab. 5.3. The desired rotational speed and the corresponding load torque used in the dynamic simulations are depicted in Fig. 5.10. In cases where the machine is no longer capable of following the predefined trajectory due to a fault, the desired rotational speed is scaled down to remain within the feasible operating limits. While this chapter assumes full knowledge of all required fault parameters, chapter 7 presents two methods for the parameter estimation: one for partial demagnetization and one for phase short circuits.

Fault	Description	Section
Open phase fault	<i>Eliminate the affected phase from the set of the healthy phases considered by the feedforward and feedback control.</i>	6.1
Phase short circuit	<i>Eliminate the affected phase from the set of the healthy phases considered by the feedforward and feedback control; compensate the faulty torque ripple by an analytical prediction.</i>	6.2
Interturn short circuit	<i>Reduce the short circuit current within the faulty coil by an effective field weakening; this is achieved by predicting the PM flux linkage of the faulty coil turns.</i>	6.3
Demagnetization	<i>Adapt the Fourier coefficients of the magnetization based on the spatial course of demagnetization.</i>	6.4
High resistance connection	<i>Modificate the weight of the affected phase in the optimization problem for determining the optimal currents in the feedforward control.</i>	6.5
Multi-fault	<i>Merge the handlings described above into a unified, generalized formulation; this enables the simultaneous consideration and compensation of multiple faults.</i>	6.6

Table 6.1: Overview over all fault handling strategies developed in this work.

6.1 Open phase faults

An open phase in the plant model is efficiently considered by removing the corresponding phase from the differential equation system (4.17) describing the current dynamics.

Modifications in the control

For the compensation, the affected phase must be eliminated from the control system. The necessary adjustments to both the feedforward and the feedback control are described in the following.

Modifications in the feedforward control

The optimization problem of the feedforward control, used to determine the desired currents, must be reformulated to account for an open phase. It is given by:

$$\begin{aligned}
&\text{minimize } K(\vec{J}_h) = \frac{1-\alpha}{2} \vec{J}_h^T \vec{J}_h + \frac{\alpha}{2} \left(\vec{J}_h + \frac{\mathcal{L}_h^{-1} \vec{\Phi}_{M,h}}{S_{SL}} \Big|_{des} \right)^T \left(\vec{J}_h + \frac{\mathcal{L}_h^{-1} \vec{\Phi}_{M,h}}{S_{SL}} \Big|_{des} \right) \\
&\text{subject to } h_1(\vec{J}_h) = \vec{1}^T \vec{J}_h = 0 \\
&\quad h_2(\vec{J}_h) = \frac{\ell\pi}{\mu_0} \vec{P}_h^T \Big|_{des} \vec{J}_h + \frac{\ell\pi}{\mu_0} Q \Big|_{des} - T_{des} = 0
\end{aligned} \tag{6.1}$$

where each vector and matrix denoted with the index h contains the according quantities of all remaining, healthy coils. By solving this minimization problem, the combined equation system including the derivatives of the currents corresponds to the one under prefault conditions (5.8), with the exception of the eliminated faulty phases:

$$\begin{pmatrix} \mathbf{M}_h & \mathbf{0} \\ \dot{\mathbf{M}}_h & \mathbf{M}_h \end{pmatrix} \begin{pmatrix} \vec{x}_h \\ \dot{\vec{x}}_h \end{pmatrix} = \begin{pmatrix} \vec{b}_h \\ \dot{\vec{b}}_h \end{pmatrix}, \quad (6.2)$$

with

$$\mathbf{M}_h = \begin{pmatrix} \mathbf{I} & \vec{1} & \frac{\ell\pi}{\mu_0} \vec{P}_h|_{des} \\ \vec{1}^T & 0 & 0 \\ \frac{\ell\pi}{\mu_0} \vec{P}_h^T|_{des} & 0 & 0 \end{pmatrix}, \quad \vec{x}_h = \begin{pmatrix} \vec{J}_h \\ \lambda_1 \\ \lambda_2 \end{pmatrix}, \quad \vec{b}_h = \begin{pmatrix} -\alpha \frac{\mathcal{L}_h^{-1} \vec{\Phi}_{M,h}}{S_{SL}}|_{des} \\ 0 \\ T_{des} - \frac{\ell\pi}{\mu_0} Q|_{des} \end{pmatrix} \quad (6.3)$$

and

$$\dot{\mathbf{M}}_h = \begin{pmatrix} \mathbf{0} & \vec{0} & \frac{\ell\pi}{\mu_0} \dot{\vec{P}}_h|_{des} \\ \vec{0}^T & 0 & 0 \\ \frac{\ell\pi}{\mu_0} \dot{\vec{P}}_h^T|_{des} & 0 & 0 \end{pmatrix}, \quad \dot{\vec{x}}_h = \begin{pmatrix} \dot{\vec{J}}_h \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \end{pmatrix}, \quad \dot{\vec{b}}_h = \begin{pmatrix} -\alpha \frac{\mathcal{L}_h^{-1} \dot{\vec{e}}_h}{S_{SL}}|_{des} \\ 0 \\ -\frac{\ell\pi}{\mu_0} \dot{Q}|_{des} \end{pmatrix}. \quad (6.4)$$

Solving equation system (6.2), the desired currents $\vec{J}_{des,h}$ and their time derivatives $\dot{\vec{J}}_{des,h}$ are obtained, which are then used to calculate the desired voltages:

$$\vec{u}_{des,h}(\alpha) = \mathcal{R}_h \vec{i}_{des,h}(\alpha) + \mathcal{L}_h \frac{d\vec{i}_{des,h}(\alpha)}{dt} + \vec{e}_h(\theta_{R,des}). \quad (6.5)$$

To precalculate the field weakening LUT the same methodology as described in subsection 5.2.2 can be used. However it should be noted that the two space vectors (positive and negative) of the according switches need to be eliminated for each faulty phase.

Modifications in the feedback control

To enable the feedback control to operate under open phase conditions, the corresponding phase must be excluded from the optimization. The adjusted optimization problem for determining the optimal distributed voltage corrections $\Delta \vec{u}_h$ is formulated as follows:

$$\begin{aligned} & \text{minimize } K(\Delta \vec{u}_h) = \frac{1}{2}(\Delta \vec{u}_h + \Delta \vec{v}_h)^T(\Delta \vec{u}_h + \Delta \vec{v}_h) \\ & \text{subject to } h(\Delta \vec{u}_h) = \vec{P}_h^T \Big|_{des} \frac{\mathcal{L}_h^{-1}}{S_{SL}} \Delta \vec{u}_h - \Delta \Xi'_h = 0 \\ & \text{where } \Delta \vec{v}_h = \mathcal{L}_h \frac{\Delta \vec{J}_h}{\Delta t} S_{SL} \end{aligned} \quad (6.6)$$

Again each vector denoted with the index h contains the according quantities of all remaining healthy coils. The required correction $\Delta \Xi'_h$ is calculated based on equation (5.29), excluding the contribution of the faulty phase. By solving this minimization problem, the optimal distributed voltage corrections can be obtained as follows:

$$\Delta \vec{u}_h = S_{SL} \left(\frac{\Delta \Xi'_h + \vec{P}_h^T \Big|_{des} \frac{\Delta \vec{J}_h}{\Delta t}}{\vec{P}_h^T \Big|_{des} (\mathcal{L}_h^{-1})^2 \vec{P}_h \Big|_{des}} \mathcal{L}_h^{-1} \vec{P}_h \Big|_{des} - \mathcal{L}_h \frac{\Delta \vec{J}_h}{\Delta t} \right). \quad (6.7)$$

The input voltage vector for the power electronics, considering the open phase condition, is then given by:

$$\vec{u}_{in,h} = \vec{u}_{des,h} - \Delta \vec{u}_h. \quad (6.8)$$

Evaluation

In the following, the performance of the developed control system is evaluated under the presence of an open phase fault. For this purpose, one phase of the PMSM is assumed to be permanently disconnected, and the control system is adapted accordingly as described above.

Feedforward control

To evaluate the feedforward control under consideration of an open phase it is used to predict the optimal desired currents and voltages at two different operating points¹. In the following simulations, the desired voltage is transformed to the complex plane. Thereby a circle in the complex plane, which is equal to one electrical period, corresponds to voltages

¹In the plots of this section showing the desired currents $\vec{i}_{des}$, the actual currents $\vec{i}_{mes}$ or the torque capabilities $\frac{\ell \pi}{\mu_0} \vec{P}$, the open phase is represented by the **green curve**.

which are purely sinusoidal and evenly distributed along the phases. The greater the ripple, the higher the amplitudes of higher frequent harmonics. The more elliptical the voltages are, the higher is their asymmetrical distribution. To visualize compliance with the voltage limit of the power electronics, the resulting decagon of the space vectors is drawn. The desired voltage must not exceed this decagon.

In Fig. 6.2a the resulting field weakening LUT is shown. Comparing this field weakening LUT with the one for the healthy machine (see Fig. 5.4a), it can be seen that the operating area (and especially the field weakening area) under consideration of the electrical limitations is clearly reduced. This reduction is a direct consequence of the open phase, which reduces the feasible solution space. Due to the loss of one coil, the waveforms of the torque capabilities from the remaining healthy coils shown in Fig. 6.2f are unevenly distributed. As a result, the desired currents have different waveforms for each coil with a lower torque per ampere ratio (see Fig. 6.2c and Fig. 6.2d), and the desired voltage has an elliptical trajectory (Fig. 6.2b).

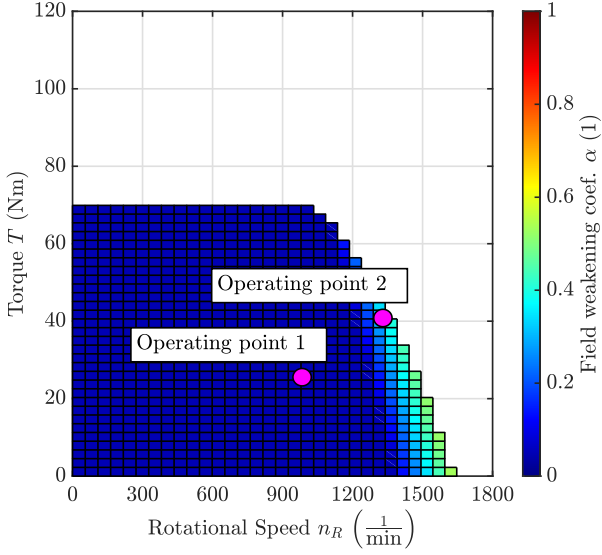
Overall control system

The simulation results of the overall control system under dynamic conditions are displayed in Fig. 6.3¹. It is assumed that the open phase fault occurs at $t_{fault} = 2.0s$ and the control system is adapted to the fault after a delay of 10ms ($t_{adapt} = 2.01s$). Since the control is no longer able to follow the desired trajectory shown in Fig. 5.10 due to the decreased operating area (see Fig. 6.2a), the desired rotational speed is scaled down by a factor of 0.8.

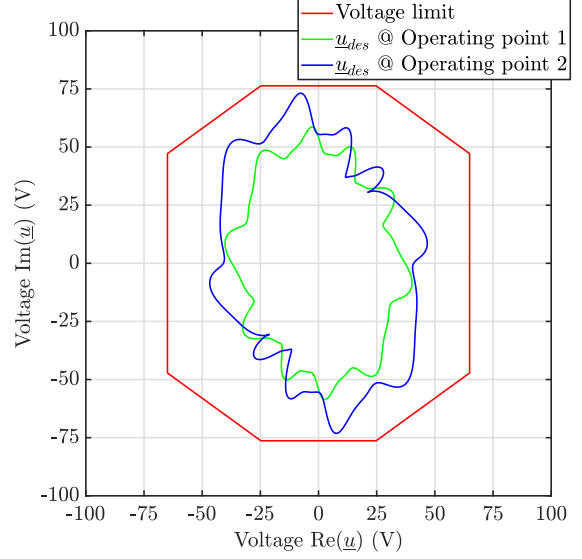
By observing the actual torque and the rotor angle deviation during simulation (see Fig. 6.3a and Fig 6.3c), clear peaks can be identified at the time of fault occurrence. These peaks are a result of the simulated delayed fault adaption of the control system, highlighting the importance of a robust fault tolerant control strategy. After the fault adaptation at $t \approx t_{adapt}$ the actual currents in Fig. 6.3e are significantly increased compared to the desired ones (Fig. 6.3g). This is a result of the feedback control, which re-stabilizes the system.

To gain further insight into the behavior of the field weakening strategy, the waveform of field weakening coefficient α is displayed in Fig. 6.3b. It can be seen that α is nonzero at each time period with increased rotational speeds (see Fig. 6.3d) in order to reach these operating points.

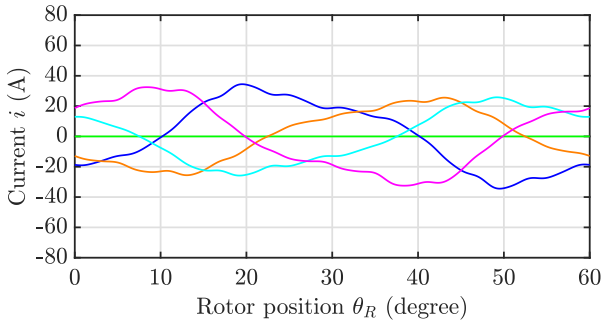
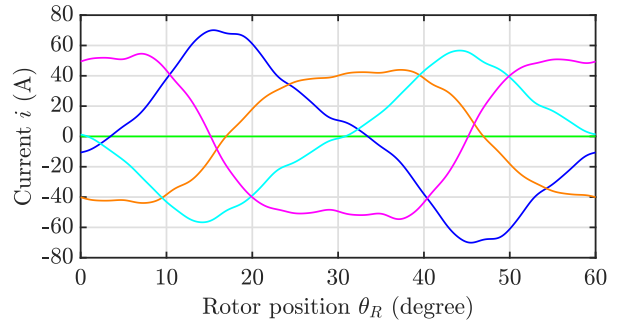
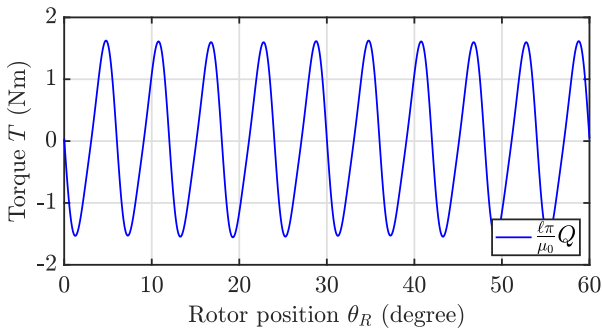
Fig. 6.3f shows the actual currents at a time interval, where there occurs a step in the rotational speed and thereby in the desired torque. A comparison with the desired current waveforms in Fig. 6.3h reveals that the feedback control enables fast transitions in the currents, whereby an accurate tracking of the trajectory under transient conditions is achieved even in the case of an open phase.



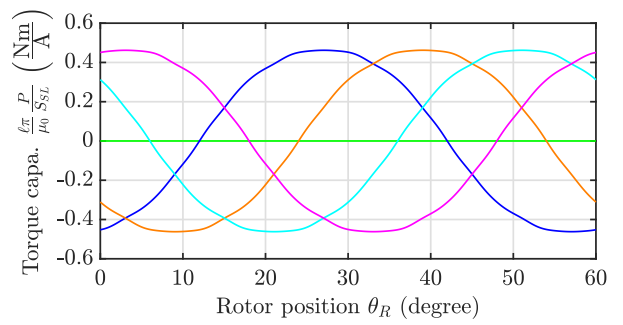
(a) Field weakening LUT.

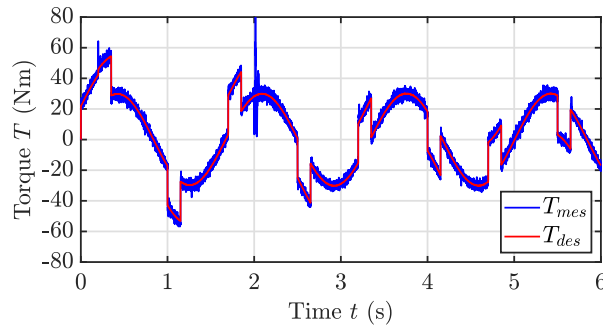


(b) Desired voltages in the complex plane.

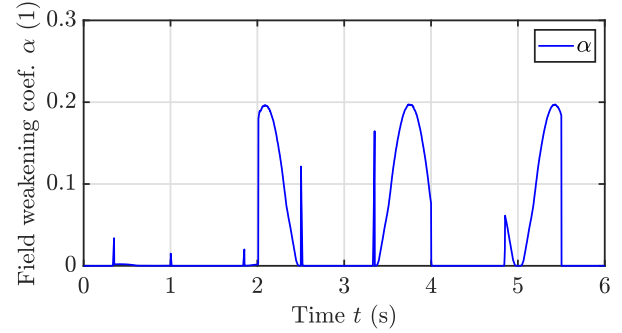
(c) Desired currents $\vec{i}_{des}$ at operating point 1.(d) Desired currents $\vec{i}_{des}$ at operating point 2.

(e) Predicted cogging torque.

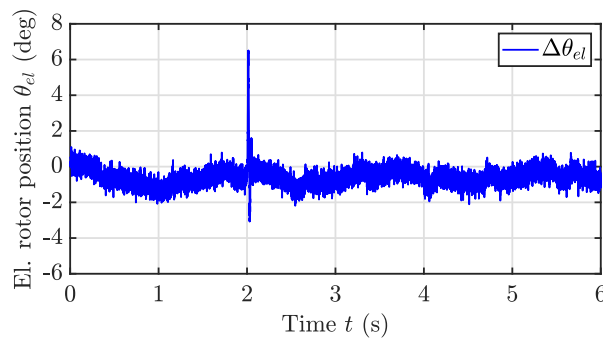
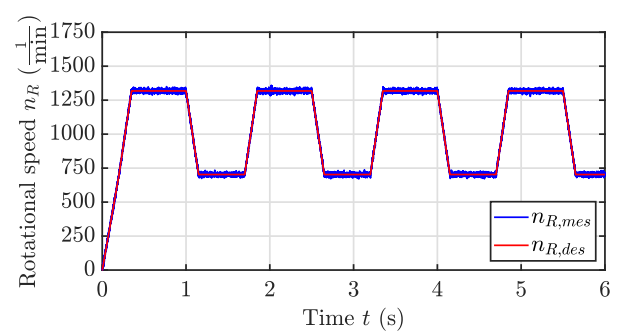
(f) Predicted torque capabilities $\frac{\ell_{\pi}}{\mu_0} \vec{P}$.Figure 6.2: **Open phase** fault tolerant operation of the feedforward control.



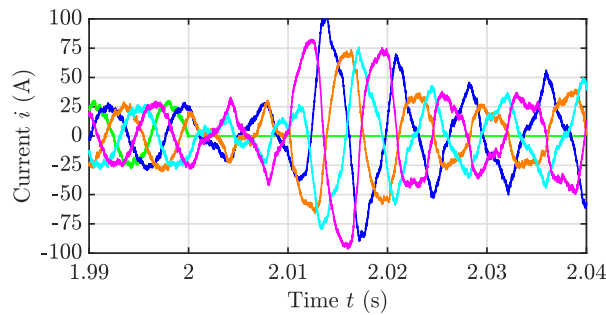
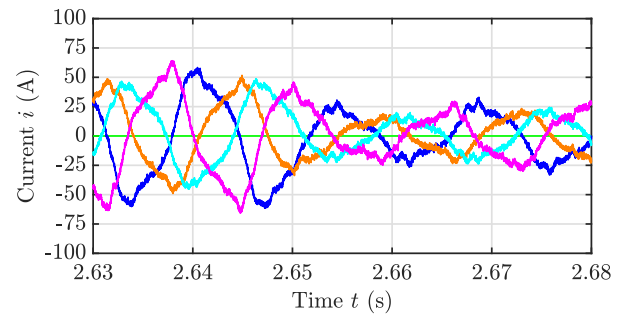
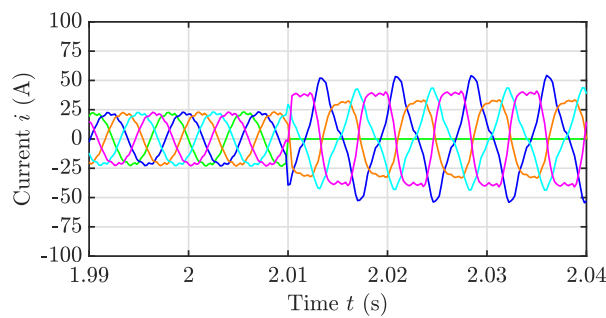
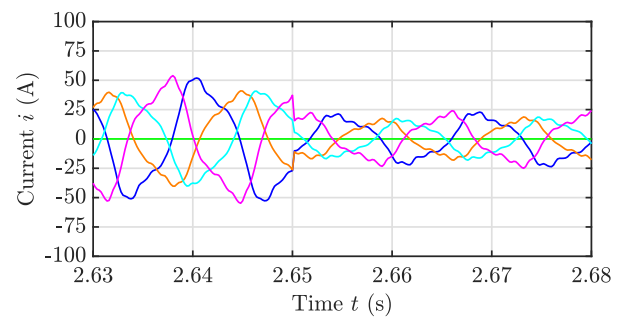
(a) Actual and desired torque.



(b) Field weakening coefficient.

(c) Deviation of the rotor angle ($\Delta\theta_{el} = p\Delta\theta_R$).

(d) Actual and desired rotational speed.

(e) Actual currents $\vec{i}_{mes}$ at the time period of fault occurrence.(f) Actual currents $\vec{i}_{mes}$ at a time period after fault adaptation.(g) Desired currents $\vec{i}_{des}$ at the time period of fault occurrence.(h) Desired currents $\vec{i}_{des}$ at a time period after fault adaptation.Figure 6.3: **Open phase** fault tolerant operation of the overall control system.

6.2 Phase short circuits

(The following section is based on [72].)

For the modeling of a phase short circuit, it is assumed that the affected phase is short-circuited through resistance R_{ps} and electrically isolated from both the power electronics and the current sensor due to a triggered fuse. This leads to the equivalent circuit depicted in Fig. 6.4a.

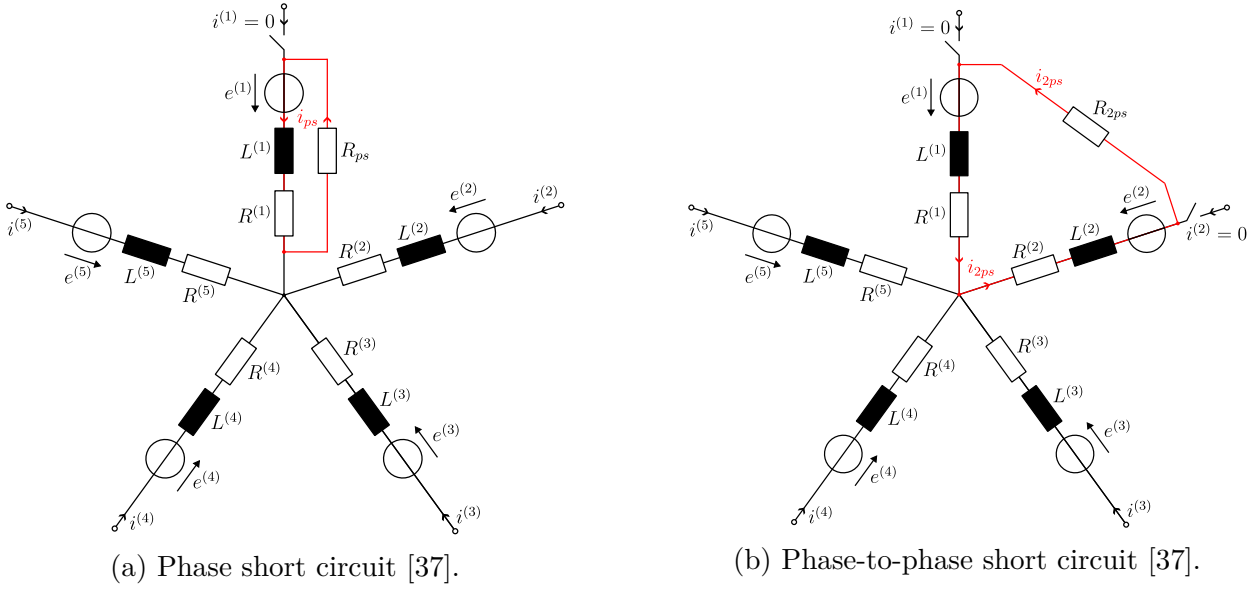


Figure 6.4: Equivalent circuits illustrating two types of phase short circuits; the **short circuit current path** is highlighted in red [72].

With negligible mutual inductances the faulty coil is completely decoupled from the remaining healthy phases. Consequently, the current dynamics of the short-circuited coil can be described by the following equation:

$$0 = (R^{(f)} + R_{ps})i_{ps} + L^{(f)}\omega_R \frac{\partial i_{ps}}{\partial \theta_R} + e^{(f)}(\theta_R), \quad (6.9)$$

where the back-EMF $e^{(f)}$ is defined in (4.7) and (f) denotes the number of the faulty coil. According to (6.9), the current is only caused by the back-EMF and is by that solely depending on the rotor angle θ_R in steady state. To solve (6.9) efficiently, (4.7) for calculating the back-EMF can be rewritten:

$$e^{(f)}(\theta_R) = \omega_R \sum_{k \in \mathbb{N}} (\alpha_k \sin(k\theta_R) + \beta_k \cos(k\theta_R)) \quad (6.10)$$

with α_k and β_k the effective back-EMF's Fourier coefficients defined in appendix C.1. By inserting (6.10) in (6.9), the differential equation can be solved by common solution proce-

dures. Thereby, the resulting current density can also be described as a Fourier series

$$J_{ps}(\theta_R, R_{ps}) = \sum_{k \in \mathbb{N}} (\zeta_k(R_{ps}) \sin(k\theta_R) + \eta_k(R_{ps}) \cos(k\theta_R)) \quad (6.11)$$

where $J_{ps} = \frac{i_{ps}}{S_{SL}}$ and Fourier coefficients

$$\begin{aligned} \zeta_k(R_{ps}) &= \omega_R \frac{-\beta_k L^{(f)} \omega_R k - \alpha_k (R^{(f)} + R_{ps})}{(R^{(f)} + R_{ps})^2 + (L^{(f)} \omega_R k)^2} \frac{1}{S_{SL}}, \\ \eta_k(R_{ps}) &= \omega_R \frac{+\alpha_k L^{(f)} \omega_R k - \beta_k (R^{(f)} + R_{ps})}{(R^{(f)} + R_{ps})^2 + (L^{(f)} \omega_R k)^2} \frac{1}{S_{SL}}. \end{aligned} \quad (6.12)$$

With (6.11) the current density flowing through the faulty coil can be precisely obtained in dependence of the rotor position θ_R and the short circuit resistance R_{ps} .

In the next step, the aim is to predict the torque ripple produced by this current density J_{ps} . Thereby the torque capability $P^{(f)}$ of the short circuited coil (f) can be expressed by

$$P^{(f)}(\theta_R) = \sum_{k \in \mathbb{N}} (\sigma_k \sin(k\theta_R) + \nu_k \cos(k\theta_R)) \quad (6.13)$$

with coefficients σ_k and ν_k also defined in appendix C.1. The product of the torque capability $P^{(f)}$ and the short circuit current density J_{ps} then gives the faulty torque ripple T_{ps} :

$$T_{ps}(\theta_R, R_{ps}) = \frac{\ell\pi}{\mu_0} \underbrace{\sum_{k \in \mathbb{N}} (\sigma_k \sin(k\theta_R) + \nu_k \cos(k\theta_R))}_{P^{(f)}(\theta_R)} \underbrace{\sum_{k' \in \mathbb{N}} (\zeta_{k'}(R_{ps}) \sin(k'\theta_R) + \eta_{k'}(R_{ps}) \cos(k'\theta_R))}_{J_{ps}(\theta_R, R_{ps})}. \quad (6.14)$$

By multiplying these two Fourier series one obtains²

$$T_{ps}(\theta_R, R_{ps}) = \sum_{v \in \mathbb{N}^0} T_{ps,v}(\theta_R, R_{ps}) = \sum_{v \in \mathbb{N}^0} (\xi_v(R_{ps}) \sin(v\theta_R) + \rho_v(R_{ps}) \cos(v\theta_R)) \quad (6.15)$$

²For further explanations see section C.1 in the appendix C.

with the resulting coefficients restricted to non-negative v

$$\begin{aligned}\xi_v(R_{ps}) &= \frac{\ell\pi}{2\mu_0} \sum_{k \in \mathbb{N}} \sum_{k' \in \mathbb{N}} [(+\sigma_k \eta_{k'}(R_{ps}) + \nu_k \zeta_{k'}(R_{ps}))\delta_{k+k',v} + (+\sigma_k \eta_{k'}(R_{ps}) - \nu_k \zeta_{k'}(R_{ps}))\delta_{k-k',v} \\ &\quad - (+\sigma_k \eta_{k'}(R_{ps}) - \nu_k \zeta_{k'}(R_{ps}))\delta_{k-k',-v}(1 - \delta_{k-k',v=0})] \\ \rho_v(R_{ps}) &= \frac{\ell\pi}{2\mu_0} \sum_{k \in \mathbb{N}} \sum_{k' \in \mathbb{N}} [(-\sigma_k \zeta_{k'}(R_{ps}) + \nu_k \eta_{k'}(R_{ps}))\delta_{k+k',v} + (+\sigma_k \zeta_{k'}(R_{ps}) + \nu_k \eta_{k'}(R_{ps}))\delta_{k-k',v} \\ &\quad + (+\sigma_k \zeta_{k'}(R_{ps}) + \nu_k \eta_{k'}(R_{ps}))\delta_{k-k',-v}(1 - \delta_{k-k',v=0})].\end{aligned}\quad (6.16)$$

and $\delta_{a,b}$ the Kronecker delta. The trigonometric relations³

$$\sin(k\theta_R) \cos(k'\theta_R) = \frac{1}{2} [\sin((k+k')\theta_R) + \sin((k-k')\theta_R)] \quad (6.17)$$

$$\cos(k\theta_R) \cos(k'\theta_R) = \frac{1}{2} [\cos((k+k')\theta_R) + \cos((k-k')\theta_R)] \quad (6.18)$$

$$\sin(k\theta_R) \sin(k'\theta_R) = \frac{1}{2} [\cos((k-k')\theta_R) - \cos((k+k')\theta_R)] \quad (6.19)$$

lead to nonzero torque ripple harmonics $T_{ps,v}$ for $v = (0, 2, 4, \dots)p$, see Fig. 6.5. $T_{ps,v=0}$ is the mean value, which is proportional to the ohmic losses in the faulty coil.

Since both $\{\sigma_k, \nu_k\}$ and $\{\zeta_k, \eta_k\}$ have approximately a $\frac{1}{k^2}$ -dependency, the resulting coefficients $\{\xi_v, \rho_v\}$ of the torque ripple harmonics decay fast towards zero. For this reason the most relevant harmonic of the faulty torque ripple $T_{ps,v}$ is the first one, $v = 2p$ (which is consistent with the experimental results shown in [39]). In the case of a partial demagnetization the torque ripple harmonics become nonzero for all $v \in \mathbb{N}^0$. However, as it can be observed in Fig. 6.5, the harmonic $v = 2p$ still remains dominant.

A phase-to-phase short circuit (see Fig. 6.4b) can be modeled by considering both faulty coils for predicting the torque ripple

$$T_{2ps}(\theta_R, R_{2ps}) = \frac{\ell\pi}{\mu_0} \left(P^{(f_1)}(\theta_R) - P^{(f_2)}(\theta_R) \right) J_{2ps}(\theta_R, R_{2ps}). \quad (6.20)$$

This equation can then be rearranged in the form of (6.15). As a result, the methodology for the compensation of a phase short circuit derived in this section 6.2 can be applied analogously to a phase-to-phase short circuit.

³ $k \pm k'$ can only result in even multiples of p in the healthy case, with $k, k' \in \{(1, 3, 5, \dots)p\}$.

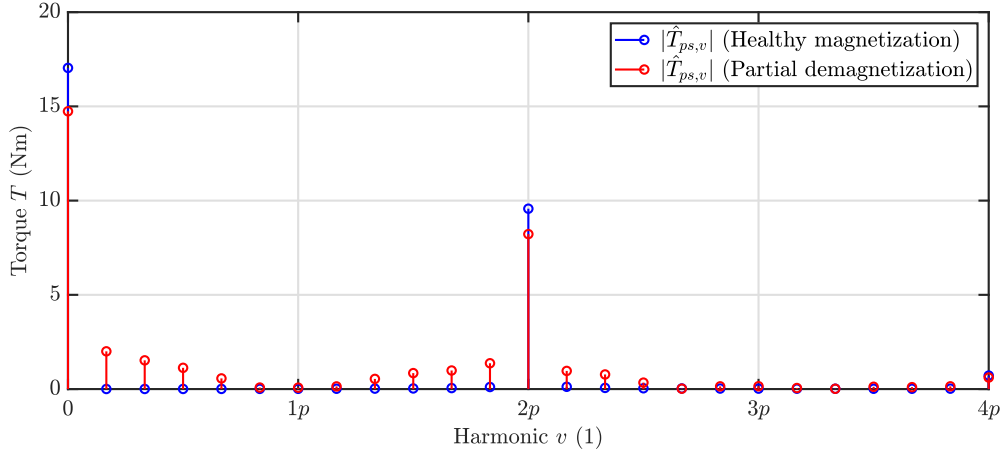


Figure 6.5: Spectrum of the faulty torque ripple T_{ps} in the case of a **healthy magnetization** and in the case of **partial demagnetization** (one magnet fully demagnetized).

Modifications in the control

Analogously to the compensation of an open phase fault (see section 6.1), the faulty phase affected by a phase short circuit must be excluded from the control system. In addition, the faulty torque ripple T_{ps} have to be compensated. This is achieved based on the analytical equation (6.15).

As stated previously, it is assumed in this chapter that all required fault parameters are known. Since the short circuit resistance is mostly unknown in practice, which however has a significant impact on the (also unknown) short circuit current, an algorithm is developed to estimate this parameter during operation, as detailed in section 7.1.

Modifications in the feedforward control

The adjusted optimization problem of the feedforward control for determining the desired currents under consideration of a phase short circuit reads:

$$\begin{aligned}
 &\text{minimize } K(\vec{J}_h) = \frac{1-\alpha}{2} \vec{J}_h^T \vec{J}_h + \frac{\alpha}{2} \left(\vec{J}_h + \frac{\mathcal{L}_h^{-1} \vec{\Phi}_{M,h}}{S_{SL}} \Big|_{des} \right)^T \left(\vec{J}_h + \frac{\mathcal{L}_h^{-1} \vec{\Phi}_{M,h}}{S_{SL}} \Big|_{des} \right) \\
 &\text{subject to } h_1(\vec{J}_h) = \vec{1}^T \vec{J}_h = 0 \\
 &\quad h_2(\vec{J}_h) = \frac{\ell\pi}{\mu_0} \vec{P}_h^T \Big|_{des} \vec{J}_h + \frac{\ell\pi}{\mu_0} Q \Big|_{des} + T_{ps} \Big|_{des} (R_{ps}) - T_{des} = 0
 \end{aligned} \tag{6.21}$$

Equality constraint h_2 demands for the currents of the healthy coils to produce the desired torque T_{des} , while compensating the faulty torque ripple $T_{ps}(R_{ps})$ and the cogging torque $\frac{\ell\pi}{\mu_0} Q$; T_{ps} is calculated based on the short circuit resistance R_{ps} by (6.15). Using the iterative

adaptation algorithm explained in section 7.1 and updating $R_s^{(i)} \rightarrow R_s^{(i+1)}$ in the feedforward control after completion of each iteration ($i+1$), the compensation of T_s can be successively improved in practice without knowledge of the real short circuit resistance.

Solving this minimization problem (6.21), the resulting equation system including the derivatives of the currents is identical to the one used for compensating an open phase (6.2), except for a modification in the vector $\vec{b}_h$ and its time derivative $\dot{\vec{b}}_h$ to account for the torque ripple T_{ps} :

$$\vec{b}_{h,ps} = \begin{pmatrix} -\alpha \frac{\mathcal{L}^{-1} \vec{\Phi}_{M,h}}{S_{SL}} \Big|_{des} \\ 0 \\ T_{des} - T_{ps} \Big|_{des} - \frac{\ell\pi}{\mu_0} Q \Big|_{des} \end{pmatrix}, \quad \dot{\vec{b}}_{h,ps} = \begin{pmatrix} -\alpha \frac{\mathcal{L}^{-1} \dot{\vec{e}}_h}{S_{SL}} \Big|_{des} \\ 0 \\ -\dot{T}_{ps} \Big|_{des} - \frac{\ell\pi}{\mu_0} \dot{Q} \Big|_{des} \end{pmatrix}. \quad (6.22)$$

The time derivative of T_{ps} is included in $\dot{\vec{b}}_{h,ps}$ to enhance the accuracy of the feedforward control. This derivative can be obtained by differentiating equation (6.15) with respect to the rotor angle, yielding $\dot{T}_{ps} = \omega_R \frac{\partial T_{ps}}{\partial \theta_R}$. Based on the resulting equation system, the desired currents $\vec{J}_{des,h}$ and their time derivatives $\dot{\vec{J}}_{des,h}$ are computed. These values are then used to calculate the desired voltages $\vec{u}_{des,h}$ according to equation (6.5).

To precalculate the field weakening LUT the methodology described in subsection 5.2.2 needs to be modified. The reason for this adjustment lies in the nonlinear dependency of the rotational speed on the short circuit current (see equations (6.11), (6.12)) and thereby on the short circuit torque T_{ps} . Hence the maximal rotational speed within the base speed range $\tilde{\omega}_{R,max}$ can no longer be calculated by (5.13). Nevertheless, only minor modifications to the existing methodology are necessary in order to generate the field weakening LUT while accounting for a phase short circuit.

While in the healthy case only the field weakening area was scanned to determine the required values of the field weakening coefficient α (see also Fig. 5.3), this is now necessary to be performed for the entire area. Therefor the maximal torque T_{max} under consideration of the current limit is first calculated by (5.12) for a rotational speed $\tilde{\omega}_{R,max} \rightarrow 0$. Subsequently, the operating area is successively scanned at discrete values lower than T_{max} and greater than $\tilde{\omega}_{R,max}$ for torque and rotational speed. At each of these discrete operating points the field weakening coefficient α is determined by the algorithm⁴ explained in the subsection 5.2.2. This adjusted methodology allows the nonlinear influence of the rotational speed on the short circuit torque to be efficiently incorporated.

⁴It should be noted that – analogously to an open phase – the two space vectors (positive and negative) of the according switches need to be eliminated for each faulty phase.

Modifications in the feedback control

Since deviations $\{\Delta\theta_R, \Delta\omega_R\}$ also cause an error in the prediction of the faulty torque ripple, ΔT_{ps} , it needs to be considered in the feedback control.

With $T_{ps}(\theta_{R,des}, R_{ps}) = \frac{\ell\pi}{\mu_0} P^{(f)}(\theta_{R,des}) J_{ps}(\theta_{R,des}, R_{ps})$ one obtains the following deviation dynamics specific for the faulty phase:

$$\mathcal{J}_m \frac{d^2 \Delta\theta_R}{dt^2} = \frac{\pi\ell}{\mu_0} \left(\frac{\partial P^{(f)}}{\partial \theta_R} J_{ps}(R_{ps}) + P^{(f)} \frac{\partial J_{ps}(R_{ps})}{\partial \theta_R} \right) \Delta\theta_R = \Delta T_{ps}. \quad (6.23)$$

Doing the time derivative, deviations of the rotational speed $\Delta\omega_R$ are considered as well

$$\begin{aligned} \mathcal{J}_m \frac{d^3 \Delta\theta_R}{dt^3} = \frac{\pi\ell}{\mu_0} \left[\left(\frac{\partial^2 P^{(f)}}{\partial \theta_R^2} J_{ps}(R_{ps}) + 2 \frac{\partial P^{(f)}}{\partial \theta_R} \frac{\partial J_{ps}(R_{ps})}{\partial \theta_R} + P^{(f)} \frac{\partial^2 J_{ps}(R_{ps})}{\partial \theta_R^2} \right) \omega_{R,des} \Delta\theta_R \right. \\ \left. + \left(\frac{\partial P^{(f)}}{\partial \theta_R} J_{ps}(R_{ps}) + P^{(f)} \frac{\partial J_{ps}(R_{ps})}{\partial \theta_R} \right) \Delta\omega_R \right]. \end{aligned} \quad (6.24)$$

Using this deviation dynamics in combination with the full motor dynamics (see subsection 5.3.2), the required correction can be determined as follows:

$$\Delta\Xi'_{h,ps} = \Delta\Xi'_h + \Delta\Xi'_{ps,\Delta\theta_R} + \Delta\Xi'_{ps,\Delta\omega_R} \quad (6.25)$$

where

$$\Xi'_{ps,\Delta\theta_R} = - \left(\frac{\partial^2 P^{(f)}}{\partial \theta_R^2} J_{ps}(R_{ps}) + 2 \frac{\partial P^{(f)}}{\partial \theta_R} \frac{\partial J_{ps}(R_{ps})}{\partial \theta_R} + P^{(f)} \frac{\partial^2 J_{ps}(R_{ps})}{\partial \theta_R^2} \right) \omega_{R,des} \Delta\theta_R \quad (6.26)$$

$$\Xi'_{ps,\Delta\omega_R} = - \left(\frac{\partial P^{(f)}}{\partial \theta_R} J_{ps}(R_{ps}) + P^{(f)} \frac{\partial J_{ps}(R_{ps})}{\partial \theta_R} \right) \Delta\omega_R \quad (6.27)$$

The required correction $\Delta\Xi'_h$ is determined based on equation (5.29), excluding the eliminated faulty phase. Again, the iterative adaptation algorithm explained in section 7.1 can be used to update $R_s^{(i)} \rightarrow R_s^{(i+1)}$ after completion of each iteration ($i+1$) to successively improve the accuracy of the feedback control without knowledge of the real short circuit resistance.

Finally, this leads to the following minimization problem for calculating the optimal distributed voltage corrections under consideration of a phase short circuit:

$$\begin{aligned}
& \text{minimize } K(\Delta \vec{u}_h) = \frac{1}{2}(\Delta \vec{u}_h + \Delta \vec{v}_h)^T(\Delta \vec{u}_h + \Delta \vec{v}_h) \\
& \text{subject to } h(\Delta \vec{u}_h) = \vec{P}_h^T \Big|_{des} \frac{\mathcal{L}_h^{-1}}{S_{SL}} \Delta \vec{u}_h - \Delta \Xi'_{h,ps} = 0 \\
& \text{where } \Delta \vec{v}_h = \mathcal{L}_h \frac{\Delta \vec{J}_h}{\Delta t} S_{SL}.
\end{aligned} \tag{6.28}$$

The solution of the minimization problem is identical to (6.7), apart from the adjusted required correction $\Delta \Xi'_{h,ps}$.

Evaluation

In the following, the performance of the developed control system is evaluated under the presence of a phase short circuit. For this purpose, one phase of the PMSM is assumed to be short circuited with a short circuit resistance $R_{ps} = 0.4\Omega$, and the control system is adapted accordingly as described above.

Feedforward control

To evaluate the feedforward control under a phase short circuit condition, it is used to predict the optimal desired currents and voltages at two different operating points⁵.

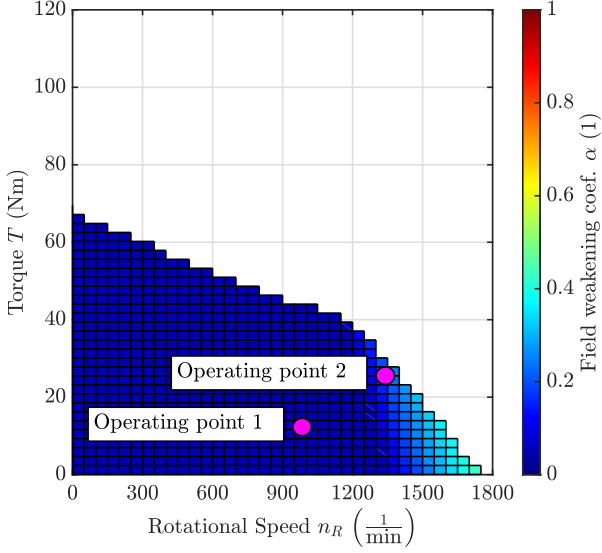
The resulting field weakening LUT is presented in Fig. 6.6a. It can be observed that the maximum torque in the base speed range is no longer constant. This variation arises as described above from the compensation of the faulty torque ripple T_{ps} , which increases non-linearly with the rotational speed. As an additional consequence, the field weakening region is significantly reduced, since the current limit would otherwise be exceeded. Similar to the open phase, a phase short circuit results in an uneven spatial distribution of the remaining healthy coils (see Fig. 6.6f), whereby the desired voltage (Fig. 6.6b) and currents (Fig. 6.6c and Fig. 6.6d) are distributed asymmetrically. Additional to their asymmetric distribution, the amplitudes of the desired currents vary periodically in dependence of the faulty torque ripple shown in Fig. 6.6e in order to achieve its compensation.

⁵In the plots of this section showing the desired currents $\vec{i}_{des}$, the actual currents $\vec{i}_{mes}$ or the torque capabilities $\frac{\ell\pi}{\mu_0} \vec{P}$, the phase short circuited phase is represented by the **green curve**.

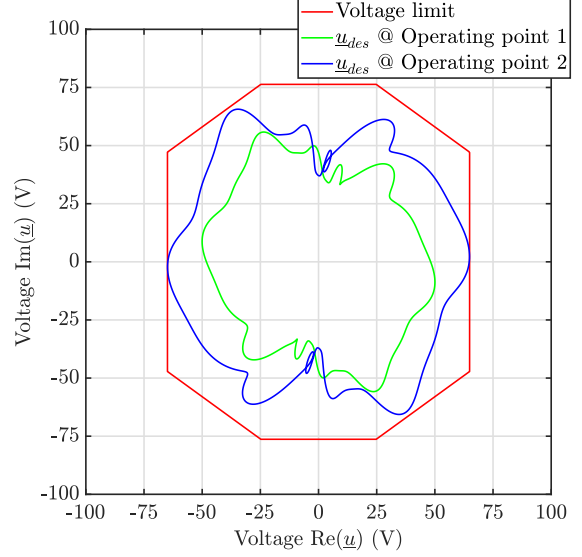
Overall control system

The simulation results of the overall control system under dynamic conditions are displayed in Fig. 6.7⁵. It is assumed that the phase short circuit occurs at $t_{fault} = 2.0\text{s}$ and the control system is adapted to the fault after a delay of 10ms ($t_{adapt} = 2.01\text{s}$). Since the control is no longer able to follow the desired trajectory shown in Fig. 5.10 due to the decreased operating area (see Fig. 6.6a), the desired rotational speed is reduced by a factor of 0.75.

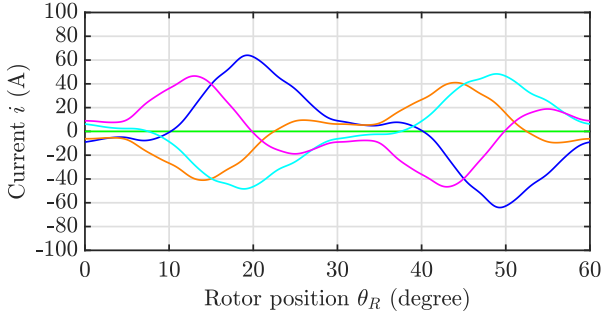
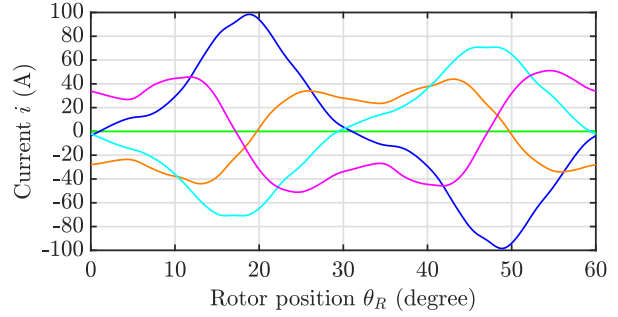
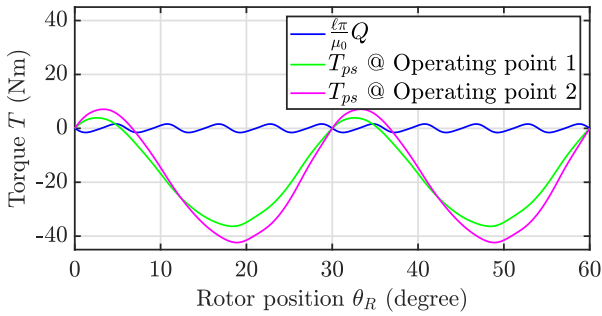
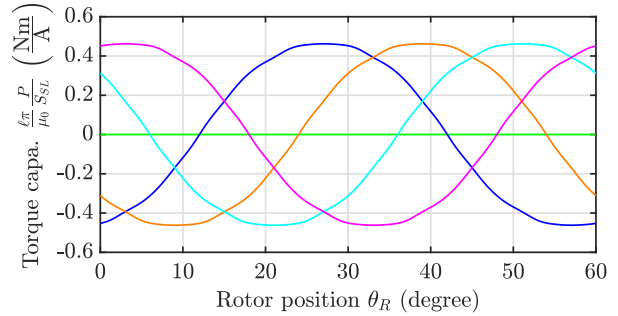
By observing the actual torque and the rotor angle deviation (see Fig. 6.7a and Fig 6.7c), clear peaks can be identified at the time of fault occurrence $t = t_{fault}$. These peaks are a result of the simulated delayed fault adaption of the control system. Once the control system is fault-adapted at $t \approx t_{adapt}$, large current amplitudes appear in the actual phase currents (see Fig. 6.7e). This is a result of the feedback control, which re-stabilizes the system. After the adaptation, the control tracks accurately the desired trajectory, while compensating T_{ps} (see Fig. 6.7g and 6.7h). The slightly increased torque ripple in the actual torque (see Fig. 6.7a) compared to the prefault conditions is on the one hand a result of the more complex current waveforms, which lead to increased deviations of the actual currents to the desired ones. And on the other hand a result of transients, which are not considered in the feedforward control for predicting the faulty torque ripple.

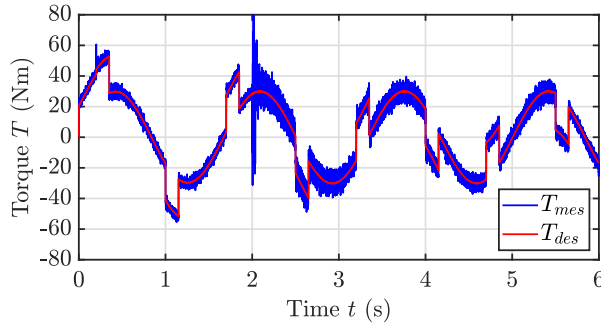


(a) Field weakening LUT.

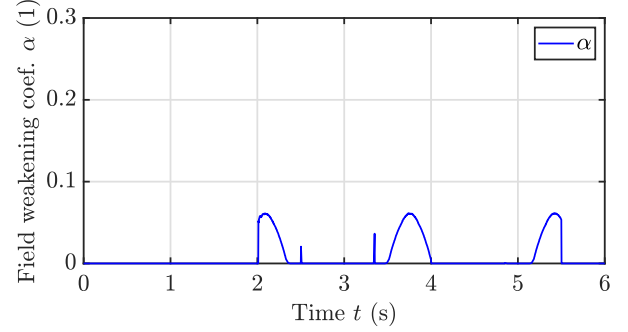


(b) Desired voltages in the complex plane.

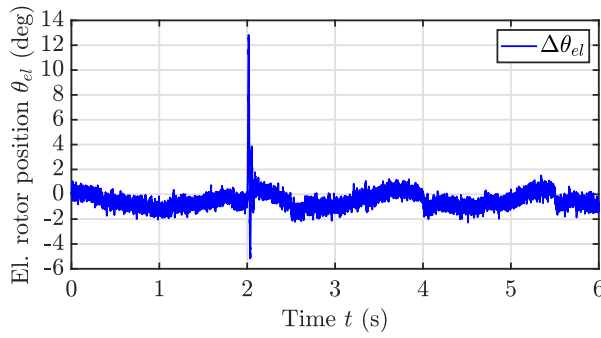
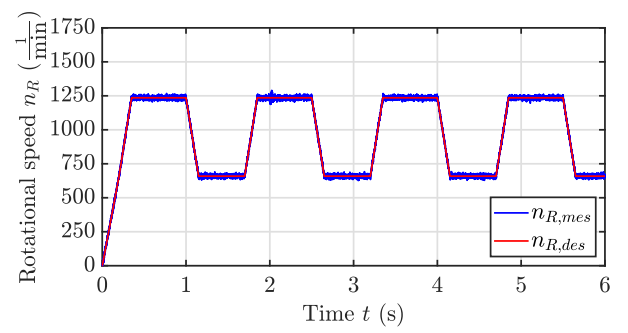
(c) Desired currents $\vec{i}_{des}$ at operating point 1.(d) Desired currents $\vec{i}_{des}$ at operating point 2.(e) Predicted cogging torque $\frac{\ell\pi}{\mu_0}Q$ and faulty torque ripple T_{ps} .(f) Predicted torque capabilities $\frac{\ell\pi}{\mu_0}\vec{P}$.Figure 6.6: **Phase short circuit** tolerant operation of the feedforward control.



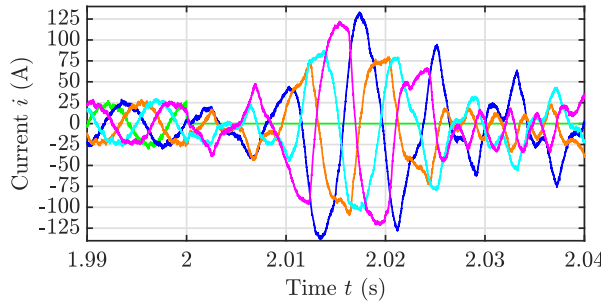
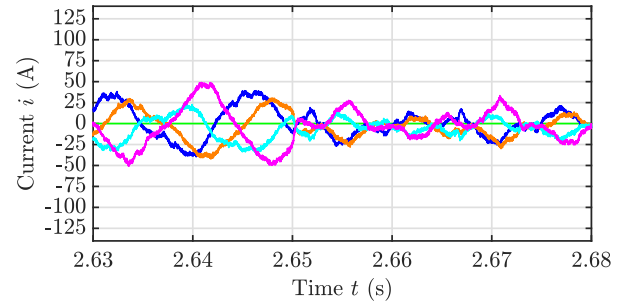
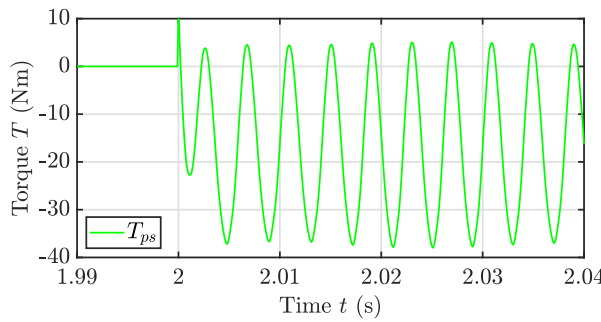
(a) Actual and desired torque.



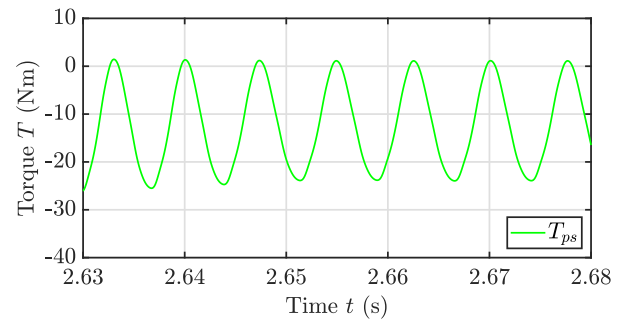
(b) Field weakening coefficient.

(c) Deviation of the rotor angle ($\Delta\theta_{el} = p\Delta\theta_R$).

(d) Actual and desired rotational speed.

(e) Actual currents $\vec{i}_{mes}$ at the time period of fault occurrence.(f) Actual currents $\vec{i}_{mes}$ at a time period after fault adaptation.

(g) Faulty torque ripple at the time period of fault occurrence.



(h) Faulty torque ripple at a time period after fault adaptation.

Figure 6.7: **Phase short circuit** fault tolerant operation of the overall control system.

6.3 Interturn short circuits

(The following section is based on [73].)

In the case of an interturn short circuit, the faulty coil splits into multiple sub-coils, since then there are multiple current densities in the affected slots (see Fig. 6.8a). Thereby, radii $r_{S,1}$ and $r_{S,2}$ define the interturn short circuit -position and -thickness, $n_{turn}^{(f,1)}$ and $n_{turn}^{(f,3)}$ the number of winding turns from the still operational parts of the coil and $n_{turn}^{(f,2)}$ the number of winding turns from the short circuited part of the coil. This results in the electrical circuit shown in Fig. 6.8b. Due to the required consideration of multiple current densities in the affected slots, the particular solution needs to be adjusted [55], see appendix C.2. Based on this, the equations for calculating the inductances, PM-flux linkages, back-EMFs and torque need to be adjusted as well.

Inductances

The inductance of the operational and the short circuited parts of the coil, L_g and L_s , as well as the mutual coupling M_{sg} of these coils are to be determined. The operational part of the coil L_g is effectively equal to the self inductance of partial subdomains $I^{(a,1)}$ and $I^{(a,3)}$ for the slots $a \in S^{(f)}$ ⁶ in combination with their mutual coupling

$$L_g = L_1 + L_3 + 2M_{13}. \quad (6.29)$$

The inductance of the short circuited part of the coil is equal to the self inductance of partial subdomains $I^{(a,2)}$, $a \in S^{(f)}$

$$L_s = L_2. \quad (6.30)$$

The mutual inductance between L_g and L_s is equal to the mutual coupling of partial subdomains $I^{(a,2)}$ to $I^{(a,1)}$ and partial subdomains $I^{(a,2)}$ to $I^{(a,3)}$, $a \in S^{(f)}$

$$M_{sg} = M_{21} + M_{23}. \quad (6.31)$$

The procedure for determining these inductances is described in the appendix C.2.2.

⁶ $S^{(f)}$ is the set of the slots from faulty coil (f).

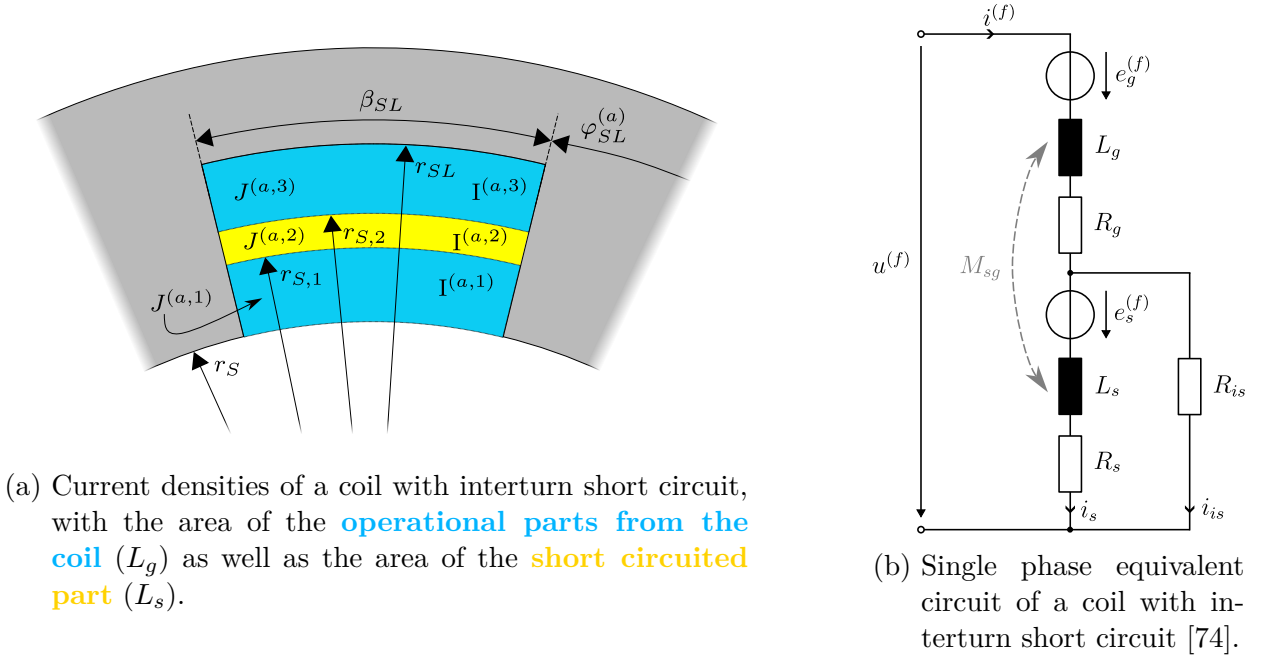


Figure 6.8: Relevant current densities and corresponding equivalent circuit used to model interturn short circuits [73].

PM flux linkage & back-EMF

The PM flux linkage of the short circuited coil turns from faulty coil (f) can be calculated by⁷

$$\Phi_{M,s}^{(f)}(\theta_R) = \Phi_M^{(f,2)}(\theta_R) = \ell n_{turn}^{(f,2)} (a_0^{(I,fi)} - a_0^{(I,fr)}) . \quad (6.32)$$

Thereby the back-EMF is obtained by

$$e_s^{(f)}(\theta_R) = \omega_R \frac{\partial \Phi_{M,s}^{(f)}(\theta_R)}{\partial \theta_R} = \omega_R \ell n_{turn}^{(f,2)} \frac{\partial (a_0^{(I,fi)} - a_0^{(I,fr)})}{\partial \theta_R} . \quad (6.33)$$

Analogously, the back-EMF of the operational part from coil (f) is equal to

$$e_g^{(f)}(\theta_R) = e^{(f,1)}(\theta_R) + e^{(f,3)}(\theta_R) = \omega_R \ell (n_{turn}^{(f,1)} + n_{turn}^{(f,3)}) \frac{\partial (a_0^{(I,fi)} - a_0^{(I,fr)})}{\partial \theta_R} . \quad (6.34)$$

Current dynamics

In the case of an interturn short circuit the affected coils splits up in the electrical network shown in Fig. 6.8b. With the calculated inductances and back-EMFs, the current dynamics

⁷For further explanations see section C.2.3 in the appendix C.

are described by the following equations [74]:

$$u_{in}^{*(f)} = R_g i^{(f)} + R_{sc}(i^{(f)} - i_s) + L_g \frac{di^{(f)}}{dt} + e_g^{(f)} + M_{sg} \frac{di_s}{dt} \quad (6.35)$$

$$0 = R_s i_s - R_{sc}(i^{(f)} - i_s) + L_s \frac{di_s}{dt} + e_s^{(f)} + M_{sg} \frac{di^{(f)}}{dt} \quad (6.36)$$

with

$$R_g = \frac{n_{turn}^{(f,1)} + n_{turn}^{(f,3)}}{n_{turn}} R^{(f)} \quad (6.37)$$

$$R_s = \frac{n_{turn}^{(f,2)}}{n_{turn}} R^{(f)}. \quad (6.38)$$

Incorporating these equations into the equation system of the 'healthy' current dynamics (4.17), the interturn short circuit is efficiently modeled.

Torque

The torque contribution from each coil (c) is of the format

$$T^{(c)} = \frac{\ell\pi}{\mu_0} \underbrace{n_{turn} h^{(c)}(\theta_R) S_{SL}}_{P^{(c)}(\theta_R)}, \underbrace{\frac{i^{(c)}}{S_{SL}}}_{J^{(c)}} \quad (6.39)$$

where $h^{(c)}(\theta_R)$ indicates a term with θ_R -dependence. This equation shows that the decisive quantity for torque generation is the current flowing through the coil turns. Therefore, the produced torque of the faulty coil (f) with interturn short circuit can be obtained by calculating an effective current density

$$J^{(f)} = \frac{\frac{n_{turn}^{(f,2)}}{n_{turn}} i_s + \frac{n_{turn}^{(f,1)} + n_{turn}^{(f,3)}}{n_{turn}} i^{(f)}}{S_{SL}}. \quad (6.40)$$

Employing this effective current density, one can still use the 'healthy' torque capability of the affected coil with interturn short circuit, which simplifies the notation.

Modifications in the control

The concept for compensating an interturn short circuit is to apply an adjusted field weakening to the affected coil. This adjustment reduces the effective PM flux linkage in the faulty coil turns and thereby lowering the induced short circuit current.

Modifications in the feedforward control

The short circuited coil turns (yellow area in Fig. 6.8a) can be field weakened by the operational ones (blue area in Fig. 6.8a) through their mutual coupling M_{sg} . This field weakening is achieved by determining a current for the affected coil, which extinguishes the PM flux linkage $\Phi_{M,s}^{(f)}$ of the short circuited coil turns predicted by the Subdomain Model:

$$i_{des}^{(f)} = -\frac{\Phi_{M,s}^{(f)}}{M_{sg}|_{des}}. \quad (6.41)$$

As a consequence, $i_s = 0$ in the equivalent circuit shown in Fig. 6.8b for the case of a negligible short circuit resistance ($R_{is} = 0$). The effective current density for calculating the torque contribution of the faulty coil (f) is then equal to

$$J_{des}^{(f)} = \frac{n_{turn}^{(f,1)} + n_{turn}^{(f,3)} i_{des}^{(f)}}{n_{turn} S_{SL}}. \quad (6.42)$$

While the current density of the faulty coil is determined by the field weakening requirement, the desired currents for the remaining healthy coils can be calculated by solving the following minimization problem:

$$\begin{aligned} & \text{minimize } K(\vec{J}_h) = \frac{1-\alpha}{2} \vec{J}_h^T \vec{J}_h + \frac{\alpha}{2} \left(\vec{J}_h + \frac{\mathcal{L}_h^{-1} \vec{\Phi}_{M,h}}{S_{SL}} \Big|_{des} \right)^T \left(\vec{J}_h + \frac{\mathcal{L}_h^{-1} \vec{\Phi}_{M,h}}{S_{SL}} \Big|_{des} \right) \\ & \text{subject to } h_1 = \vec{1}^T \vec{J}_h + J_{des}^{(f)} = 0 \\ & h_2 = \frac{\ell\pi}{\mu_0} \left(\vec{P}_h^T \Big|_{des} \vec{J}_h + P^{(f)} \Big|_{des} J_{des}^{(f)} + Q \Big|_{des} \right) - T_{des} = 0 \end{aligned} \quad (6.43)$$

Solving (6.43), the resulting equation system including the derivatives of the currents is identical to the one used for compensating an open phase (6.2), except for a modification in the vector $\vec{b}_h$ and its time derivative $\dot{\vec{b}}_h$ to account for the predefined desired current density

$J_{des}^{(f)}$ of the faulty coil:

$$\vec{b}_{is} = \begin{pmatrix} -\alpha \frac{\mathcal{L}^{-1} \vec{\Phi}_{M,h}}{S_{SL}} \Big|_{des} \\ -J_{des}^{(f)} \\ T_{des} - \frac{\ell\pi}{\mu_0} (Q \Big|_{des} + P^{(f)} \Big|_{des} J_{des}^{(f)}) \end{pmatrix}, \quad \vec{b}_{is} = \begin{pmatrix} -\alpha \frac{\mathcal{L}^{-1} \vec{e}_h}{S_{SL}} \Big|_{des} \\ -j_{des}^{(f)} \\ -\frac{\ell\pi}{\mu_0} (\dot{Q} \Big|_{des} + P^{(f)} \Big|_{des} j_{des}^{(f)} + \dot{P}^{(f)} \Big|_{des} J_{des}^{(f)}) \end{pmatrix}$$

with

$$\frac{di_{des}^{(f)}}{dt} = -\frac{e_s^{(f)}}{M_{sg}} \Big|_{des}, \quad (6.44)$$

$$j_{des}^{(f)} = \frac{n_{turn}^{(f,1)} + n_{turn}^{(f,3)}}{n_{turn}} \frac{di_{des}^{(f)}}{dt} \frac{1}{S_{SL}}. \quad (6.45)$$

Solving the resulting equation system yields the desired currents $\vec{J}_{des,h}$ and their time derivatives $\dot{\vec{J}}_{des,h}$, which are then used to compute the desired voltages:

$$\vec{u}_{des,h}(\alpha) = \mathcal{R}_h \vec{i}_{des,h}(\alpha) + \mathcal{L}_h \frac{d\vec{i}_{des,h}(\alpha)}{dt} + \vec{e}_h(\theta_{R,des}) \quad (6.46)$$

$$u_{des}^{(f)} = R_g i_{des}^{(f)} + L_g \frac{di_{des}^{(f)}}{dt} + e_g^{(f)}(\theta_{R,des}). \quad (6.47)$$

To precalculate the field weakening LUT the methodology described in subsection 5.2.2 can be used.

Modifications in the feedback control

The adjusted optimization problem of the feedback control for calculating the optimal distributed voltage corrections under consideration of an interturn short circuit reads:

$$\begin{aligned} & \text{minimize } K(\Delta \vec{u}_h) = \frac{1}{2} (\Delta \vec{u}_h + \Delta \vec{v}_h)^T (\Delta \vec{u}_h + \Delta \vec{v}_h) \\ & \text{subject to } h_1(\Delta \vec{u}_h) = \vec{P}_h^T \Big|_{des} \frac{\mathcal{L}_h^{-1}}{S_{SL}} \Delta \vec{u}_h + P^{(f)} \Big|_{des} \frac{\Delta u^{(f)}}{L_{ef} S_{SL}} - \Delta \Xi' = 0 \\ & \quad h_2(\Delta \vec{u}_h) = \vec{1}^T \Delta \vec{u}_h + \Delta u^{(f)} = 0 \\ & \quad \text{where } \Delta \vec{v}_h = \mathcal{L}_h \frac{\Delta \vec{J}_h}{\Delta t} S_{SL}. \end{aligned} \quad (6.48)$$

In addition to the equality constraint h_1 , which demands for the voltage corrections of the healthy coils the compensation of $\Delta \Xi'$ while accounting for the voltage correction of the faulty coil, a second constraint h_2 is considered. This enforces an equilibrium of all voltage corrections, which ensures an electrical potential of the star point close to zero and by that

an electrical decoupling of the coils.

The voltage correction $\Delta u^{(f)}$ for the faulty coil is directly calculated based on the changed dynamics due to the interturn short circuit to enforce a minimal deviation $\Delta i^{(f)}$ and thereby a lowest possible short circuit current i_s . Based on the circuit shown in Fig. 6.8b, the following discrete dynamics of a deviation in the currents can be obtained:

$$\Delta u^{(f)} = -L_g \frac{\Delta i^{(f)}}{\Delta t} - M_{sg} \frac{\Delta i_s}{\Delta t} \quad (6.49)$$

$$0 = -L_s \frac{\Delta i_s}{\Delta t} - M_{sg} \frac{\Delta i^{(f)}}{\Delta t}. \quad (6.50)$$

The back-EMF and the voltage drop at the coil's resistance do not need to be considered here since they are already taken into account in the feedforward control. Using this dynamics one obtains the required voltage correction for the faulty coil (f)

$$\Delta u^{(f)} = - \underbrace{\left(L_g - \frac{M_{sg}^2}{L_s} \right)}_{L_{ef}} \frac{\Delta J^{(f)}}{\Delta t} S_{SL}. \quad (6.51)$$

Summarizing, the voltage correction for the faulty coil $\Delta u^{(f)}$ to compensate a deviation from the desired current $i_{des}^{(f)}$ can be determined by (6.51). Based on this, the voltage corrections for the healthy coils $\Delta \vec{u}_h$ to compensate the required correction $\Delta \Xi'$ are obtained by solving the minimization problem (6.48), whose solution can be displayed in matrix notation:

$$\begin{pmatrix} \mathbf{I} & \vec{1} & \frac{\mathcal{L}_h^{-1} \vec{P}_h}{S_{SL}} \Big|_{des} \\ \frac{\vec{P}_h^T}{S_{SL}} \Big|_{des} & 0 & 0 \\ \vec{1}^T & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta \vec{u}_h \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} -\mathcal{L}_h \frac{\Delta \vec{J}_h}{\Delta t} S_{SL} \\ \Delta \Xi' + P^{(f)} \Big|_{des} \frac{\Delta J^{(f)}}{\Delta t} \\ L_{ef} \frac{\Delta J^{(f)}}{\Delta t} S_{SL} \end{pmatrix}. \quad (6.52)$$

Evaluation

In the following, the performance of the developed control system is evaluated under the presence of an interturn short circuit in one coil. Due to the fact that it is hardly possible to locate the exact position of the interturn short circuit and in order to investigate the robustness of the fault tolerant control, a displacement between the *estimated* and the *real* interturn short circuit position is simulated. While the estimated interturn short circuit is assumed to be in the middle of the slot, the actual position is at its top. Moreover, a low short circuit impedance is assumed, as this scenario represents the most severe fault condition [12]. The resulting fault parameters used for this simulation are summarized in Tab. 6.2. To account for the increased current demand required to extinct the PM flux

linkage, the maximal allowed RMS of the currents is raised from 60A to 120A.

Feedforward control

To evaluate the feedforward control under consideration of an interturn short circuit it is used to predict the optimal desired currents and voltages at different operating points⁸. The resulting field weakening LUT is shown in Fig. 6.9a. Unlike in the healthy case, where a hyperbolic decrease of the maximal torque in dependence of the rotational speed can typically be observed in the field weakening area, this behavior is not present here. As previously discussed, the task of the desired current for the faulty coil (f) is to extinct the PM flux linkage in the shorted coil turns. Since the amplitude of the PM flux linkage remains invariably constant (independently of the desired torque and rotational speed), the required current amplitude for the faulty coil must also remain constant across the entire speed range (see green curves in Fig. 6.9c and Fig. 6.9d). Thereby and due to the fact that the full extinction of the PM flux linkage requires the highest current, the torque limit is effectively determined by the desired current of the faulty coil.

Fig. 6.9c shows the desired currents at an operating point where no field weakening is required ($\alpha = 0$). It can be observed that the currents for the healthy coils are high in relation to the desired torque and have a complex waveform. Since the currents for the healthy coils are calculated based on a maximal torque per ampere strategy (see optimization problem (6.43)), they should optimally follow the waveforms of their torque capabilities and associated back-EMFs. In contrast, $i_{des}^{(f)}$ is phase shifted by 180° with respect to the PM flux linkage to achieve its effective extinction. This leads to an asymmetrical placement of the faulty coil in the polyphase system, as the field weakening mechanism acts exclusively on a single coil. Due to the star connection, which enforces current equilibrium, this asymmetry affects all phases. As a result, the generation of the desired torque requires higher overall current amplitudes, thus reducing the effective torque-per-ampere ratio. Moreover, because the extinction current in the faulty coil is phase-shifted by 90° with respect to its torque capability, it produces an oscillating torque component. This oscillation must be compensated by the healthy coils, further increasing their current demand and waveform complexity.

In contrast, if field weakening is required, the desired currents for the healthy coils are increasingly shifted to their PM flux linkages. This reduces accordingly the asymmetrical distribution of all currents and thereby the complexity of their waveforms. This can be clearly observed in Fig. 6.9d.

⁸In the plots of this section showing the desired currents $\vec{i}_{des}$, the actual currents $\vec{i}_{mes}$ or the torque capabilities $\frac{\ell\pi}{\mu_0}\vec{P}$, the interturn short circuited phase is represented by the **green curve**.

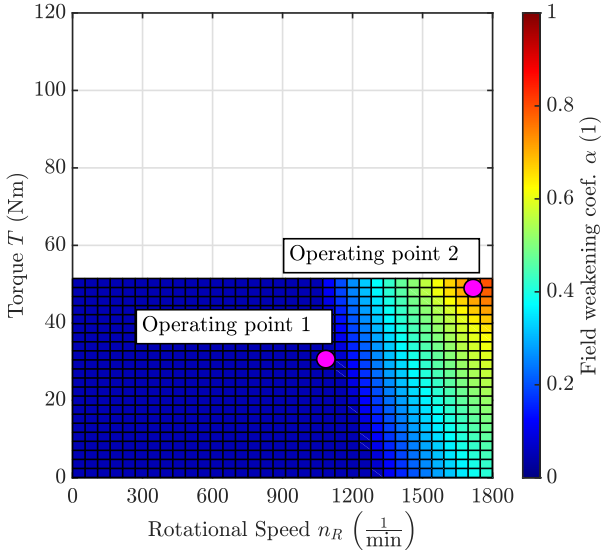
Table 6.2: Relevant parameters of the interturn short circuit

	Parameter	Real	Estimate	Unit
Winding turns	$n_{turn}^{(f,1)}$	28	13	–
	$n_{turn}^{(f,2)}$	3	3	–
	$n_{turn}^{(f,3)}$	4	19	–
Radii	r_S	61.75	61.75	mm
	$r_{S,1}$	81.9	72.9	mm
	$r_{S,2}$	82.9	73.9	mm
	r_{SL}	85.0	85.0	mm
Resistances	R_h	105	91	m Ω
	R_s	10	–	m Ω
	R_{sc}	1	–	m Ω
Inductances	L_g	0.40	0.44	mH
	L_s	0.0051	0.0043	mH
	M_{sg}	0.043	0.042	mH

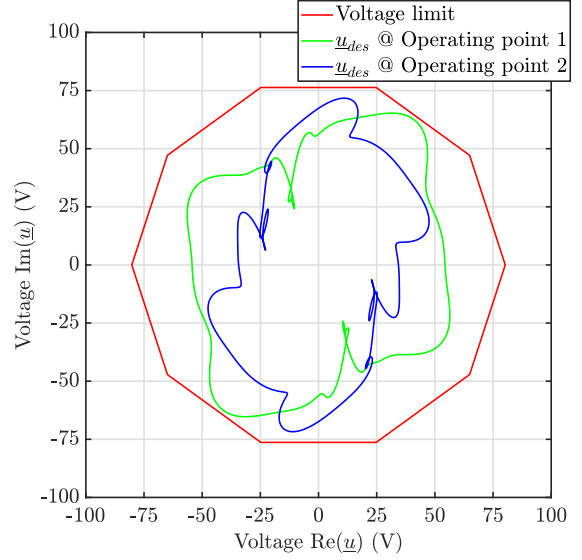
Overall control system

The simulation results of the overall control system under dynamic conditions are displayed in Fig. 6.10⁸. It is assumed that the interturn short circuit occurs at $t_{fault} = 2.0s$ and that the control system is adapted to the fault after a delay of 40ms ($t_{adapt} = 2.04s$).

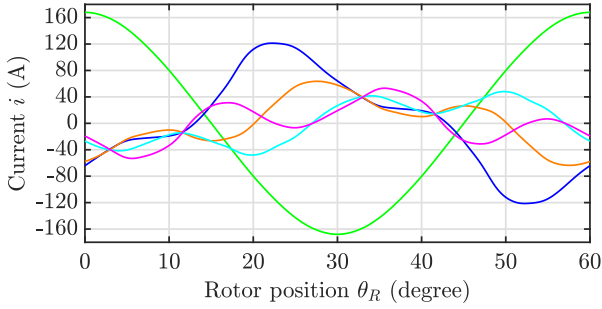
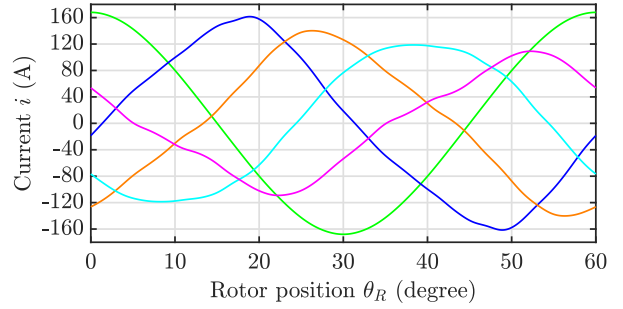
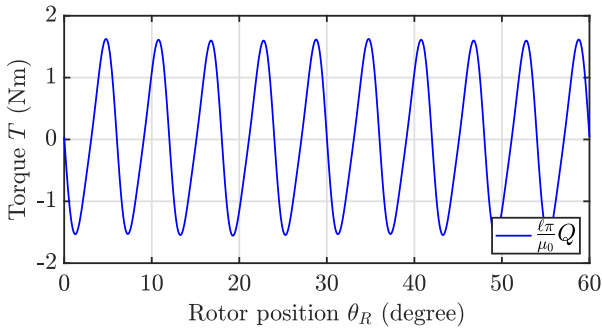
The curve of the short circuit current i_s is shown in Fig. 6.10g at fault occurrence. There, a strong increase in the amplitude can be seen for $2.0s < t < 2.04s$, which also causes a ripple in the actual torque (see Fig. 6.10a). This is the time period where the control system is not yet adapted to the fault, demonstrating the criticality of interturn short circuits. After the fault adaptation of the control at $t = 2.04s$ this short circuit current is quickly reduced from about 450A down to a range of 50A to 75A. The remaining short circuit current in Fig. 6.10g and Fig. 6.10h for $t > 2.04s$ is firstly caused by the prediction error due to the discrepancy in the *estimated* parameter set, in particular the assumed displacement of the interturn short circuit position. Secondly high frequent current harmonics in the phase current $i^{(f)}$ generated by the simulated inverter induce a voltage in the short circuited coil turns. By observing Fig. 6.10a and Fig. 6.10c, it can be seen that there is no relevant increase in the actual torque ripple and rotor position deviation after the fault adaptation, demonstrating full operational capability while compensating the interturn short circuit.



(a) Field weakening LUT.



(b) Desired voltages in the complex plane.

(c) Desired currents $\vec{i}_{des}$ at operating point 1.(d) Desired currents $\vec{i}_{des}$ at operating point 2.

(e) Predicted cogging torque.

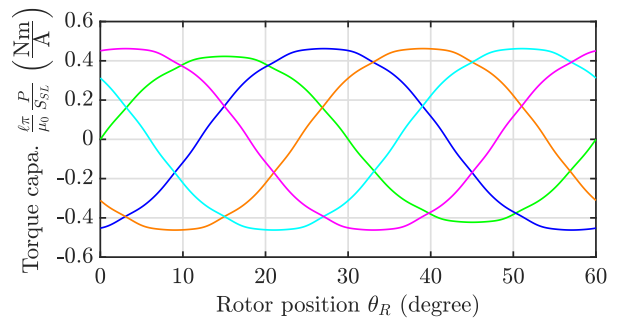
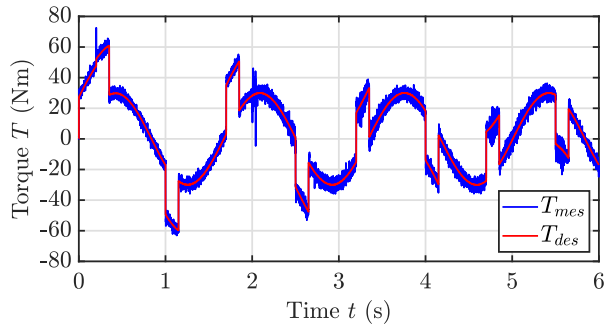
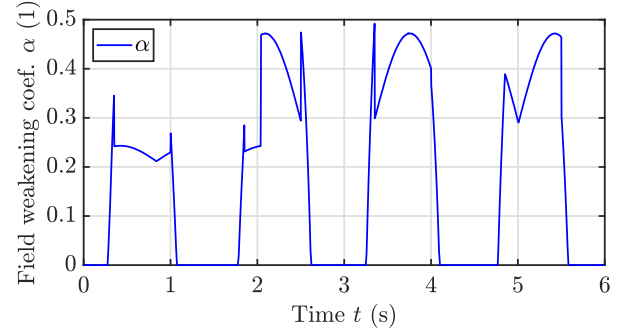
(f) Predicted torque capabilities $\frac{\ell\pi}{\mu_0} \vec{P}$.

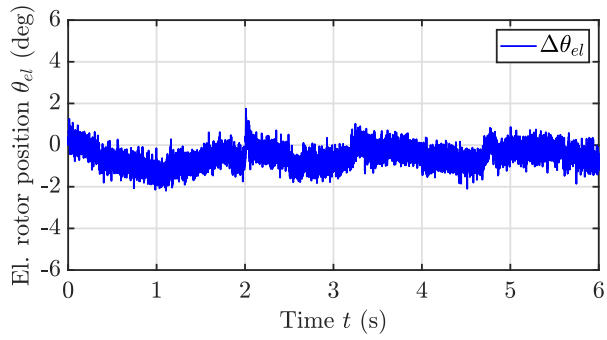
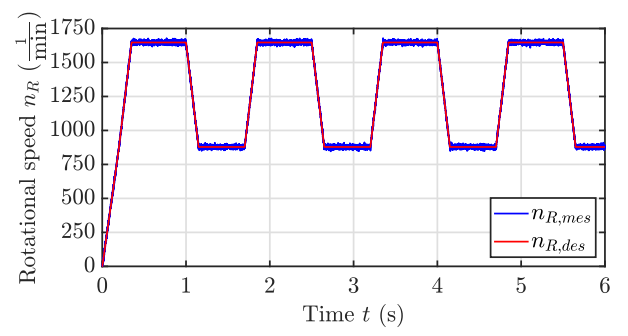
Figure 6.9: Interturn short circuit tolerant operation of the feedforward control.



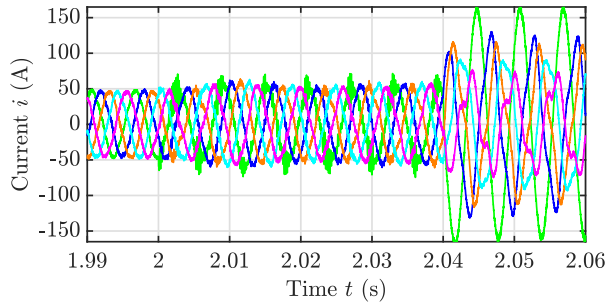
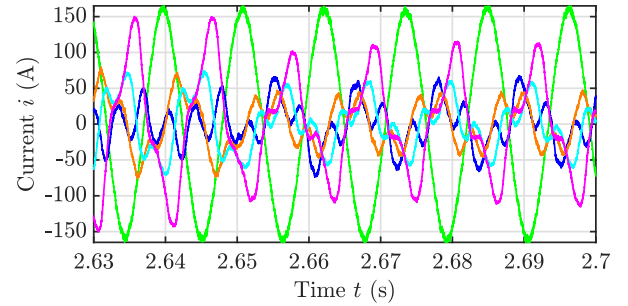
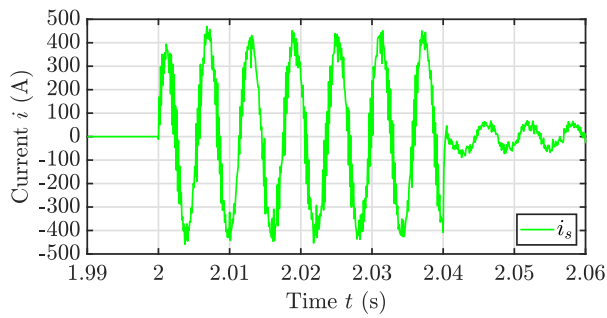
(a) Actual and desired torque.



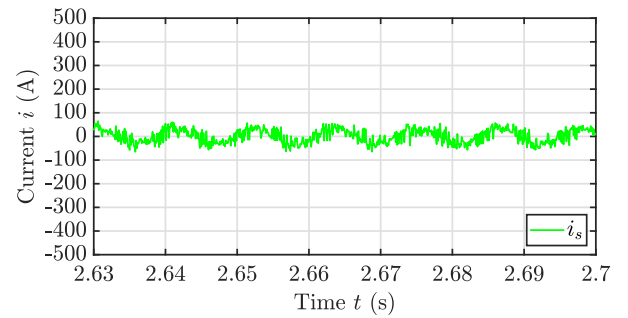
(b) Field weakening coefficient.

(c) Deviation of the rotor angle ($\Delta\theta_{el} = p\Delta\theta_R$).

(d) Actual and desired rotational speed.

(e) Actual currents $\vec{i}_{mes}$ at the time period of fault occurrence.(f) Actual currents $\vec{i}_{mes}$ at a time period after fault adaptation.

(g) Short circuit current at the time period of fault occurrence.



(h) Short circuit current at a time period after fault adaptation.

Figure 6.10: **Interturn short circuit** fault tolerant operation of the overall control system.

6.4 Demagnetization

To model a demagnetization fault with the Subdomain Model, simply the Fourier coefficients (4.1) of the magnetization have to be updated

$$\tilde{M}_r(\theta_R) = \sum_{k \in \mathbb{N}} \frac{\mu_0}{r_R} \left[-\tilde{a}_k^{(M)}(\theta_R) \sin(k\varphi) + \tilde{c}_k^{(M)}(\theta_R) \cos(k\varphi) \right]. \quad (6.53)$$

where $\{\tilde{a}_k^{(M)}, \tilde{c}_k^{(M)}\}$ are the re-calculated Fourier coefficients according to the spatial course of demagnetization. For instance, a demagnetized magnet is efficiently considered by effectively using a superposition of two Fourier series: the first one describes the arrangement of the healthy magnets and the second one describes the partial demagnetization. This results in the following solution for the Fourier coefficients $\{\tilde{a}_k^{(M)}, \tilde{c}_k^{(M)}\}$:

$$\begin{aligned} \tilde{a}_k^{(M)} &= -r_R \mu_0 [+(M_k + \tilde{M}_k^{(c)}) \sin(k\theta_R) + \tilde{M}_k^{(s)} \cos(k\theta_R)], \\ \tilde{c}_k^{(M)} &= +r_R \mu_0 [(M_k + \tilde{M}_k^{(c)}) \cos(k\theta_R) - \tilde{M}_k^{(s)} \sin(k\theta_R)], \end{aligned} \quad (6.54)$$

with

$$\begin{aligned} \tilde{M}_k^{(c)} &= \frac{\hat{M}_{demag}}{k\pi} [+ \sin(k(\beta_{demag} + \varphi_{demag})) - \sin(k\varphi_{demag})], \\ \tilde{M}_k^{(s)} &= \frac{\hat{M}_{demag}}{k\pi} [- \cos(k(\beta_{demag} + \varphi_{demag})) + \cos(k\varphi_{demag})], \end{aligned} \quad (6.55)$$

$\hat{M}_{demag}$, β_{demag} and φ_{demag} the demagnetization -amplitude, -width and -position. Since the magnetization becomes asymmetrical due to a partial demagnetization, Fourier coefficients $\{\tilde{a}_k^{(M)}, \tilde{c}_k^{(M)}\}$ become nonzero for all $k \in \mathbb{N}^0$ ⁹.

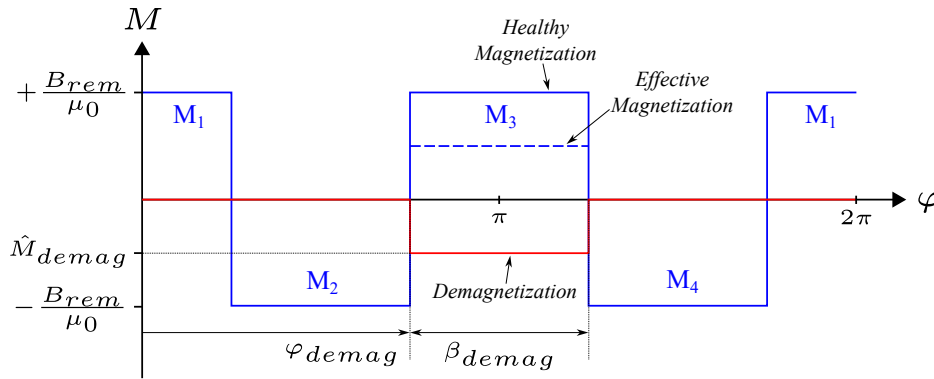


Figure 6.11: Visualized Fourier expansion of the **healthy magnetization** and the **demagnetization** for the case of a partial demagnetization [69].

⁹Note that a more general particular solution for the case of $k = 1$ has then to be considered, see appendix A.2.3.

In Fig. 6.11 the resulting spatial magnetization curves are shown for the example of a machine with two pole pairs and 50% demagnetization of the third magnet.

Modifications in the control

Inserting the updated Fourier coefficients of the magnetization $\{\tilde{a}_k^{(M)}, \tilde{c}_k^{(M)}\}$ in the equation for the PM flux linkage (4.6), back-EMF (4.7) and the cogging torque as well as the torque capabilities in (4.14), the relevant quantities in the control system are efficiently adjusted to the specified demagnetization fault¹⁰.

Modifications in the feedforward control

By inserting the quantities $\{\tilde{\Phi}_M, \tilde{e}, \tilde{Q}, \tilde{P}\}$ into the minimization problem for the healthy machine (5.5), the optimal currents can be predicted under consideration of a demagnetization fault:

$$\begin{aligned} \text{minimize } K(\vec{J}) &= \frac{1-\alpha}{2} \vec{J}^T \vec{J} + \frac{\alpha}{2} \left(\vec{J} + \frac{\mathcal{L}^{-1} \tilde{\Phi}_M}{S_{SL}} \Big|_{des} \right)^T \left(\vec{J} + \frac{\mathcal{L}^{-1} \tilde{\Phi}_M}{S_{SL}} \Big|_{des} \right) \\ \text{subject to } h_1(\vec{J}) &= \vec{1}^T \vec{J} = 0 \\ h_2(\vec{J}) &= \frac{\ell\pi}{\mu_0} \tilde{P}^T \Big|_{des} \vec{J} + \frac{\ell\pi}{\mu_0} \tilde{Q} \Big|_{des} - T_{des} = 0 \end{aligned} \quad (6.56)$$

Solving this minimization problem, the resulting combined equation system, which includes the derivatives of the currents, is structurally equivalent to (5.8), except for the adjusted quantities $\{\tilde{\Phi}_M, \tilde{e}, \tilde{Q}, \tilde{P}\}$. Based on this adapted equation system, the desired currents $\vec{J}_{des}$ and their time derivatives $\dot{\vec{J}}_{des}$ can be efficiently computed. These quantities are then used to determine the desired voltages:

$$\vec{u}_{des}(\alpha) = \mathcal{R} \vec{i}_{des}(\alpha) + \mathcal{L} \frac{d\vec{i}_{des}(\alpha)}{dt} + \vec{e}(\theta_{R,des}). \quad (6.57)$$

To precalculate the field weakening LUT the methodology described in subsection 5.2.2 can be used.

¹⁰The thereby updated quantities are also denoted with a tilde.

Modifications in the feedback control

The adjusted optimization problem of the feedback control for determining the optimal distributed voltage corrections under consideration of a demagnetization fault is formulated as follows:

$$\begin{aligned}
 & \text{minimize } K(\Delta\vec{u}) = \frac{1}{2}(\Delta\vec{u} + \Delta\vec{v})^T(\Delta\vec{u} + \Delta\vec{v}) \\
 & \text{subject to } h(\Delta\vec{u}) = \vec{P}^T \Big|_{des} \frac{\mathcal{L}^{-1}}{S_{SL}} \Delta\vec{u} - \Delta\tilde{\Xi}' = 0 \\
 & \text{where } \Delta\vec{v} = \mathcal{L} \frac{\Delta\vec{J}}{\Delta t} S_{SL}
 \end{aligned} \tag{6.58}$$

with solution equal to (5.40), apart from the updated quantities $\{\vec{P}, \Delta\tilde{\Xi}'\}$.

Evaluation

In the following, the performance of the developed control system is evaluated under the presence of partial demagnetization. For this purpose, one magnet is assumed to be fully demagnetized, and the control system is adapted accordingly as described above.

Feedforward control

To evaluate the feedforward control under consideration of demagnetization it is used to predict the optimal desired currents and voltages at two different operating points.

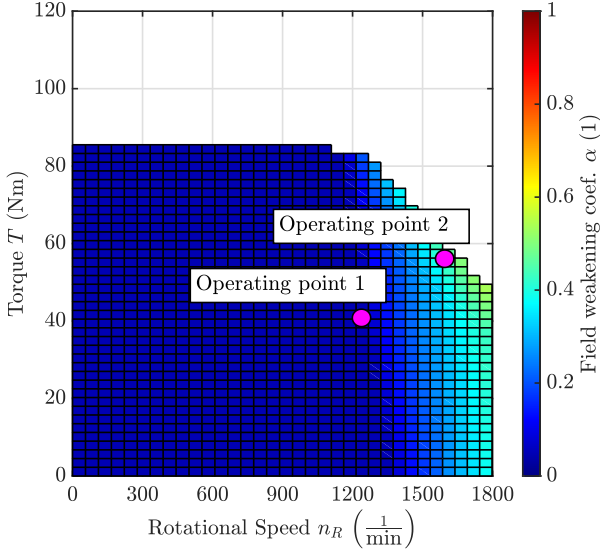
By observing the field weakening LUT (see Fig. 6.12a), it can be seen that the demagnetization fault has a low influence on the operating area compared to the other faults, demonstrating an efficient compensation. Due to the partial demagnetization, the electrical period becomes identical to the mechanical one. During a complete mechanical rotation, the torque capability of each coil exhibits a noticeable drop at the angular positions where the demagnetized magnet passes by (see Fig. 6.12f). At such intervals the desired current of the affected coil tends to be reduced in order to achieve either maximum torque per ampere (see Fig. 6.12c) or an optimal field weakening (see Fig. 6.12d). Due to the asymmetrical flux distribution caused by the partial demagnetization, the cogging torque increases significantly (see Fig. 6.12e) and must therefore be compensated. As a result, the amplitudes of the desired currents vary periodically in accordance with the cogging torque.

Overall control system

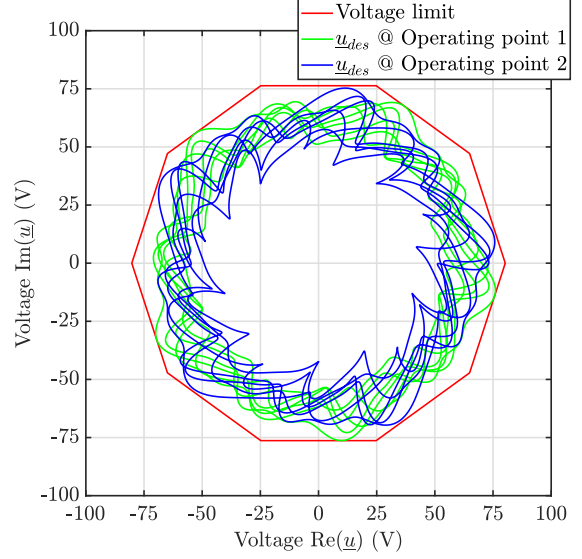
The simulation results of the overall control system under dynamic conditions are displayed in Fig. 6.13. It is assumed that the partial demagnetization occurs at $t_{fault} = 2.0\text{s}$ and that the control system is adapted to the fault after a delay of 40ms ($t_{adapt} = 2.04\text{s}$).

By examining the actual currents shown in Fig. 6.13e, it becomes apparent that there is no significant change in their waveforms during the interval between fault occurrence and fault adaptation, $t_{fault} < t < t_{adapt}$. This behavior is achieved through the cost function K in the optimization problem (6.58): by largely canceling out voltage deviations $\Delta\vec{v}$ with the applied voltage corrections $\Delta\vec{u}$, the actual currents tend to follow the desired ones (for further explanations, see subsection 5.3.3). This effect is also evident in Fig. 6.13g, which shows these voltage corrections $\Delta\vec{u}$ of the feedback control. There, the amplitude of one specific voltage correction increases periodically whenever the demagnetized magnet passes the corresponding coil, compensating (in part) the reduced back-EMF due to the demagnetization. This property of the feedback control is also used for the diagnosis of partial demagnetization, see section 7.2.

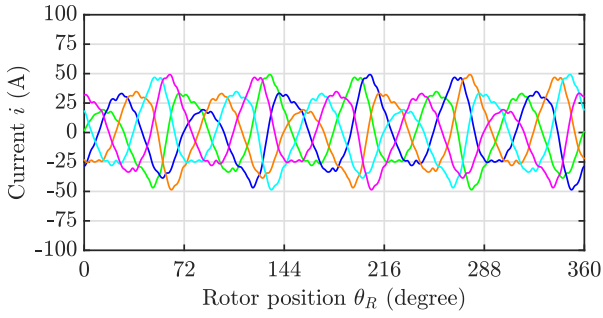
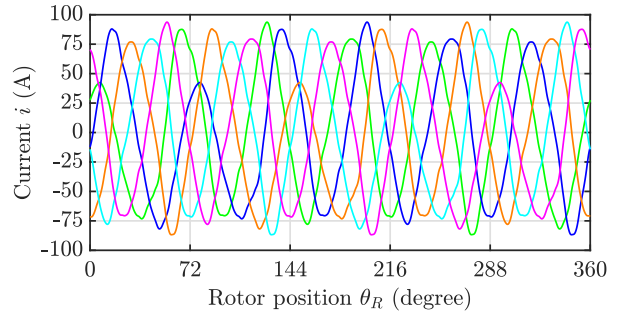
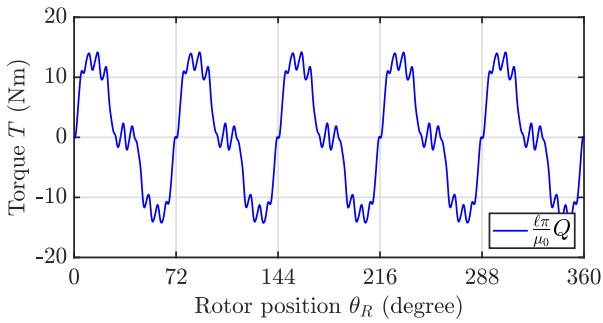
After the fault adaptation for $t > t_{adapt}$, no increase in the voltage corrections is recognizable anymore (see Fig. 6.13g and Fig. 6.13h), since the desired currents are then calculated optimally for the faulty machine with its demagnetized magnet. Thereby the fault adapted control achieves an accurate tracking of the desired trajectory without any relevant deviations of the rotor position (see Fig. 6.13c) or any increase in the torque ripple (see Fig. 6.13a).



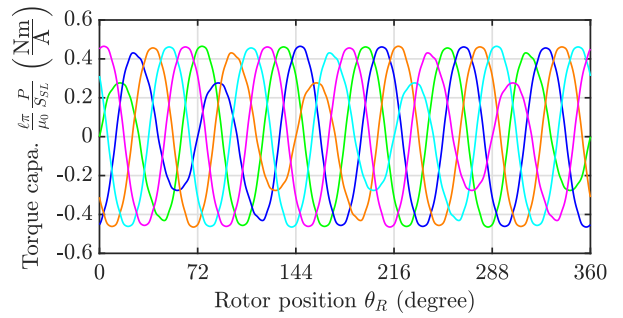
(a) Field weakening LUT.

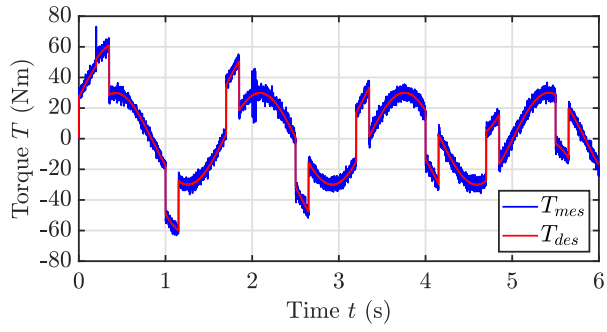


(b) Desired voltages in the complex plane.

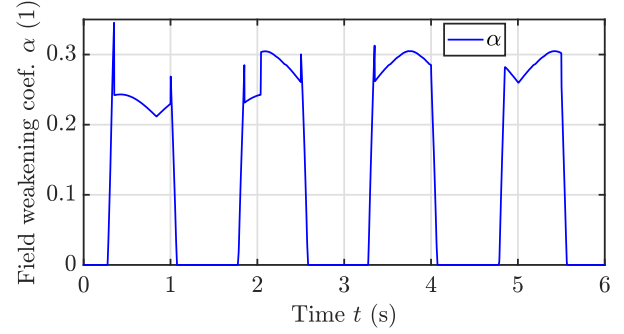
(c) Desired currents $\vec{i}_{des}$ at operating point 1.(d) Desired currents $\vec{i}_{des}$ at operating point 2.

(e) Predicted cogging torque.

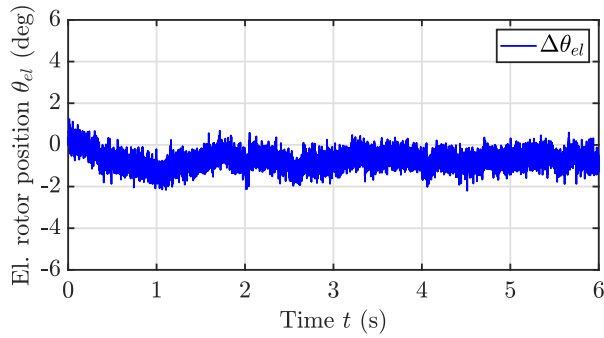
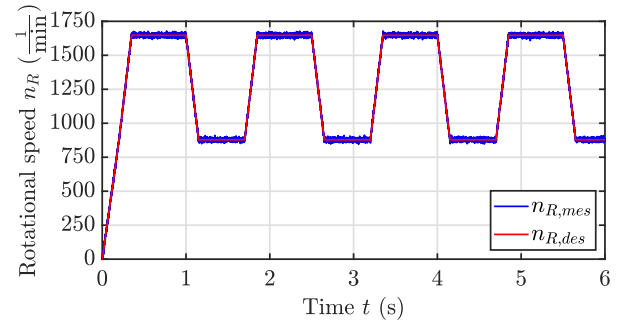
(f) Predicted torque capabilities $\frac{\ell_{\pi}}{\mu_0} \vec{P}$.Figure 6.12: **Partial demagnetization** tolerant operation of the feedforward control.



(a) Actual and desired torque.



(b) Field weakening coefficient.

(c) Deviation of the rotor angle ($\Delta\theta_{el} = p\Delta\theta_R$).

(d) Actual and desired rotational speed.

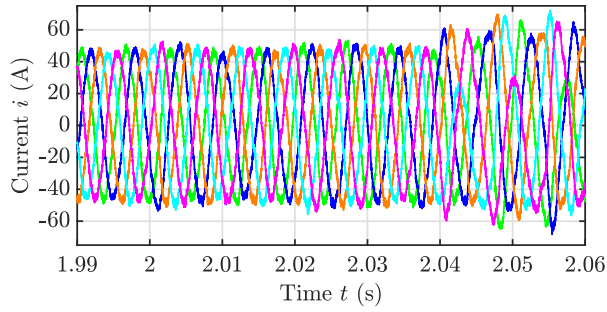
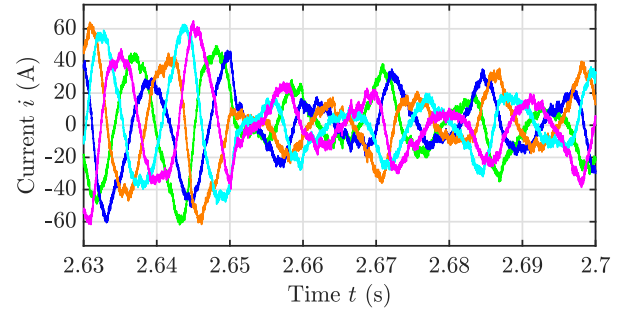
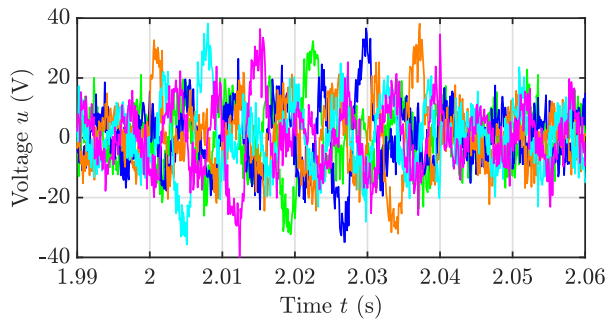
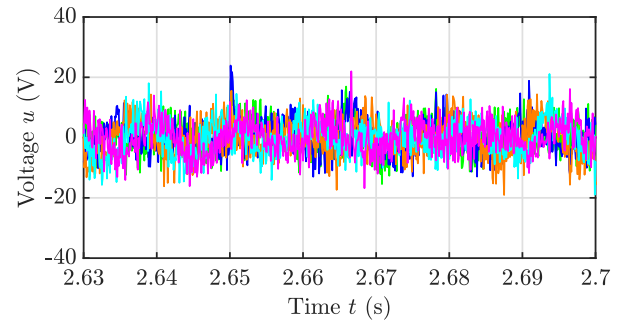
(e) Actual currents $\vec{i}_{mes}$ at the time period of fault occurrence.(f) Actual currents $\vec{i}_{mes}$ at a time period after fault adaptation.(g) Voltage corrections $\Delta\vec{u}$ at the time period of fault occurrence.(h) Voltage corrections $\Delta\vec{u}$ at a time period after fault adaptation.

Figure 6.13: **Partial demagnetization** fault tolerant operation of the overall control system.

6.5 High resistance connections

A high resistance connection in the plant model can be efficiently considered by adjusting the effective resistance of the affected phase in the equation system of the current dynamics (4.17).

Modifications in the control

To ensure minimal ohmic losses and avoid an overheating of the faulty phase with a high resistance connection, the affected phase is re-weighted in the optimization problem of the feedforward control.

Modifications in the feedforward control

The adjusted optimization problem of the feedforward control for determining the desired currents under consideration of a high resistance connection reads:

$$\begin{aligned}
 & \text{minimize } K(\vec{J}) = \frac{1}{2} \vec{J}^T (\mathbf{Z} - \alpha) \vec{J} + \frac{\alpha}{2} \left(\vec{J} + \frac{\mathcal{L}^{-1} \vec{\Phi}_M}{S_{SL}} \Big|_{des} \right)^T \left(\vec{J} + \frac{\mathcal{L}^{-1} \vec{\Phi}_M}{S_{SL}} \Big|_{des} \right) \\
 & \text{subject to } h_1(\vec{J}) = \vec{1}^T \vec{J} = 0 \\
 & \quad h_2(\vec{J}) = \frac{\ell \pi}{\mu_0} \vec{P}^T \Big|_{des} \vec{J} + \frac{\ell \pi}{\mu_0} Q \Big|_{des} - T_{des} = 0
 \end{aligned} \tag{6.59}$$

with $\mathbf{Z}$ the re-weighting matrix

$$\mathbf{Z} = \text{diag} \left(\zeta^{(1)} \quad \zeta^{(2)} \quad \dots \quad \zeta^{(c)} \quad \dots \quad \zeta^{(n_{coil})} \right), \tag{6.60}$$

$$\zeta^{(c)} = \begin{cases} \frac{R^{(c)} + R_{hr}}{R^{(c)}} & \text{for } c = f \\ 1 & \text{else} \end{cases} \tag{6.61}$$

(f) denotes the index of the faulty phase or coil affected by a high-resistance connection, and R_{hr} represents the value of the additional fault-induced resistance. The matrix $\mathbf{Z}$ increases the weighting of the affected phase proportionally to the additional resistance, causing the cost function K to reduce the RMS value of the desired current and thus minimizing the ohmic losses.

Solving the corresponding minimization problem, the resulting equation system remains equivalent to the one of the healthy machine (5.8), aside from a modification in matrix $\mathbf{M}$

to incorporate the re-weighting matrix $\mathbf{Z}$:

$$\mathbf{M}_{\text{hr}} = \begin{pmatrix} \mathbf{Z} & \vec{1} & \frac{\ell\pi}{\mu_0}\vec{P}_{des} \\ \vec{1}^T & 0 & 0 \\ \frac{\ell\pi}{\mu_0}\vec{P}_{des}^T & 0 & 0 \end{pmatrix} \quad (6.62)$$

Based on this adapted equation system, the desired currents $\vec{J}_{des}$ and their time derivatives $\frac{d\vec{J}_{des}}{dt}$ can be efficiently computed. These quantities are then used to determine the desired voltages:

$$\vec{u}_{des}(\alpha) = \mathcal{R}'\vec{i}_{des}(\alpha) + \mathcal{L}\frac{d\vec{i}_{des}(\alpha)}{dt} + \vec{e}(\theta_{R,des}). \quad (6.63)$$

where $\mathcal{R}' = \mathbf{Z}\mathcal{R}$ is the updated resistance matrix based on the specified high resistance connection. To precalculate the field weakening LUT, the methodology described in subsection 5.2.2 can be used.

Modifications in the feedback control

By replacing the resistance matrix $\mathcal{R}$ by the updated resistance matrix $\mathcal{R}' = \mathbf{Z}\mathcal{R}$ in equation (5.30) for calculating the required correction $\Delta\Xi'_{\Delta\theta_R}$ and (5.32) for calculating $\Delta\Xi'_{\Delta\theta_R}$, the feedback control is adapted to the according high resistance connection.

Evaluation

In the following, the performance of the developed control system is evaluated under the presence of a high resistance connection in one phase. For this purpose, it is assumed that the overall resistance of the affected phase increased to four times the value of a healthy phase ($R^{(f)} + R_{hr} = 0.1\Omega + 0.3\Omega = 0.4\Omega$). The control system is adapted accordingly as described above.

Feedforward control

To evaluate the feedforward control under consideration of a high resistance connection it is used to predict the optimal desired currents and voltages at two different operating points¹¹. In Fig. 6.14a it can be seen that the operating area is larger than the one under open phase fault (see Fig. 6.2a) because of the higher dimension of the search space. This effect is particularly noticeable in the field weakening region. To ensure minimal copper losses, the

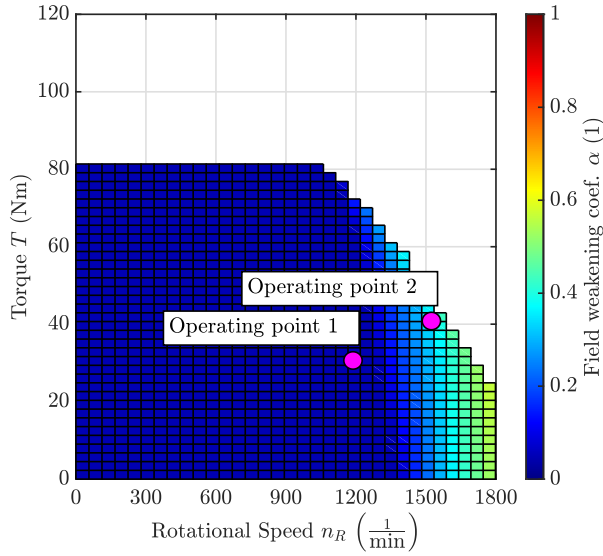
¹¹In the plots of this section showing the desired currents $\vec{i}_{des}$, the actual currents $\vec{i}_{mes}$ or the torque capabilities $\frac{\ell\pi}{\mu_0}\vec{P}$, the phase with high resistance connection is represented by the **green curve**.

RMS value of the desired current in the affected coil is significantly reduced, as illustrated in Fig. 6.14c and Fig. 6.14d. As a result of the reweighting strategy, the healthy coils are required to compensate the reduced torque contribution of the faulty phase. This leads to slightly increased current amplitudes and more complex waveforms in the healthy coils.

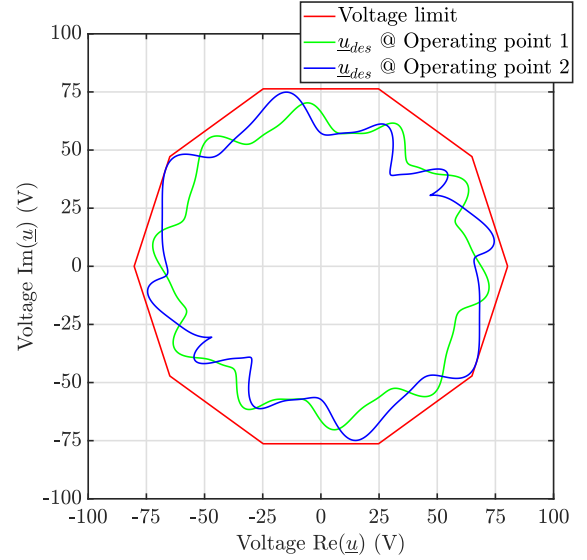
Overall control system

Fig. 6.15 presents the simulation results of the overall control system under dynamic operating conditions¹¹. It is assumed that the high resistance connection fault occurs at $t_{fault} = 2.0$ s and that the control system is adapted to the fault after a delay of 40 ms ($t_{adapt} = 2.04$ s).

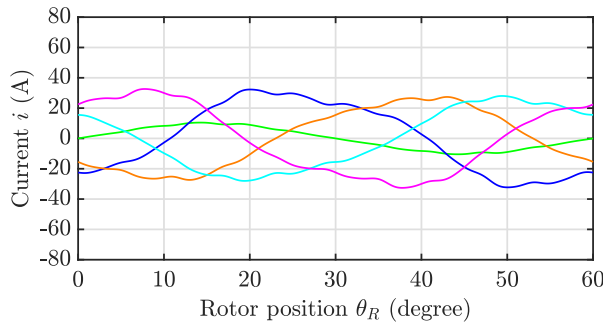
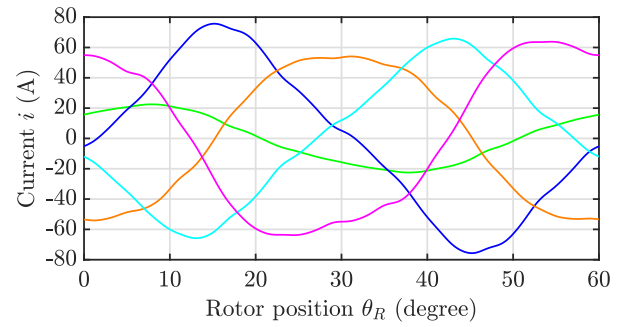
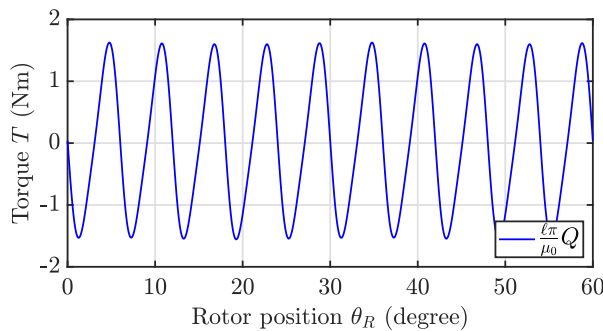
The results demonstrate that the control system is capable of efficiently compensating the fault by decreasing the current of the faulty coil (see Fig. 6.15e and 6.15f). No significant increase in the torque ripple (Fig. 6.15a) or deviation in the rotor position (Fig. 6.15c) is observed, indicating a robust and stable system behavior in the presence of a high resistance connection.



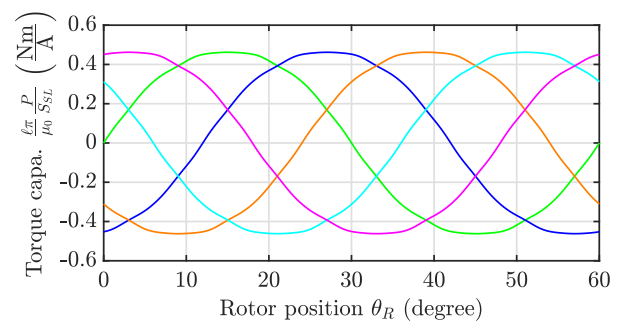
(a) Field weakening LUT.

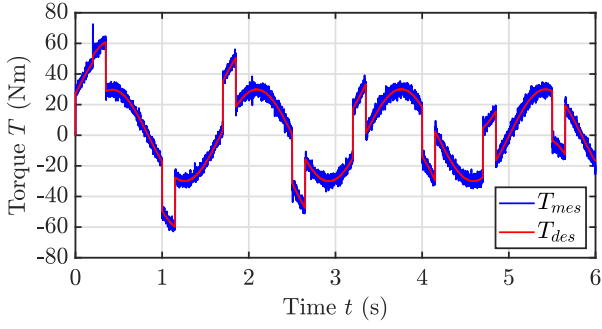


(b) Desired voltages in the complex plane.

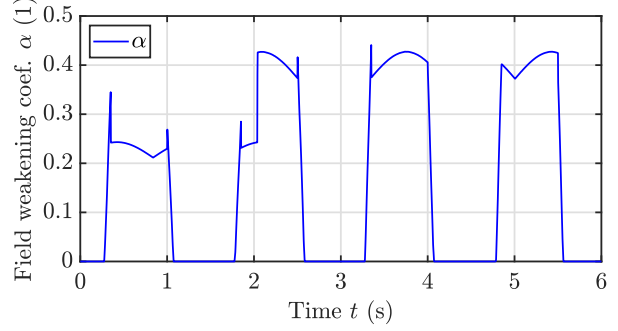
(c) Desired currents $\vec{i}_{des}$ at operating point 1.(d) Desired currents $\vec{i}_{des}$ at operating point 2.

(e) Predicted cogging torque.

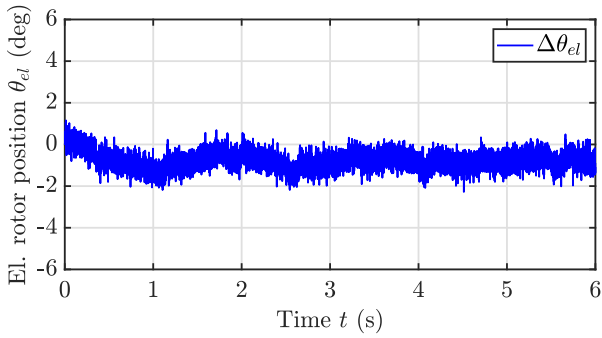
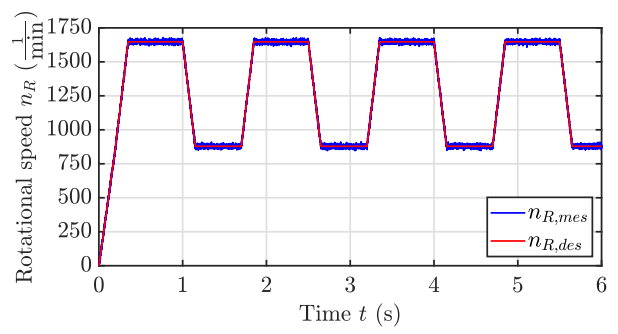
(f) Predicted torque capabilities $\frac{\ell_\pi}{\mu_0} \vec{P}$.Figure 6.14: **High resistance connection** tolerant operation of the feedforward control.



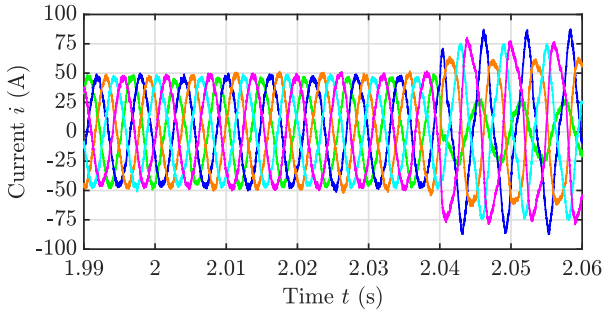
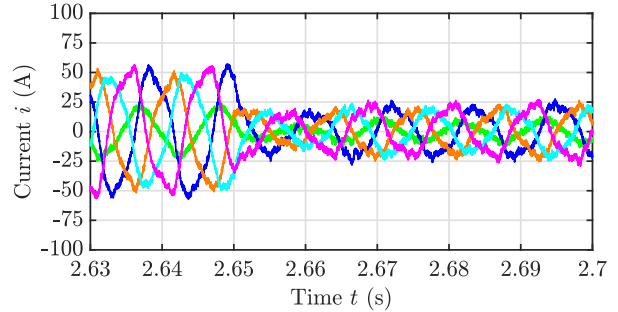
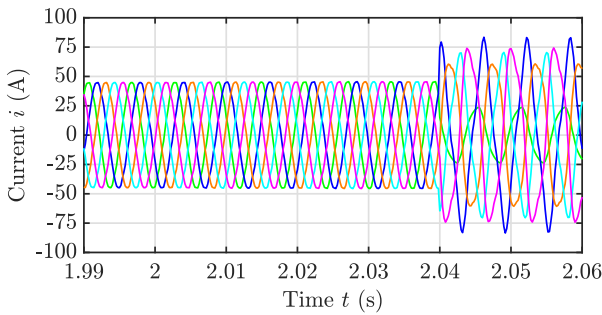
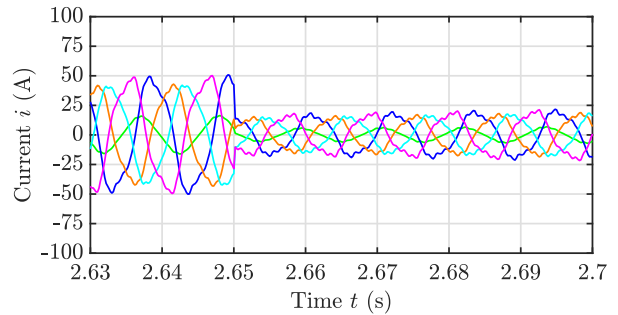
(a) Actual and desired torque.



(b) Field weakening coefficient.

(c) Deviation of the rotor angle ($\Delta\theta_{el} = p\Delta\theta_R$).

(d) Actual and desired rotational speed.

(e) Actual currents $\vec{i}_{mes}$ at the time period of fault occurrence.(f) Actual currents $\vec{i}_{mes}$ at a time period after fault adaptation.(g) Desired currents $\vec{i}_{des}$ at the time period of fault occurrence.(h) Desired currents $\vec{i}_{des}$ at a time period after fault adaptation.Figure 6.15: **High resistance connection** tolerant operation of the overall control system.

6.6 Multi-faults

All individual compensation strategies for open phases (section 6.1), phase short circuits (section 6.2), interturn short circuits (section 6.3), demagnetization faults (section 6.4), and high resistance connections (section 6.5) can be merged into a unified, generalized formulation. This enables the simultaneous consideration and compensation of multiple faults, thereby achieving a comprehensive multi-fault tolerance in the control system.

Modifications in the feedforward control

The general minimization problem of the feedforward control for determining the optimal currents under arbitrary fault conditions is formulated as follows:

$$\begin{aligned}
 \text{minimize } K(\vec{J}_{h^*}) &= \frac{1}{2} \vec{J}_{h^*}^T \mathbf{Z}_{h^*}(\alpha) \vec{J}_{h^*} + \frac{\alpha}{2} \left(\vec{J}_{h^*} + \frac{\mathcal{L}_{h^*}^{-1} \vec{\Phi}_{M,h^*}}{S_{SL}} \Big|_{des} \right)^T \left(\vec{J}_{h^*} + \frac{\mathcal{L}_{h^*}^{-1} \vec{\Phi}_{M,h^*}}{S_{SL}} \Big|_{des} \right) \\
 \text{subject to } h_1(\vec{J}_{h^*}) &= \vec{1}^T \vec{J}_{h^*} + J_{des}^{(f)} = 0 \\
 h_2(\vec{J}_{h^*}) &= \frac{\ell\pi}{\mu_0} \left(\vec{\tilde{P}}_{h^*}^T \Big|_{des} \vec{J}_{h^*} + \tilde{P}^{(f)} \Big|_{des} J_{des}^{(f)} + \tilde{Q} \Big|_{des} \right) + \tilde{T}_{ps} \Big|_{des} (R_{ps}) - T_{des} = 0 \quad (6.64)
 \end{aligned}$$

The following clarifications apply to this generalized formulation:

- All quantities marked with a tilde are calculated using the updated Fourier coefficients $\{\tilde{a}_k^{(M)}, \tilde{c}_k^{(M)}\}$, which represent the modified magnetization under the consideration of a demagnetization fault.
- Vectors with the index h^* include the corresponding quantities of both the completely healthy coils and those affected solely by a high resistance connection.
- The desired current density $J_{des}^{(f)}$ for the coil (f) with interturn short circuit is calculated based on equation (6.42) to extinct the PM flux linkage $\Phi_{M,s}^{(f)}$ of the shorted coil turns.
- The faulty torque ripple $\tilde{T}_{ps}$ caused by a phase short circuit is analytically predicted based on equation (6.15).
- Matrix $\mathbf{Z}_{h^*}(\alpha)$, which is calculated by (6.60), re-weights the phase affected by a high resistance connection.

In summary, the equality constraint h_1 enforces current equilibrium among all healthy coils, coils solely affected by high resistance connections, and those with interturn short circuits, in accordance with the assumed star connection. The second constraint h_2 ensures that the

resulting currents generate the desired torque T_{des} while compensating the cogging torque $\frac{\ell\pi}{\mu_0}\tilde{Q}$, the faulty torque ripple $\tilde{T}_{ps}$, and taking into account the torque contribution of the coil affected by the interturn short circuit.

By solving the optimization problem (6.64), the resulting equation system - including the time derivatives of the currents - becomes equivalent to

$$\begin{pmatrix} \tilde{\mathbf{M}}_{\mathbf{h}^*} & \mathbf{0} \\ \dot{\tilde{\mathbf{M}}}_{\mathbf{h}^*} & \tilde{\mathbf{M}}_{\mathbf{h}^*} \end{pmatrix} \begin{pmatrix} \vec{x}_{h^*} \\ \vec{\dot{x}}_{h^*} \end{pmatrix} = \begin{pmatrix} \vec{b}_{h^*} \\ \vec{\dot{b}}_{h^*} \end{pmatrix}, \quad (6.65)$$

with

$$\tilde{\mathbf{M}}_{\mathbf{h}^*} = \begin{pmatrix} \mathbf{Z}_{h^*} & \vec{1} & \frac{\ell\pi}{\mu_0}\vec{\tilde{P}}_{h^*}|_{des} \\ \vec{1}^T & 0 & 0 \\ \frac{\ell\pi}{\mu_0}\vec{\tilde{P}}_{h^*}^T|_{des} & 0 & 0 \end{pmatrix}, \quad \vec{x}_{h^*} = \begin{pmatrix} \vec{J}_{h^*} \\ \lambda_1 \\ \lambda_2 \end{pmatrix}, \quad \vec{b}_{h^*} = \begin{pmatrix} -\alpha \frac{\mathcal{L}_{h^*}^{-1} \vec{\Phi}_{M,h^*}}{S_{SL}}|_{des} \\ -J_{des}^{(f)} \\ T_{des} - \tilde{T}_{ps}|_{des} - \frac{\ell\pi}{\mu_0}(\tilde{Q}|_{des} + \tilde{P}^{(f)}|_{des} J_{des}^{(f)}) \end{pmatrix} \quad (6.66)$$

and

$$\dot{\tilde{\mathbf{M}}}_{\mathbf{h}^*} = \begin{pmatrix} \mathbf{0} & \vec{0} & \frac{\ell\pi}{\mu_0}\vec{\dot{\tilde{P}}}_{h^*}|_{des} \\ \vec{0}^T & 0 & 0 \\ \frac{\ell\pi}{\mu_0}\vec{\dot{\tilde{P}}}_{h^*}^T|_{des} & 0 & 0 \end{pmatrix}, \quad \vec{\dot{x}}_{h^*} = \begin{pmatrix} \dot{\vec{J}}_{h^*} \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \end{pmatrix}, \quad \vec{\dot{b}}_{h^*} = \begin{pmatrix} -\alpha \frac{\mathcal{L}_{h^*}^{-1} \dot{\vec{e}}_{h^*}}{S_{SL}}|_{des} \\ -\dot{J}_{des}^{(f)} \\ -\dot{\tilde{T}}_{ps}|_{des} - \frac{\ell\pi}{\mu_0}(\dot{\tilde{Q}}|_{des} + \dot{\tilde{P}}^{(f)} j_{des}^{(f)} + \dot{\tilde{P}}^{(f)} J_{des}^{(f)}) \end{pmatrix} \quad (6.67)$$

By this equation system (6.65), the desired currents $\vec{J}_{des,h^*}$ and their time derivatives $\dot{\vec{J}}_{des,h^*}$ can be efficiently computed. These quantities are then used to determine the desired voltages:

$$\begin{aligned} \vec{u}_{des,h^*}(\alpha) &= \mathcal{R}'_{h^*} \vec{i}_{des,h^*}(\alpha) + \mathcal{L}_{h^*} \frac{d\vec{i}_{des,h^*}(\alpha)}{dt} + \vec{e}_{h^*}(\theta_{R,des}) \\ u_{des}^{(f)} &= R_g i_{des}^{(f)} + L_g \frac{di_{des}^{(f)}}{dt} + e_g^{(f)}(\theta_{R,des}). \end{aligned} \quad (6.68)$$

To precalculate the field weakening LUT the same methodology as described in subsection 5.2.2 can be used. However, if a phase short circuit is considered, the adjusted methodology described in section 6.2 must be used.

Modifications in the feedback control

The generalized minimization problem of the feedback control for determining the optimal distributed voltage under arbitrary fault conditions is formulated as follows:

$$\begin{aligned}
& \text{minimize } K(\Delta \vec{u}_{h^*}) = \frac{1}{2}(\Delta \vec{u}_{h^*} + \Delta \vec{v}_{h^*})^T (\Delta \vec{u}_{h^*} + \Delta \vec{v}_{h^*}) \\
& \text{subject to } h_1(\Delta \vec{u}_{h^*}) = \vec{P}_{h^*}^T \Big|_{des} \frac{\mathcal{L}_{h^*}^{-1}}{S_{SL}} \Delta \vec{u}_{h^*} + \tilde{P}^{(f)} \Big|_{des} \frac{\Delta u^{(f)}}{L_{ef} S_{SL}} - \Delta \tilde{\Xi}'_{h^*,ps} = 0, \\
& \quad h_2(\Delta \vec{u}_{h^*}) = \vec{1}^T \Delta \vec{u}_{h^*} + \Delta u^{(f)} = 0, \\
& \text{where } \Delta \vec{v}_{h^*} = \mathcal{L}_{h^*} \frac{\Delta \vec{J}_{h^*}}{\Delta t} S_{SL}
\end{aligned} \tag{6.69}$$

Again, all vectors denoted with the index h^* contain both the quantities of the completely healthy coils and the ones solely affected by a high resistance connection. All quantities indicated with a tilde are calculated based on the updated Fourier coefficients $\{\tilde{a}_k^{(M)}, \tilde{c}_k^{(M)}\}$. The required correction $\Delta \tilde{\Xi}'_{h^*,ps}$ can be determined based on (6.25) under consideration of the updated resistance matrix $\mathcal{R}'_{h^*}$ to take into account both a phase short circuit and a high resistance connection. The voltage correction $\Delta u^{(f)}$ for the faulty coil (f) with interturn short circuit is calculated by (6.51).

In summary, equality constraint h_1 demands for the voltage corrections $\Delta \vec{u}_{h^*}$ the compensation of $\Delta \tilde{\Xi}'_{h^*,ps}$, while accounting for the voltage correction of the interturn short circuited coil (f). Equality constraint h_2 enforces an equilibrium of all voltage corrections, which ensures an electrical potential of the star point close to zero. By that, an electrical decoupling of the coils is achieved, minimizing the short circuit current in coil (f).

By solving this generalized minimization problem, the optimal distributed voltage corrections can be obtained by:

$$\begin{pmatrix} \mathbf{I} & \vec{1} & \frac{\mathcal{L}_{h^*}^{-1} \vec{P}_{h^*}}{S_{SL}} \Big|_{des} \\ \frac{\vec{P}_{h^*}^T \mathcal{L}_{h^*}^{-1}}{S_{SL}} \Big|_{des} & 0 & 0 \\ \vec{1}^T & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta \vec{u}_{h^*} \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} -\mathcal{L}_{h^*} \frac{\Delta \vec{J}_{h^*}}{\Delta t} S_{SL} \\ \Delta \tilde{\Xi}'_{h^*,ps} - \tilde{P}^{(f)} \Big|_{des} \frac{\Delta u^{(f)}}{L_{ef} S_{SL}} \\ -\Delta u^{(f)} \end{pmatrix}. \tag{6.70}$$

Thereby, the input voltages for the power electronics are formed as follows:

$$\vec{u}_{in,h^*} = \vec{u}_{des,h^*} - \Delta \vec{u}_{h^*} \tag{6.71}$$

$$u_{in}^{(f)} = u_{des}^{(f)} - \Delta u^{(f)} \tag{6.72}$$

This enables the optimal compensation of all considered fault combinations, including open

phases, phase short circuits, interturn short circuits, demagnetization, and high resistance connections.

Evaluation

Since the multi-fault tolerant operation of the control system represents the main contribution of this work, it is investigated more detailed in chapter 8.

6.7 Discussion

In this chapter, fault tolerant control strategies for compensating motor-intrinsic electrical and magnetic faults were developed and analyzed. For each considered fault type, the following initial conclusions can be drawn:

Open phase faults: By excluding the affected phase from the optimization problem for predicting the desired currents, this fault can be efficiently compensated. Since this control strategy is well established in the literature [35–38], the contribution lies not in the fault handling itself, but in its integration into the presented control framework.

Phase short circuits: Similar to the compensation of open phase faults, the short-circuited phase is excluded from the optimization problem. Additionally, the resulting torque ripple caused by the fault is explicitly incorporated. Again, this strategy is already established in the literature [35–38]. The contribution of the present approach lies in the usage of the Maxwell equations, which enables a physically accurate modeling of the electromagnetic interactions induced by the fault. As a result, a precise and robust compensation is achieved.

Interturn short circuits: The developed approach is similar to [40, 41] and related works, where field weakening strategies are employed to effectively reduce the induced voltage in the short circuited turns. A benefit of this method in general is the significant reduction of the short circuit current. For the developed approach this is particularly effective in the field weakening region. In contrast, in the base speed range, the asymmetry caused by the ‘single-phase field weakening’ leads to more complex current waveforms. Since the extinction of the PM flux linkage requires high phase currents, the approach is most suitable for cases with low short circuit impedance, where the short circuit current would otherwise grow to critical values. For higher impedances, the resulting fault current may remain within acceptable limits, whereby an active compensation would be unnecessary.

Demagnetization: Among the considered (single) fault types, the developed compensation strategy for demagnetization faults represents the most notable advancement over the current state of the art due to the scarce number of existing publications considering this fault [5]. The compensation is efficiently achieved by inserting the updated Fourier coefficients of the magnetization, based on the spatial course of demagnetization, into the optimization problems of both the feedforward and feedback control. As a result, the method enables the consideration of arbitrary, symmetric and asymmetric demagnetization profiles.

High resistance connection: By integrating a re-weighting matrix into the cost function of the feedforward control, the copper losses are efficiently minimized in the presence of a high resistance connection. While more detailed approaches exist in the literature, such as [9], the contribution lies again in the integration of the fault handling into the presented control framework.

For the field weakening strategy, it is necessary to precalculate a corresponding LUT for each specific fault case. For this purpose, a neural network could be trained on a sufficiently large dataset that covers a wide range of fault scenarios. In the event of a fault, the trained network could then be used to generate the appropriate LUT based on the detected fault, enabling real-time adaptation of the field weakening strategy.

7 Fault adaptation and diagnosis

While in chapter 6 it was assumed that all required fault parameters for the adaptation are known, this information needs to be estimated by an appropriate algorithm in practice. In the following methodologies for the adaptation of the control to phase short circuits and the diagnosis of partial demagnetization are presented.

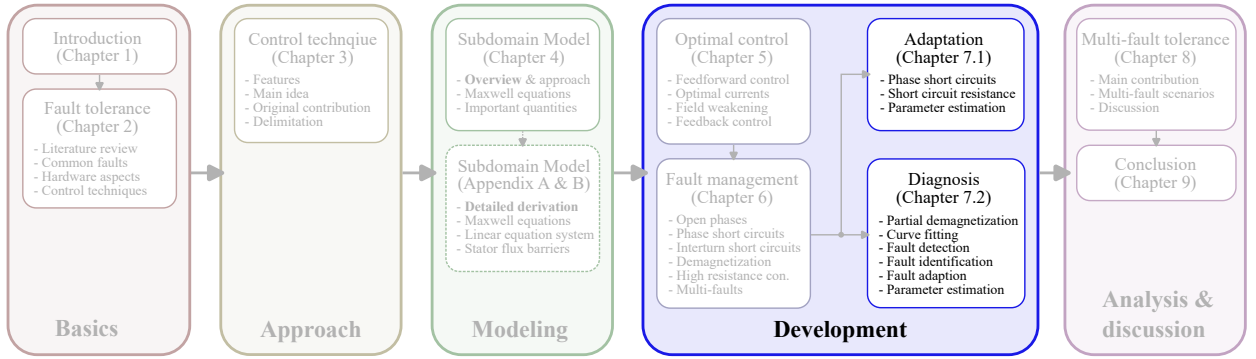


Figure 7.1: Structural overview of the work, with the sections of this chapter 7 highlighted.

7.1 Adaptation to phase short circuits

(The following section is based on [72].)

Since the only unknown in (6.15) for predicting the faulty torque ripple T_{ps} is the short circuit resistance R_{ps} , which in turn has a significant influence on T_{ps} , an algorithm is developed to iteratively adapt this parameter. This is achieved by extracting a characteristic harmonic of the measured torque with a Fast Fourier Transform (FFT) in steady state operation.

7.1.1 Methodology

As described in section 6.2, the dominant torque harmonic $T_{ps,v}$ caused by the faulty coil equals $v = 2p$. Accordingly, the first step is to monitor this characteristic harmonic using a FFT of the measured torque during steady state conditions. This allows extraction of the

amplitude $\hat{T}_{ps,2p}^{*(i)}$ and phase angle $\psi_{ps,2p}^{*(i)}$ of the $2p$ -th harmonic:

$$\hat{T}_{ps,2p}^{*(i)} \cos(2p\theta_R + \psi_{ps,2p}^{*(i)}) := T_{ps,2p}^{*(i)}(\theta_R), \quad (7.1)$$

resulting in a harmonic model $T_{ps,2p}^{*(i)}(\theta_R)$ of the faulty torque ripple. Thereby $\hat{T}_{ps,2p}^{*(i)}$ and $\psi_{ps,2p}^{*(i)}$ are updated successively for each iteration (i). Starting the first iteration ($i = 0$) at the time shortly after fault occurrence (before any compensation), $T_{ps,2p}^{*(i=0)}(\theta_R)$ is directly correlated with the real faulty torque ripple $\bar{T}_{ps,2p}(\theta_R)$. However, if the torque ripple is (partially) compensated, the measured $T_{ps,2p}^{*(i)}(\theta_R)$ is a superposition of the real value $\bar{T}_{ps,2p}(\theta_R)$ and the compensated value $T_{ps,2p}(\theta_R, R_{ps}^{(i)})$ based on its estimation

$$T_{ps,2p}^{*(i)}(\theta_R) = \bar{T}_{ps,2p}(\theta_R) - T_{ps,2p}(\theta_R, R_{ps}^{(i)}) = \Delta T_{ps,2p}^{(i)}. \quad (7.2)$$

Since an error in the estimated short circuit resistance ΔR_{ps} causes directly an erroneous compensation and thereby a remaining (measured) torque ripple $\Delta T_{ps,2p}$, they are approximately proportional to each other¹

$$\Delta T_{ps,2p}^{(i)} \approx \underbrace{\left(\frac{\partial \xi_{2p}(R_{ps}^{(i)})}{\partial R_{ps}} \sin(2p\theta_R) + \frac{\partial \rho_{2p}(R_{ps}^{(i)})}{\partial R_{ps}} \cos(2p\theta_R) \right)}_{\frac{\partial T_{ps,2p}(R_{ps}^{(i)})}{\partial R_{ps}}} \Delta R_{ps}. \quad (7.3)$$

where coefficients $\{\xi_{2p}, \rho_{2p}\}$ and thereby $T_{ps,2p}$ is linearized at the actual estimate $R_{ps}^{(i)}$. This requires effectively a derivation of coefficients (6.12) with respect to the resistance R_{ps} .

By demanding an asymptotically stable convergence of ΔR_{ps}

$$\frac{d}{dt} \Delta R_{ps} = -\frac{1}{\tau} \Delta R_{ps} \approx -\frac{1}{\frac{\partial T_{ps,2p}(R_{ps}^{(i)})}{\partial R_{ps}}} \frac{1}{\tau} \Delta T_{ps,2p}^{(i)} \quad (7.4)$$

the following time discrete dynamic is obtained

$$\frac{(R_{ps}^{(i+1)} - \bar{R}_{ps}) - (R_{ps}^{(i)} - \bar{R}_{ps})}{\Delta t} \approx -\frac{1}{\frac{\partial T_{ps,2p}(R_{ps}^{(i)})}{\partial R_{ps}}} \frac{1}{\tau} \Delta T_{ps,2p}^{(i)}. \quad (7.5)$$

¹From here on, all torque-related variables will be evaluated for actual rotor angle θ_R . For this reason the argument θ_R will be omitted.

The time constant τ indicates the speed of convergence and $\bar{R}_{ps}$ denotes the true value of the short circuit resistance. Thereby a new estimation for the short circuit resistance

$$R_{ps}^{(i+1)} = R_{ps}^{(i)} + \begin{cases} -\frac{1}{\frac{\partial T_{ps,2p}(R_{ps}^{(i)})}{\partial R_{ps}}} \frac{\Delta t}{\tau} \Delta T_{ps,2p} & \text{if } \left| \frac{\partial T_{ps,2p}(R_{ps}^{(i)})}{\partial R_{ps}} \right| > 0 \\ 0 & \text{if } \left| \frac{\partial T_{ps,2p}(R_{ps}^{(i)})}{\partial R_{ps}} \right| = 0 \end{cases} \quad (7.6)$$

can be found. This new estimation is incorporated into the feedforward and feedback control (see Fig. 7.2), improving the compensation and thus leading to $\Delta T_{ps,2p} \rightarrow 0$.

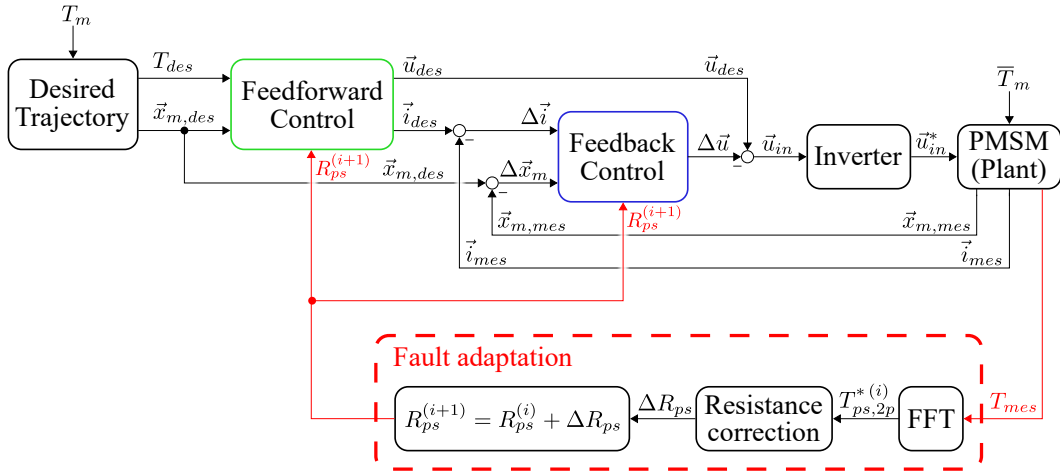


Figure 7.2: Structure of the control, with the **adaptation algorithm** highlighted [72].

For the field weakening, a dynamic interpolation of the field weakening LUTs for each estimate $R_{ps}^{(i+1)}$ would be required (e.g. by a neuronal network). Since this dynamic interpolation is not part of this work, field weakening is not considered in the following evaluation.

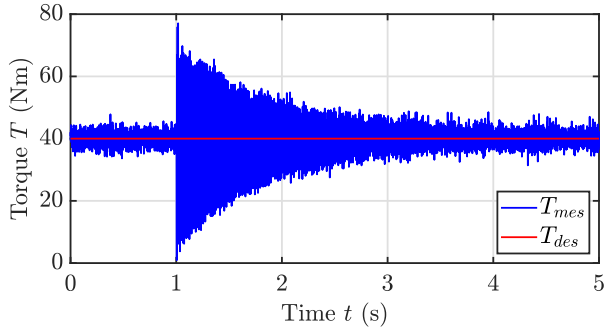
7.1.2 Evaluation

In the following, the performance of the developed control system in combination with the introduced adaptation algorithm (see Fig. 7.2) is evaluated under the presence of a phase short circuit. It is assumed that the control system is in a steady state with a (constant) desired rotational speed of $n_{des} = 1000 \frac{1}{\text{min}}$, a (constant) load torque of $T_m = 40\text{Nm}$ and the phase short circuit is appearing at $t_{fault} = 1.0\text{s}$. The real value of the short circuit resistance is set to $\bar{R}_{ps} = 0.4\Omega$ and the initial estimate to $R_{ps}^{(i=0)} = 5\Omega$.

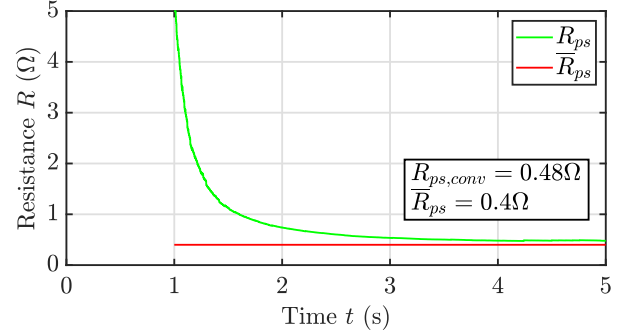
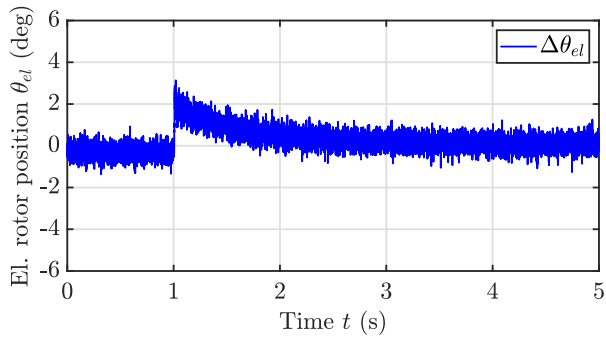
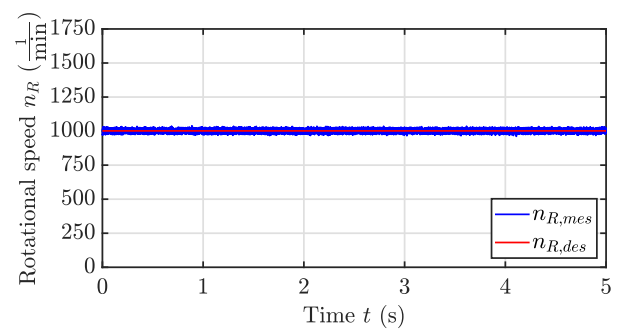
The simulation results of the overall control system are displayed in Fig 7.3². After fault occurrence, the control system is successively adapting the short circuit resistance (see Fig. 7.3b), improving the prediction (see Fig. 7.3g and 7.3h) and thus the compensation

²In the plots of this section showing the actual currents $\vec{i}_{mes}$, the phase short circuited phase is represented by the **green curve**.

(Fig. 7.3a) of the faulty torque ripple T_{ps} . Since the torque ripple (see Fig. 7.3a) after the adaption is clearly reduced, the fault adaptation is demonstrated. By examining Fig. 7.3b, it is noticeable that there is still a difference between the estimated short circuit resistance and the real one after convergence, while the real and estimated faulty torque ripple shown in Fig. 7.3h is nearly the same. Since the estimated PM flux linkage is assumed to be higher than the real one (see Tab. 5.3), the estimated faulty torque ripple would be higher as well compared to the real one at the same short circuit resistance. However, this parameter discrepancy is compensated by the iterative adaptation algorithm based on the remaining faulty torque ripple $\Delta T_{ps,2p}$ with a higher (estimated) short circuit resistance.



(a) Actual and desired torque.

(b) **Estimated** and **real** short circuit resistance.(c) Deviation of the rotor angle ($\Delta\theta_{el} = p\Delta\theta_R$).

(d) Actual and desired rotational speed.

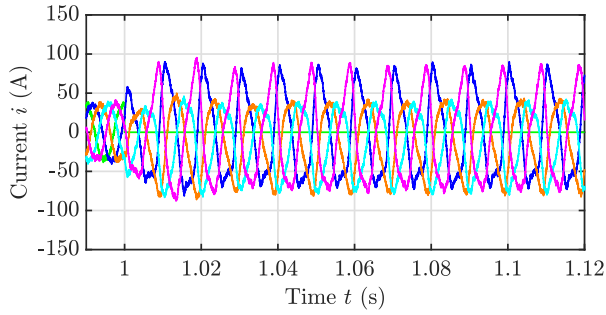
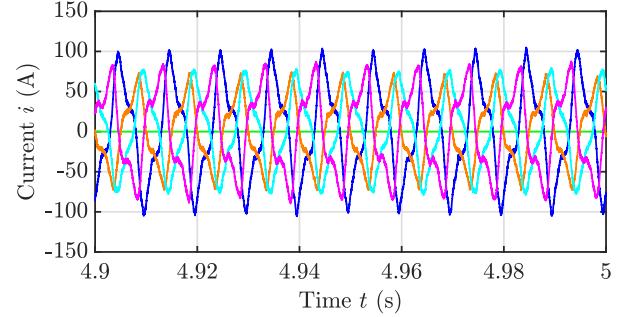
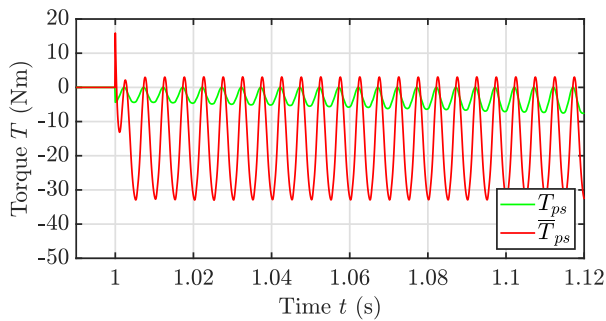
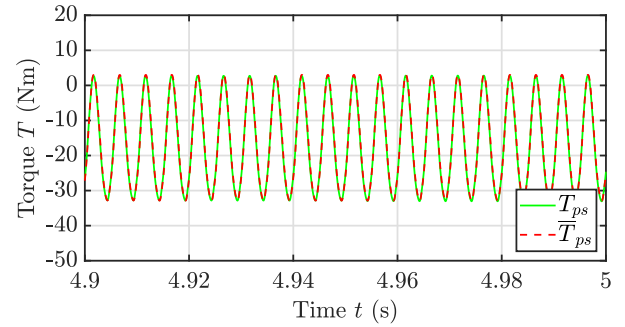
(e) Actual currents $\vec{i}_{mes}$ at the time period of fault occurrence.(f) Actual currents $\vec{i}_{mes}$ at a time period after convergence.(g) **Estimated** and **real** faulty torque ripple at the time period of fault occurrence.(h) **Estimated** and **real** faulty torque ripple at a time period after convergence.

Figure 7.3: **Phase short circuit** fault tolerant operation of the overall control system including the algorithm for adapting the short circuit resistance.

7.2 Diagnosis of partial demagnetization

(The following section is based on [69].)

As described in subsection 5.3.3, the refined feedback control reduces the electrical imbalance of the polyphase system caused by a partial demagnetization applying asymmetrical distributed voltage corrections – without requiring fault-specific modifications of the control scheme (see also Fig. 6.13g). However, since the feedback can never fully compensate this phenom by itself, fault adaptation of the overall control system is desirable. Using the Fourier series of the magnetization (6.53) with the updated Fourier coefficients $\{\tilde{a}_k^{(M)}, \tilde{c}_k^{(M)}\}$, the specified partial demagnetization is considered in the relevant quantities for the control system $\{\vec{\Phi}_M, \vec{e}, \vec{Q}, \vec{P}\}$ (see section 6.4). In practice, these Fourier coefficients $\{\tilde{a}_k^{(M)}, \tilde{c}_k^{(M)}\}$ are unknown at time of fault occurrence. The aim of the developed diagnosis system is to first estimate the demagnetization amplitude and position and then determine the corresponding change in the Fourier coefficients of the magnetization. The main idea is to quantify the asymmetry in the voltage correction $\Delta \vec{u}$ of the feedback control, since it combines the information of the deviations in the current densities $\Delta \vec{J}$, rotor position $\Delta \theta_R$ and rotational speed $\Delta \omega_R$ with the relevant geometric information. An efficient way to display this information at discrete time step n is the complex plane:

$$\Delta \underline{u}(n) = \sum_{c=1}^{n_{coil}} \Delta u^{(c)}(n) e^{j\varphi_{coil}^{(c)}} \quad (7.7)$$

with $\varphi_{coil}^{(c)}$ denoting the position of coil (c) as shown in Fig. 7.5a. In Fig. 7.4 it can be seen that symmetrically distributed sine waves along the coils with the same amplitude create a circle in the complex plane. When one sine wave has a higher amplitude than the others,

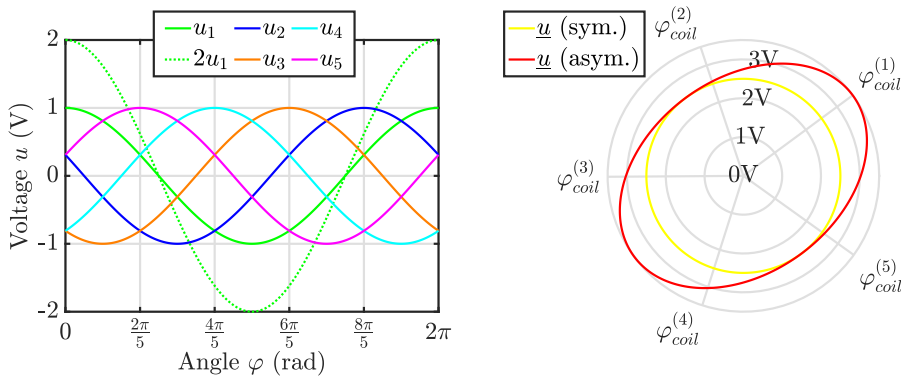


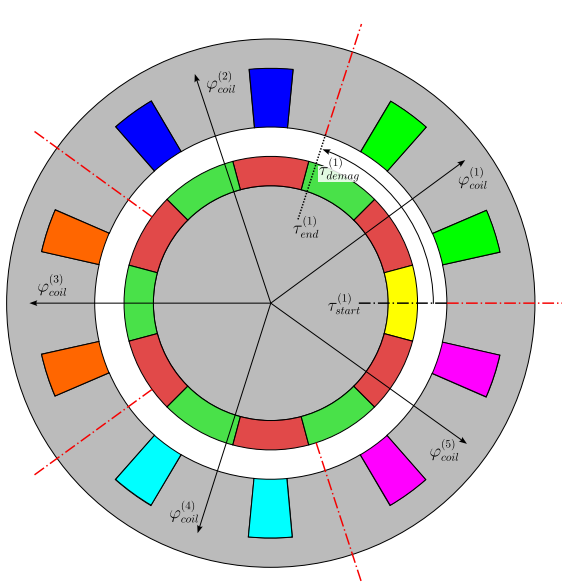
Figure 7.4: Visualization of symmetrical and asymmetrical distributed voltages. The **symmetrical** case, which is circular in the complex plane, is based on the sine waves with equal amplitudes, and a phase shift of $\frac{2\pi}{5}$ relative to one another. The **asymmetrical** case is based on the same sine waves, but with one wave having a doubled amplitude; thus it is elliptical in the complex plane [69].

the distribution becomes asymmetrical, creating an ellipse.

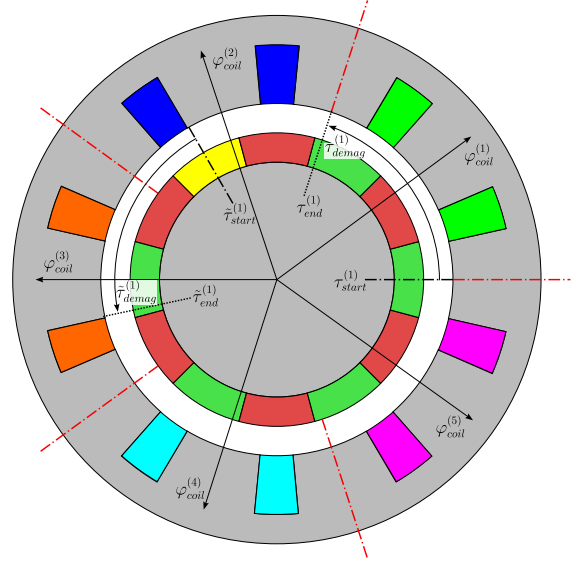
To quantify this asymmetry, the first step is to divide the electrical machine into coil zones (see Fig. 7.5a; the coil zones are separated by **red lines**). For the case that the **faulty magnet** is identical to the reference magnet³, Fig. 7.5a shows how coil zones are translated into coil periods $\tau_{demag}^{(c)}$ by the movement of the faulty magnet: the coil period $\tau_{demag}^{(c)}$ starts when the reference magnet reaches $\tau_{start}^{(c)}$ and ends when $\tau_{end}^{(c)}$ is reached. All voltage corrections $\Delta \underline{u}(n)$ sampled during this period $\tau_{demag}^{(c)}$ can be displayed in the complex plane and approximately form an ellipse which is oriented in the direction $\varphi_{coil}^{(c)}$ of coil (c) . Fig. 7.6a shows this schematically for coil period $\tau_{demag}^{(1)}$. If the demagnetized magnet is different to the reference magnet, the resulting ellipse tends to be twisted according to their angular distance, see Fig. 7.5b and Fig. 7.6b.

For an explicit quantification of the ellipticity of the voltage correction in the complex plane, FFT methods may be used. However, analyzing characteristic harmonics of the voltage correction can be subject to increased errors, because transients, high level of noise or other non-ideal effects may shift the spectrum. This is particularly relevant as the voltage correction of the feedback inherently includes the measurement noise of the currents, rotor angle and rotational speed, which is also amplified by the control. For this reason an alternative robust method based on curve fitting is developed, which is described in the next subsections.

³The reference magnet is the magnet used to define the rotor angle θ_R .

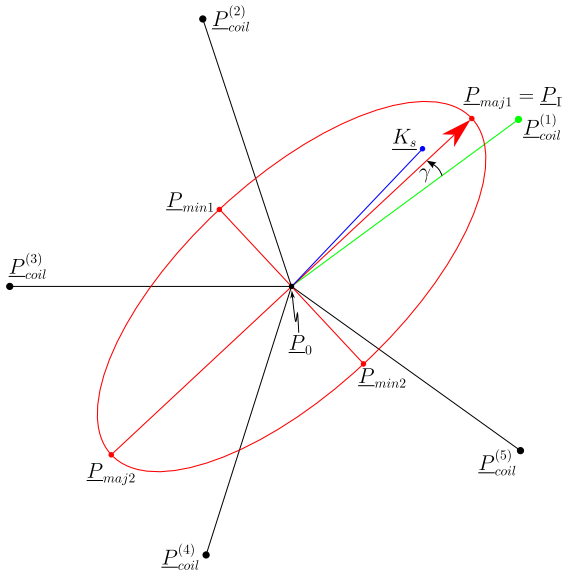


- (a) In this case the **demagnetized magnet** is the reference magnet – within one coil period $\tau_{dema}^{(c)}$ the faulty magnet moves along the corresponding coil, which results in an orientation of the ellipse in direction of that coil (c).

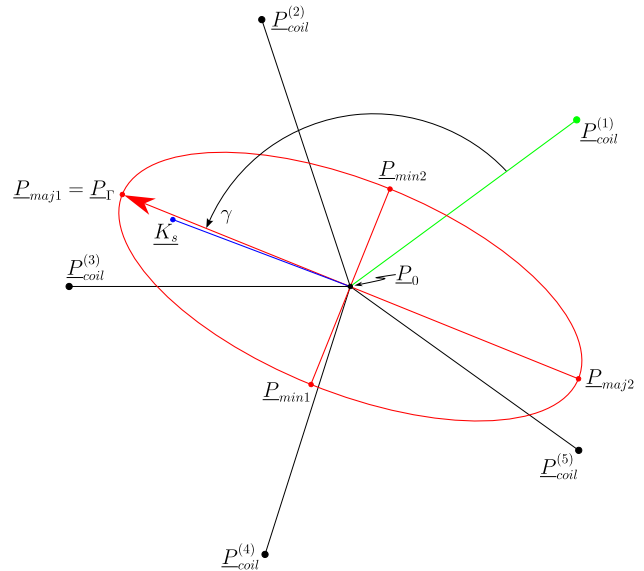


- (b) In this case the **demagnetized magnet** is the fifth magnet counterclockwise from the reference magnet – within one coil period $\tau_{dema}^{(c)}$ the faulty magnet actually moves along $\tilde{\tau}_{dema}^{(c)}$, which results in an orientation of the ellipse in the direction of the coils passed by the faulty magnet.

Figure 7.5: Exemplary electrical machines with 5 phases. The **coil zones** are separated by red lines, the **demagnetized magnet** is yellow [69].



- (a) The demagnetized magnet is the first (reference) magnet (see Fig. 7.5a).



- (b) The demagnetized magnet is the fifth magnet counterclockwise from the reference magnet (see Fig. 7.5b).

Figure 7.6: Ellipse parameters for the case of coil period $\tau_{dema}^{(1)}$ (green marked) [69].

7.2.1 Fitting of ellipses

Since the voltage correction $\Delta\vec{u}$ along one coil period is not fully elliptical (or circular) in reality, an ellipse can be fitted. The method used in this work was derived by Fitzgibbon et. al. [75], a reference publication for direct least square fitting of an ellipse to n_{mes} two-dimensional data points. Using the conic equation

$$F(\vec{a}, \vec{x}) = \vec{a}^T \vec{x} = ax^2 + bxy + cy^2 + dx + ey + f = 0$$

with $\vec{a}^T = (a \ b \ c \ d \ e \ f)$ and $\vec{x}^T = (x^2 \ xy \ y^2 \ x \ y \ 1)$ an efficient minimization problem can be defined:

$$\begin{aligned} & \text{minimize } K(\vec{a}) = \sum_{n=1}^{n_{mes}} F(\vec{a}, \vec{x}(n))^2 \\ & \text{subject to } h(\vec{a}) = \vec{a}^T \mathbf{C} \vec{a} - 1 = 0 \end{aligned}$$

where $\mathbf{C} = \begin{pmatrix} 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ (7.8)

The quadratic equality constraint h allows a non iterative algebraic solution, avoids the trivial solution $\vec{a}_{opt} = \vec{0}_6$ and enforces an ellipse to be fitted to the data points $\vec{x}$. Solving (7.8) yields the optimal (non-trivial) parameters $\vec{a}_{opt}$ of the conic equation. Thereby $\{\underline{P}_{maj1}, \underline{P}_{maj2}\}$, $\{\underline{P}_{min1}, \underline{P}_{min2}\}$ and $\underline{P}_0$ can be extracted, defining the ending points of the major and minor axes as well as the center point of the fitted ellipse in the complex plane, as sketched in Fig. 7.6a and Fig. 7.6b.

Now it is necessary to decide which of the two points $\{\underline{P}_{maj1}, \underline{P}_{maj2}\}$ provides the information about the position of the faulty magnet. For this purpose, the complex correction emphasis $\underline{K}_s$ is calculated, which is based on the RMS of each coil along one coil period:

$$\underline{K}_s = \sum_{c=1}^{n_{coil}} \sqrt{\frac{1}{n_{mes}} \sum_{n=1}^{n_{mes}} (\Delta u^{(c)}(n))^2 e^{j\varphi_{coil}^{(c)}} + \underline{P}_0}. \quad (7.9)$$

where n_{mes} is the number of stored voltage corrections within one coil period $\tau_{demag}^{(c)}$. $\underline{K}_s$ approximately points either in direction of $\underline{P}_{maj1}$ or $\underline{P}_{maj2}$, whereby the ellipse orientation

point $\underline{P}_\Gamma$ can be defined:

$$\underline{P}_\Gamma = \begin{cases} \underline{P}_{maj1} & \text{if } \Gamma > 0 \\ \underline{P}_{maj2} & \text{if } \Gamma < 0 \end{cases} \quad (7.10)$$

with

$$\Gamma = \left(\frac{\text{Re}(\underline{K}_s - \underline{P}_0)}{\text{Im}(\underline{K}_s - \underline{P}_0)} \right) \cdot \left(\frac{\text{Re}(\underline{P}_{maj1} - \underline{P}_0)}{\text{Im}(\underline{P}_{maj1} - \underline{P}_0)} \right). \quad (7.11)$$

The orientation angle γ at actual coil period $\tau_{demag}^{(c)}$ can then be determined by

$$\begin{aligned} \gamma &= \arg(\underline{P}_\Gamma - \underline{P}_0) - \arg(\underline{P}_{coil}^{(c)} - \underline{P}_0) \\ &= \arg(\underline{P}_\Gamma - \underline{P}_0) - \varphi_{coil}^{(c)} \end{aligned} \quad (7.12)$$

and the ellipticity ϵ by

$$\epsilon = \frac{|\underline{P}_{maj1} - \underline{P}_0| + |\underline{P}_{maj2} - \underline{P}_0|}{|\underline{P}_{min1} - \underline{P}_0| + |\underline{P}_{min2} - \underline{P}_0|}. \quad (7.13)$$

7.2.2 Detection

To detect a partial demagnetization the ellipticity ϵ of the voltage correction is analyzed consecutively. In the healthy (balanced) case values of ϵ close to 1 are to be expected. In the event of a demagnetization fault, the value of ϵ will increase markedly. The detection of partial demagnetization is contingent upon the definition of a specific threshold value $\epsilon_{thres.}$. To achieve an improved robustness ϵ is smoothed by a moving average. Once this threshold is exceeded, partial demagnetization is detected. Thereby it should be noted that additional effects may contribute to an electrical imbalance of the PMSM, which could then result in the erroneous triggering of the demagnetization fault handle.

7.2.3 Identification

To identify the specific demagnetized magnet the following algorithm is used:

1. Determine and store the orientation angle γ of the ellipse based on the voltage correction $\Delta \underline{u}$ for multiple coil periods $\tau_{demag}^{(c)}$ with $c \in \{1, 2, \dots, n_{coil}\}$.
2. Calculate the arithmetic average γ_{mean} over all stored orientation angles γ (for improved robustness).
3. Determine the index of the demagnetized magnet by: $i_{demag} = \text{nint}\left(\frac{\gamma_{mean}}{2\pi/n_{magnet}} + 1\right)$, where $\text{nint}(x)$ is the nearest integer function.
4. Take now the identified demagnetized magnet i_{demag} as reference magnet for the definition of the rotor angle θ_R .

If the demagnetized magnet is the reference as shown in Fig. 7.5a, the orientation of the ellipse always tends to show in coil direction, $\frac{\gamma_{mean}}{2\pi/n_{magnet}} + 1 \approx 1$ (see Fig. 7.6a). If the demagnetized magnet is not the reference, the orientation is twisted according to the angle between the reference and the demagnetized magnet. As shown in Fig. 7.5b, the fifth magnet counterclockwise from the reference magnet is demagnetized and thereby $\frac{\gamma_{mean}}{2\pi/n_{magnet}} + 1 \approx 5$ (see Fig. 7.6b).

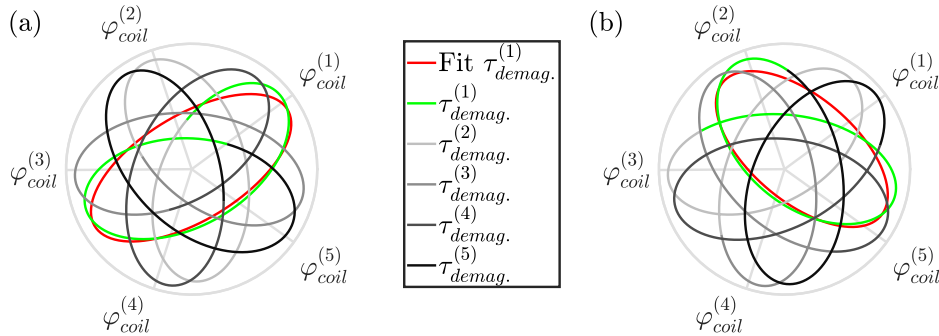


Figure 7.7: Actual (idealized) trajectory of the voltage correction in the complex plane for one full mechanical period, where the demagnetized magnet is (a) the reference magnet and (b) the fifth magnet counterclockwise from the reference. In both cases the **trajectory of coil period $\tau_{demag}^{(1)}$** is compared to the **ellipse with the least square algebraic distance** [69].

As it can be seen in Fig. 7.7, the deviation of the voltage correction along each coil period to an ideal ellipse is depending on the actual demagnetized magnet relative to the reference. In Fig. 7.7a, where the demagnetized magnet is the reference magnet, the curve of the voltage correction is close to an ellipse. Thereby the orientation angle γ can be determined most accurately. In Fig. 7.7b, where the demagnetized magnet is the fifth magnet, there is a

clear deviation to an ellipse as it is a continuous overlay of two coil periods. This shows that the closer the demagnetized magnet is to the reference, the more elliptical is the correction and thereby the smaller the error in the orientation angle. Identifying the demagnetized magnet over multiple iterations, with the method described above, the robustness can be further improved.

7.2.4 Adaptation

After the demagnetized magnet has been identified, the demagnetization amplitude $\hat{M}_{demag}$ needs to be adapted. Assuming that the difference between this estimated demagnetization amplitude $\hat{M}_{demag}$ and the actual demagnetization amplitude $\bar{M}_{demag}$, i.e. the estimation error $\Delta\hat{M}_{demag}$, is proportional to the term $(\epsilon - 1)$

$$(\hat{M}_{demag} - \bar{M}_{demag}) = \Delta\hat{M}_{demag} \approx \kappa(\epsilon - 1) \quad (7.14)$$

and demanding an asymptotically stable convergence in $\Delta\hat{M}_{demag}$ of the form

$$\frac{d}{dt}\Delta\hat{M}_{demag} = -\frac{1}{\tau}\Delta\hat{M}_{demag} \approx -\frac{\kappa}{\tau}(\epsilon - 1), \quad (7.15)$$

the following time discrete dynamic for $\tau > 0$ can be derived

$$\frac{(\hat{M}_{demag}^{(i+1)} - \bar{M}_{demag}) - (\hat{M}_{demag}^{(i)} - \bar{M}_{demag})}{t^{(i+1)} - t^{(i)}} \approx -\frac{\kappa}{\tau}(\epsilon - 1). \quad (7.16)$$

By rearranging this equation, a new estimated demagnetization amplitude $\hat{M}_{demag}^{(i+1)}$ is obtained after completion of coil period $i + 1$ and starting with $\hat{M}_{demag}^{(i=0)} = 0 \frac{\text{A}}{\text{m}}$ for $i = 0$:

$$\hat{M}_{demag}^{(i+1)} = \hat{M}_{demag}^{(i)} - \frac{\kappa\Delta t}{\tau}(\epsilon - 1). \quad (7.17)$$

By the factor $\frac{\kappa\Delta t}{\tau}$ the speed of convergence is set. The sign of κ is depending on the sign of the demagnetized magnet. If it is declared as a magnet with magnetization in the positive direction, then κ is positive, otherwise κ is negative. After each correction period, the effective magnetization (6.53) is updated based on new Fourier coefficients $\{\tilde{a}_k^{(M)}, \tilde{c}_k^{(M)}\}$ and then incorporated in the feedforward and feedback control (see also Fig. 7.8). By successive adapting the demagnetization amplitude $\hat{M}_{demag}$ to the real value $\bar{M}_{demag}$, the asymmetry in the voltage correction is reduced and becomes again more circular in the complex plane. Thereby ϵ tends to values close to one and convergence would be reached. However, in reality not only the partial demagnetization but also other effects will have influence on ϵ , whereby the adaptation of the demagnetization amplitude $\hat{M}_{demag}$ could diverge. Nevertheless this

can be simply avoided by an adjustment of the correction:

$$\hat{M}_{demag}^{(i+1)} = \hat{M}_{demag}^{(i)} - \frac{\kappa \Delta t}{\tau} (\epsilon - 1) \cos(2\gamma). \quad (7.18)$$

Since in the close-circular case after the adaptation the orientation angle γ will tend to have randomly distributed values (a circle does not have an orientation), the sign of the correction will change randomly as well due to the added factor $\cos(2\gamma)$ in (7.18). Thereby the small remaining corrections will cancel out each other over multiple periods and an asymptotic convergence is ensured.

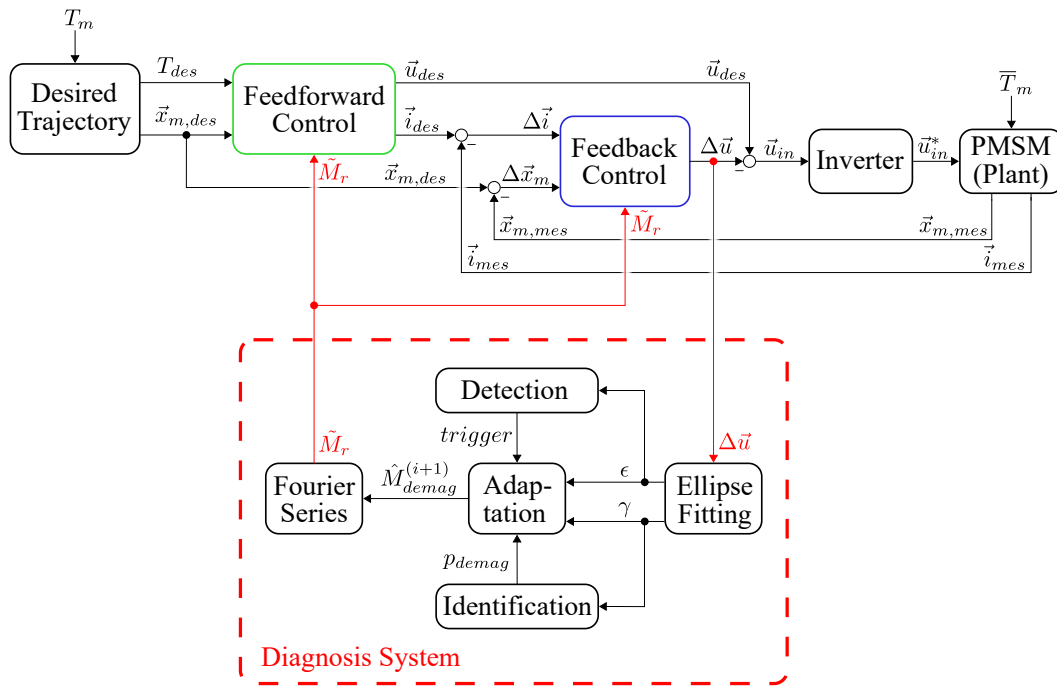


Figure 7.8: Structure of the control, with the **diagnosis system** highlighted [69].

For the field weakening, a dynamic interpolation of the field weakening LUTs for each estimate $\hat{M}_{demag}^{(i+1)}$ would be required (e.g. by a neuronal network). Since this dynamic interpolation is not part of this work, field weakening is not considered the following evaluation.

7.2.5 Evaluation

In the following, the performance of the diagnosis in combination with the control system (see Fig. 7.8) is analyzed. For this purpose, a full demagnetization of the 7-th magnet (seen from the reference magnet) is assumed at $t_{fault} = 3.0s$.

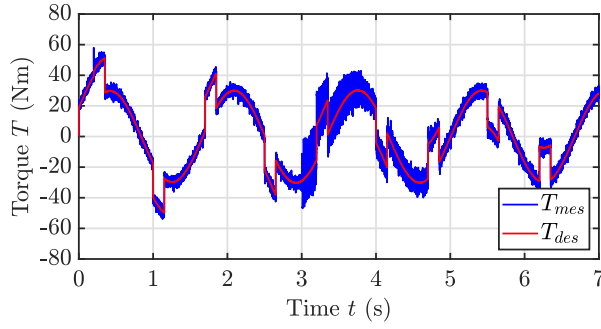
The simulation results are presented in Fig. 7.9. Due to the partial demagnetization the complex voltage corrections become elliptical (see Fig. 7.9g), which manifests in an increase of the smoothed ellipticity $\tilde{\epsilon}$ over time after fault occurrence (see Fig. 7.9d). It can be seen

that $\tilde{\epsilon}$ quickly exceeds the threshold value $\epsilon_{thres.} = 1.25$, whereby the partial demagnetization fault is detected.

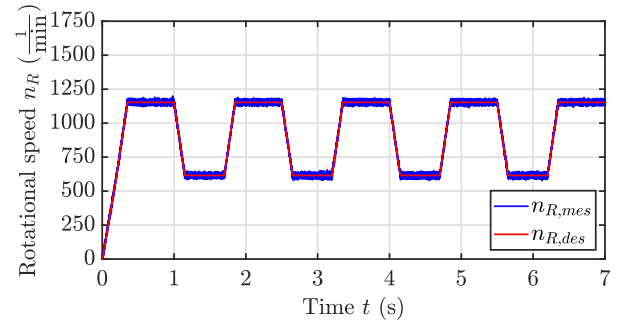
At this time the identification process starts, see Fig. 7.9e. Here, the orientation angle γ for 11 coil periods is shown as well as the value γ_{mean} obtained by averaging over these 11 periods. The average orientation angle is compared to the angles of the magnets, relative to the reference magnet. It is obvious that the determined γ_{mean} matches most with the 7-th magnet ($\varphi_{magnet}^{(7)}$) – the demagnetized magnet could be identified correctly.

After the identification of the demagnetized magnet it is set to be the reference magnet. Now, the adaptation phase starts (Fig. 7.9f). It can be observed that the demagnetization amplitude $\hat{M}_{demag}$ successively adapts to the actual, real amplitude $\hat{\bar{M}}_{demag}$. However, after converging, there is still a small difference between $\hat{M}_{demag}$ and $\hat{\bar{M}}_{demag}$. This phenomenon occurs when the voltage correction reaches a state of near-circularity in the complex plane, at which point the remaining discrepancies are rectified by the control (see also Fig. 7.9h).

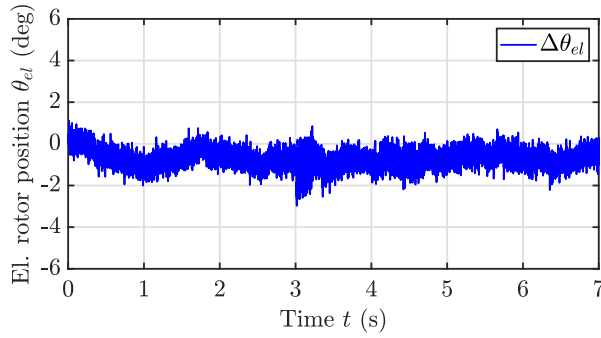
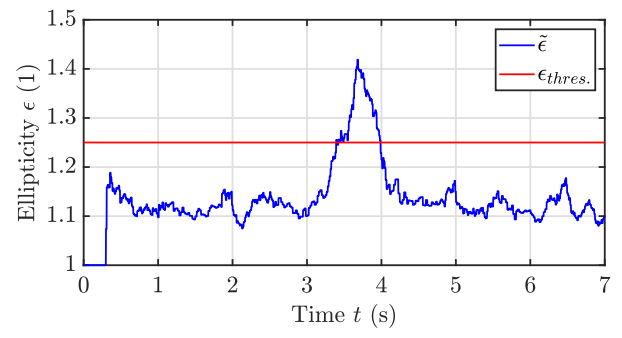
In Fig. 7.9a the actual torque produced by the PMSM is drawn and compared to the desired torque. It can be seen that the torque oscillation increases at fault occurrence ($t = 3s$). Nevertheless, after the adaptation of the demagnetization amplitude for $t > 5.0s$ is converged, the oscillation is decreased again to values close to prefault conditions.



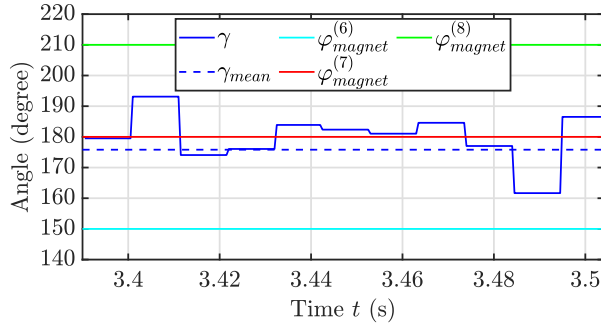
(a) Actual and desired torque.



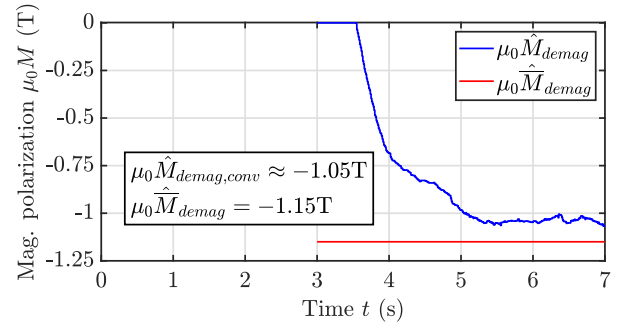
(b) Actual and desired rotational speed.

(c) Deviation of the rotor angle ($\Delta\theta_{el} = p\Delta\theta_R$).

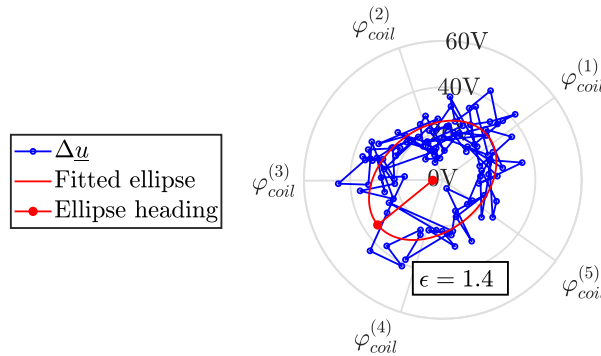
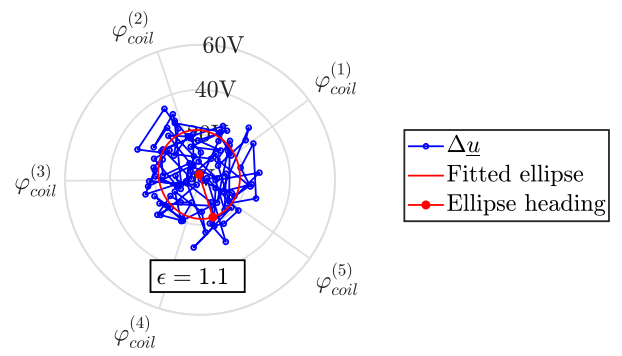
(d) Detection of partial demagnetization.



(e) Identification of partial demagnetization.



(f) Adaptation of partial demagnetization.

(g) Complex voltage corrections of coil period $\tau_{demag}^{(1)}$ after fault occurrence ($t \approx 3s$).(h) Complex voltage corrections of coil period $\tau_{demag}^{(1)}$ after convergence ($t \approx 6s$).Figure 7.9: **Diagnosis of partial demagnetization** in the overall control system.

7.3 Discussion

In this chapter, a method for the adaptation to phase short circuits and an algorithm for the diagnosis of partial demagnetization were developed and analyzed. Therefore, the following conclusions can be drawn:

Adaptation to phase short circuits: The approach is based on the monitoring of a characteristic harmonic in the torque caused by the phase short circuit, which enables an iterative refinement of the estimated short circuit resistance. While the method shares some conceptual similarities with [39], its primary advantage lies in the efficient integration into the overall control framework and the explicit use of the derived physical equations describing the fault mechanism. This ensures an effective adaptation of the established control to the fault.

Diagnosis of partial demagnetization: A review of existing literature [2, 76–83] shows that signal analysis for partial demagnetization is typically performed using FFT or wavelet-based techniques. In contrast, the method proposed here monitors the voltage correction of the feedback control in the complex plane. By fitting an ellipse into this voltage trajectory, a qualitative characteristic in the signal is quantified by the ellipticity ϵ and orientation angle γ . This approach is inherently robust to high noise and transients due to the filtering effect of curve fitting. In combination with the developed multi-fault tolerant control system, the method enables reliable detection, identification, and compensation of partial demagnetization under transient conditions.

To harness the benefits of the developed multi-fault tolerant control system in practical applications, it is essential to establish a comprehensive diagnostic system. This system must not only be able to detect single faults but also reliably identify combinations of faults and estimate the relevant physical fault parameters with sufficient accuracy. The methods presented in this chapter may be regarded as some foundational components of a comprehensive multi-fault diagnosis system capable of detecting, identifying, and adapting all relevant fault scenarios. The development of such a system remains an important subject for future research [2].

8 Analysis of the multi-fault tolerant control

In this chapter, the multi-fault tolerant operation of the proposed control system is analyzed. By efficiently accounting for all relevant combinations of the previously discussed fault types (see chapter 6), this chapter represents the main original contribution of the present work. To illustrate the capabilities of the system, three exemplary scenarios are examined in which multiple faults occur simultaneously.

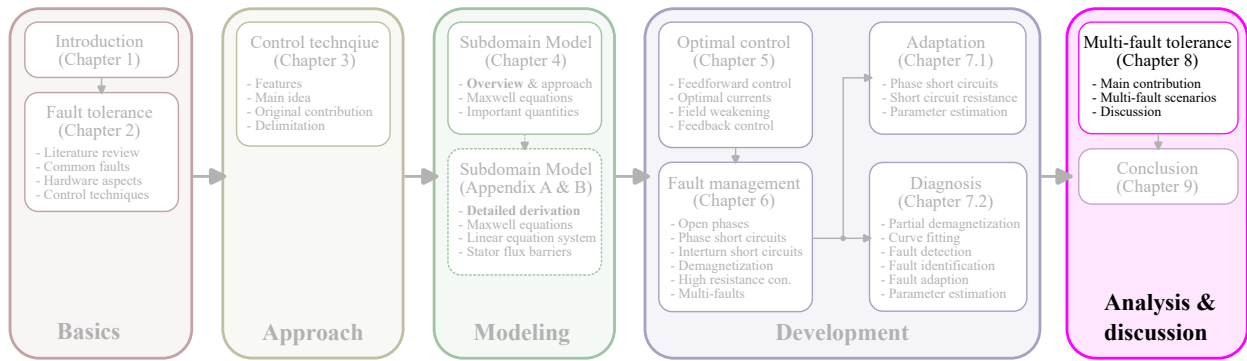


Figure 8.1: Structural overview of the work, with this chapter 8 highlighted.

8.1 Open phase and demagnetization

A realistic multi-fault scenario could be the following one: due to a short circuit in one phase or coil the corresponding fuse has been triggered, resulting in an open phase condition. There is always a small latency between the occurrence of a short circuit and the triggering of the fuse. In this time high currents may occur. This can cause demagnetization of the magnet which is near the short circuited coil because the magnetic field is strongest there. In the following analysis, it is assumed that the affected magnet is fully demagnetized.

Feedforward control

To evaluate the performance of the feedforward control compensating simultaneously an open phase and a partial demagnetization fault, it is used to predict the optimal desired currents and voltages at two different operating points¹. In both operating points the desired currents exhibit a relatively complex waveform (Fig. 8.2c and Fig. 8.2d). On the one hand this results from the uneven spatial distribution of the remaining healthy coils, and on the other hand from the partial demagnetization, which causes a drastically increase in the cogging torque (Fig. 8.2e) and noticeable drops in the torque capabilities at the angular positions where the affected magnet passes by (Fig. 8.2f). By comparing the field weakening LUT in Fig. 8.2a, with the one considering solely an open phase in Fig. 6.2a, it can be seen that the resulting operating area is only marginal smaller, demonstrating an efficient compensation of the additional partial demagnetization fault.

Overall control system

The simulation results of the overall control system under dynamic conditions are displayed in Fig. 8.3¹. It is assumed that the open phase fault and the partial demagnetization occur simultaneously at $t_{fault} = 2.0s$ and that the control system is adapted to the fault after a delay of 10ms ($t_{adapt} = 2.01s$). Since the control is no longer able to follow the desired trajectory shown in Fig. 5.10 due to the decreased operating area (see Fig. 8.2a), the desired rotational speed is scaled down by a factor of 0.9.

By observing the actual torque during simulation (see Fig. 8.3a), it can be seen that there are some high peaks in the torque ripple at $t = t_{fault}$. Similar peaks are also visible in the deviation of the rotor angle, see Fig 8.3c. These are in both cases a consequence of the simulated delayed fault adaptation. At $t = t_{adapt}$, larger deviations between the actual currents, shown in Fig. 8.3e, and the desired ones, shown in Fig. 8.3g, can be observed. This is a result of the feedback control re-stabilizing the machine. Following this transient phase, the control system operates close to pre-fault conditions, demonstrating full compensation of the open phase and partial demagnetization.

¹In the plots of this section showing the desired currents $\vec{i}_{des}$, the actual currents $\vec{i}_{mes}$ or the torque capabilities $\frac{\ell\pi}{\mu_0}\vec{P}$, the open phase is represented by the **green curve**.

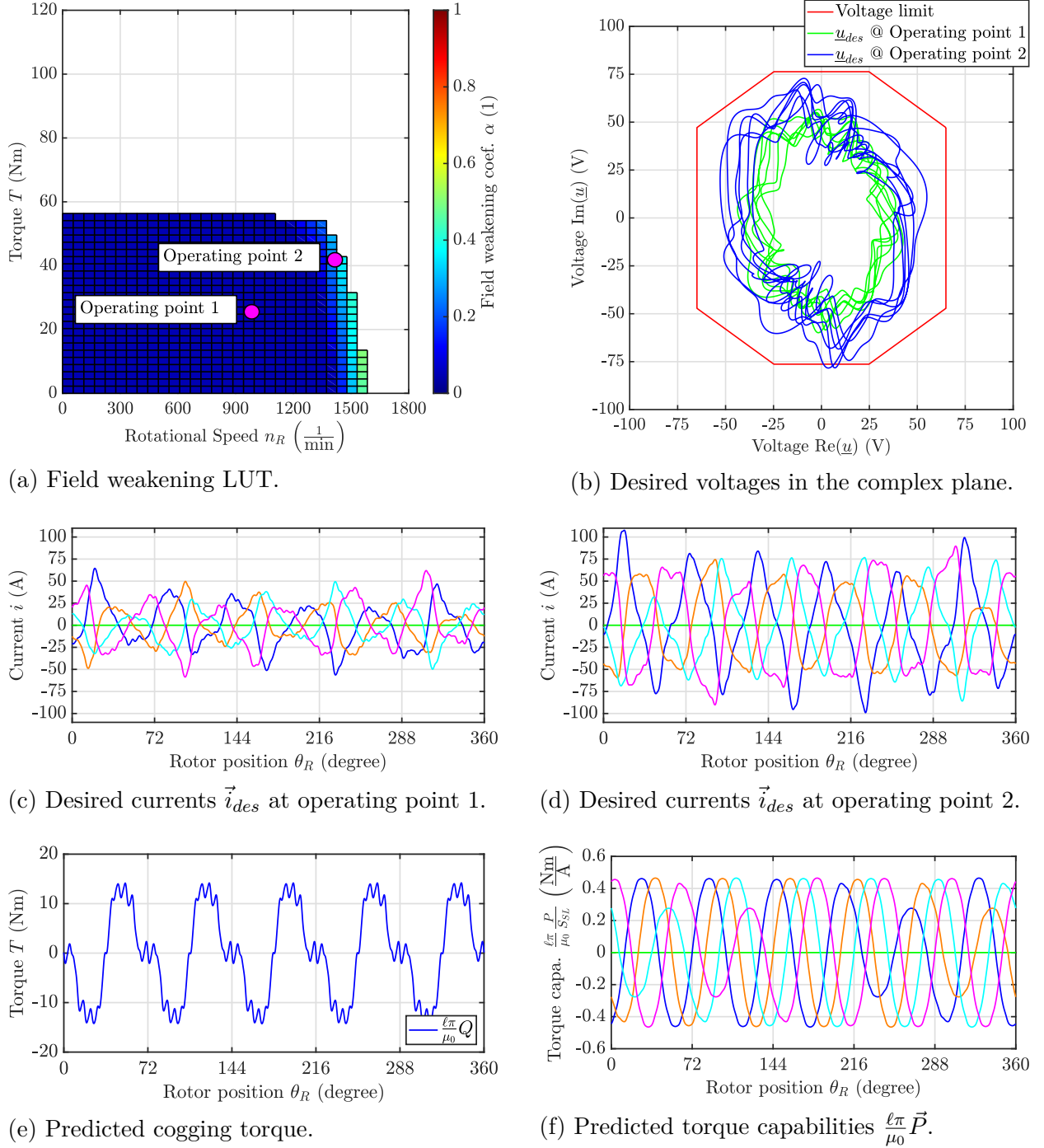
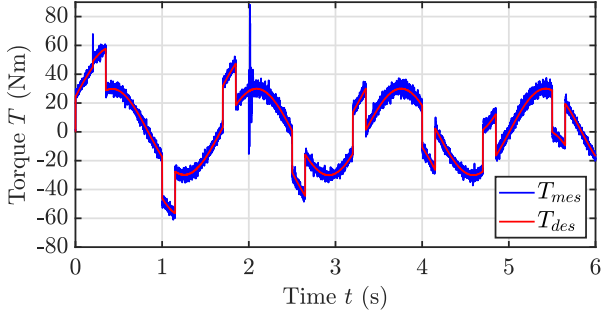
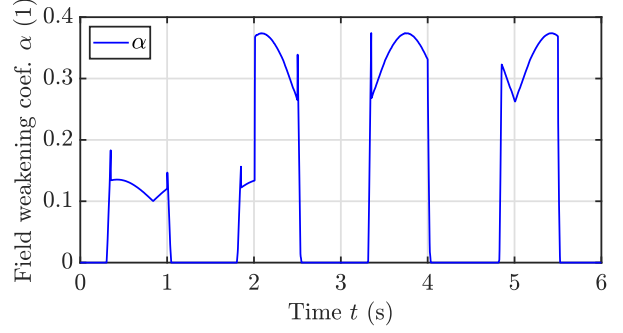


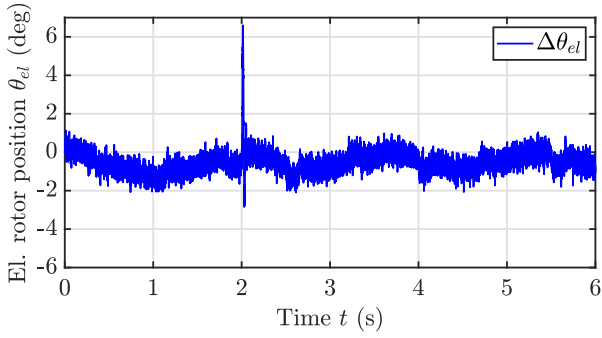
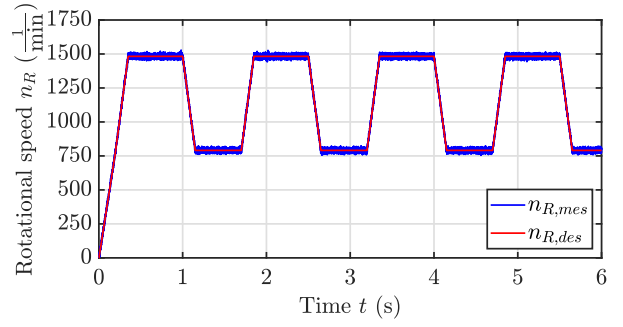
Figure 8.2: **Open phase and demagnetization** fault tolerant operation of the feedforward control.



(a) Actual and desired torque.



(b) Field weakening coefficient.

(c) Deviation of the rotor angle ($\Delta\theta_{el} = p\Delta\theta_R$).

(d) Actual and desired rotational speed.

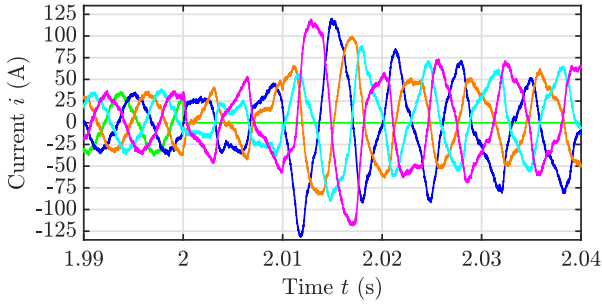
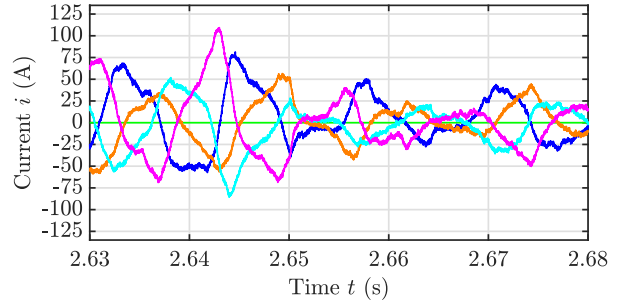
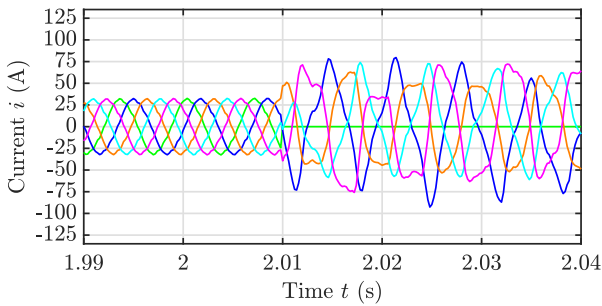
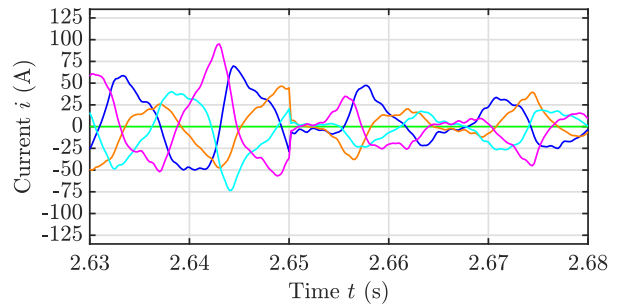
(e) Actual currents $\vec{i}_{mes}$ at the time period of fault occurrence.(f) Actual currents $\vec{i}_{mes}$ at a time period after fault adaptation.(g) Actual currents $\vec{i}_{des}$ at the time period of fault occurrence.(h) Actual currents $\vec{i}_{des}$ at a time period after fault adaptation.

Figure 8.3: **Open phase and demagnetization** fault tolerant operation of the overall control system.

8.2 Interturn short circuit, high resistance connection and demagnetization

The second scenario considers the simultaneous occurrence of an interturn short circuit in one phase, a high resistance connection in another phase, and a partial demagnetization. Although such a fault combination is unlikely in practice, the objective is to evaluate the robustness of the control system under these challenging fault conditions. Thereby the parameters for the interturn short circuit correspond to those listed in Tab. 6.2. Due to the increased current demand required for extinguishing the PM flux linkage, the maximum permitted RMS of each phase current used for calculating the field weakening LUT is raised to 120A (instead of 60A). For the partial demagnetization, it is assumed again that one magnet has been completely demagnetized and for the high resistance connection that the effective resistance of one phase increased to four times the value of a healthy phase.

Feedforward control

To evaluate the performance of the feedforward control in the described multi-fault scenario, it is used to predict the optimal desired currents and voltages at two different operating points².

Fig. 8.4a shows the resulting field weakening LUT. Compared to the one considering solely an interturn short circuit (Fig. 6.9a), it is noticeable that the operating area is only marginally smaller. However, the required values of the field weakening coefficient α to reach an operating point in the field weakening area are clearly increased in the multi-fault scenario. Since the current limit is still taken into account, these higher α values do not negatively affect the operation. Fig. 8.4c illustrates the desired currents at an operating point where no field weakening is necessary. It can be observed that the desired current of the faulty coil affected by the interturn short circuit is the highest, due to the large current required to extinct the PM flux linkage. Between rotor positions $144^\circ < \theta_R < 216^\circ$, the demagnetized magnet passes the coil with interturn short circuit, which accordingly leads to a temporarily reduction in the desired current. The current waveforms of the other coils are complex, caused by the asymmetrical phase shift of the extinction current in the faulty phase (see also section 6.3). This asymmetry also results in a highly complex trajectory of the desired voltages, as shown in Fig 8.4b. When field weakening is required (Fig. 8.4d), the asymmetry, and thus the complexity of the current waveforms is reduced. In both operating points, the current amplitude of the coil with the high resistance connection is reduced to

²In the plots of this section showing the desired currents $\vec{i}_{des}$, the actual currents $\vec{i}_{mes}$ or the torque capabilities $\frac{\ell\pi}{\mu_0}\vec{P}$, the phase with interturn short circuit is represented by the **green curve** and the phase with high resistance connection by the **orange curve**.

minimize the copper losses.

Overall control system

The simulation results of the overall control system under dynamic operating conditions are shown in Fig. 8.5². Thereby it is assumed that all three faults (interturn short circuit, high resistance connection and partial demagnetization) occur at $t_{fault} = 2.0\text{s}$ and that the control system is adapted to the fault after a delay of 40ms ($t_{adapt} = 2.04\text{s}$).

The curve of the short circuit current i_s at the time of fault occurrence is depicted in Fig. 8.5g. There, a strong increase in the amplitude can be seen for $2.0\text{s} < t < 2.04\text{s}$, which results in an increased torque ripple (Fig. 8.5a). This is the time period where the control system is not yet adapted to the fault. After the fault adaptation of the control at $t \approx 2.04\text{s}$ the short-circuit current is rapidly reduced from about 400A to 50A, leading to a mitigation of the torque ripple. Nevertheless, compared to the pre-fault conditions, an increased level of torque ripple remains present. This is a result of the highly complex current waveforms required for this multi-fault compensation, which cause larger deviations between the actual and the desired currents. By comparing the actual torque with the curve of the field weakening coefficient α , it strikes out that this increased torque ripple mainly occurs at the time intervals where no field weakening is required. This is a result of the more complex current waveforms in the base speed range (as described above).

Overall, it can be shown that the developed control system is still full operational (including the field weakening strategy) even in the event of three simultaneously occurring faults.

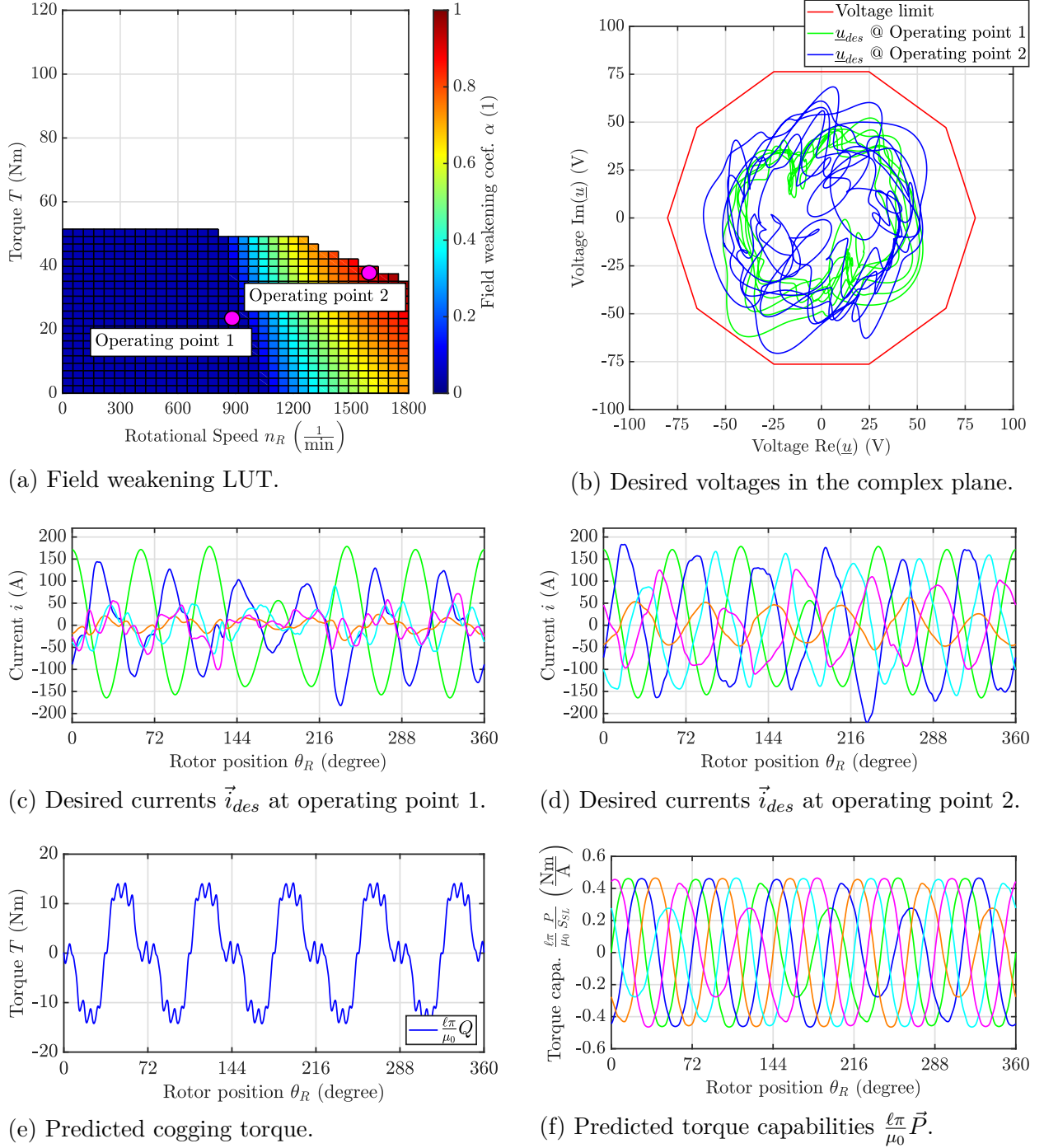
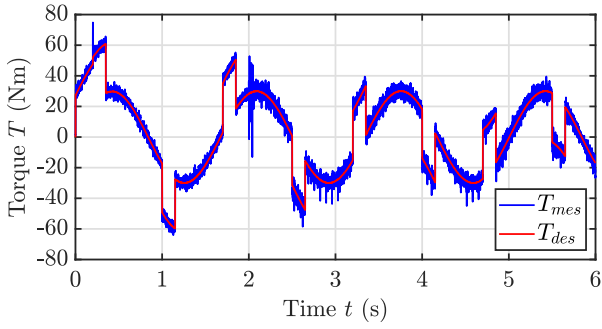
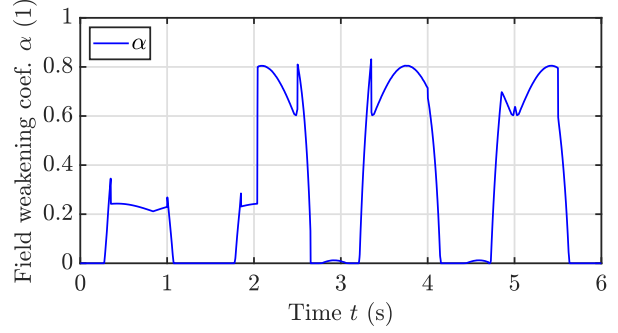


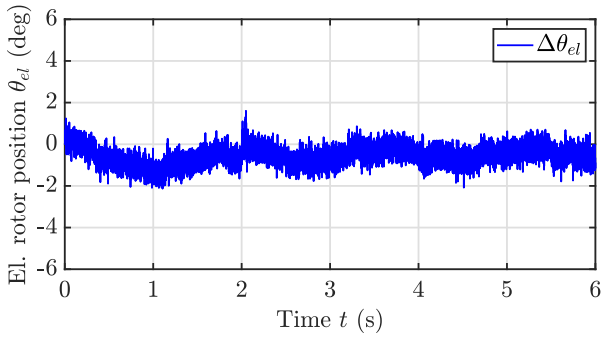
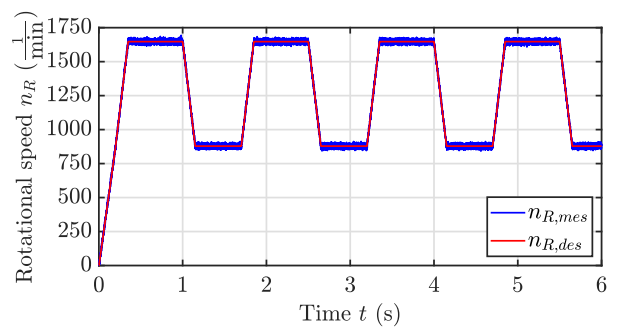
Figure 8.4: **Interturn short circuit, high resistance connection and demagnetization** fault tolerant operation of the feedforward control.



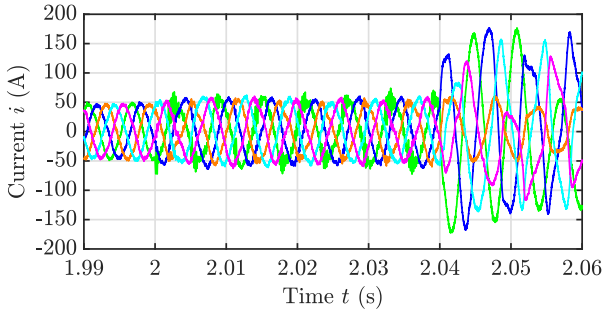
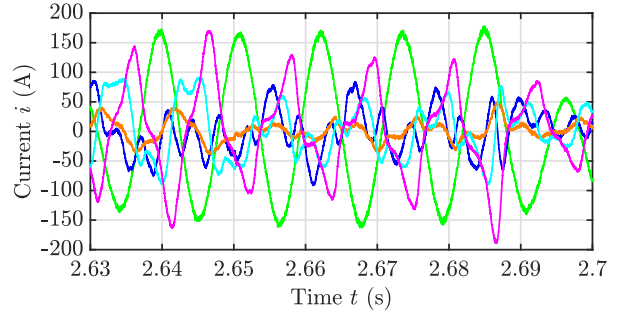
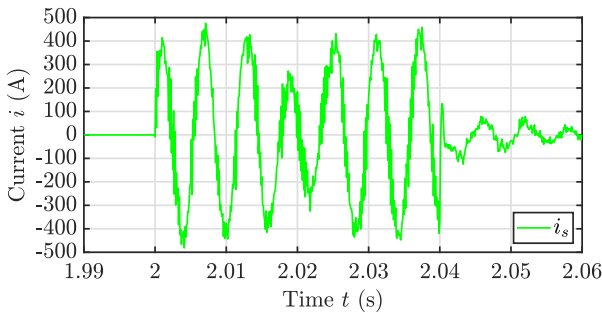
(a) Actual and desired torque.



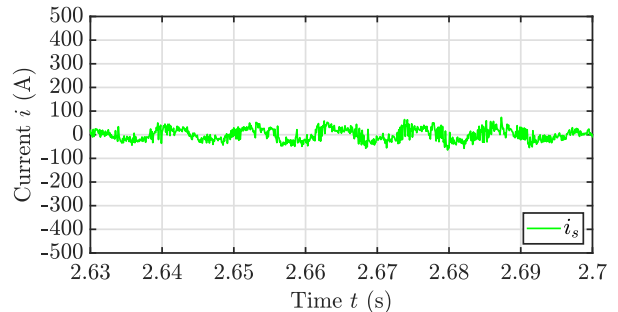
(b) Field weakening coefficient.

(c) Deviation of the rotor angle ($\Delta\theta_{el} = p\Delta\theta_R$).

(d) Actual and desired rotational speed.

(e) Actual currents $\vec{i}_{mes}$ at the time period of fault occurrence.(f) Actual currents $\vec{i}_{mes}$ at a time period after fault adaptation.

(g) Short circuit current at the time period of fault occurrence.



(h) Short circuit current at a time period after fault adaptation.

Figure 8.5: **Interturn short circuit, high resistance connection and demagnetization** fault tolerant operation of the overall control system.

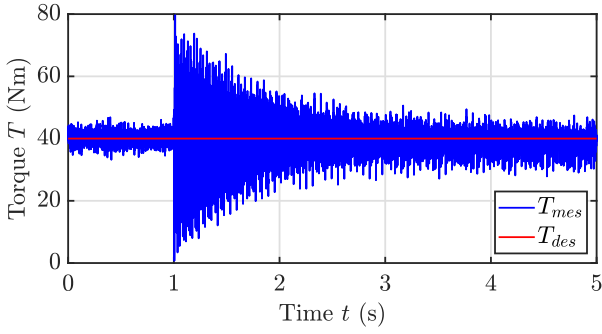
8.3 Adaptation of phase short circuit and demagnetization

In the third scenario, the performance of the developed control system in combination with the introduced adaptation algorithm (see section 7.1) is evaluated under the presence of a phase short circuit and a partial demagnetization. It is assumed that the control system is in a steady state with a (constant) desired rotational speed of $n_{des} = 1000 \frac{1}{\text{min}}$, a (constant) load torque of $T_m = 40\text{Nm}$ and the phase short circuit as well as the partial demagnetization appearing simultaneously at $t_{fault} = 1.0\text{s}$. The real value of the short circuit resistance is set to $\bar{R}_{ps} = 0.4\Omega$ and the initial estimate to $R_{ps}^{(i=0)} = 5\Omega$. For the partial demagnetization it is assumed that one magnet is fully demagnetized, that it is diagnosed when the fault occurs and that the Fourier coefficients of the magnetization are adapted accordingly.

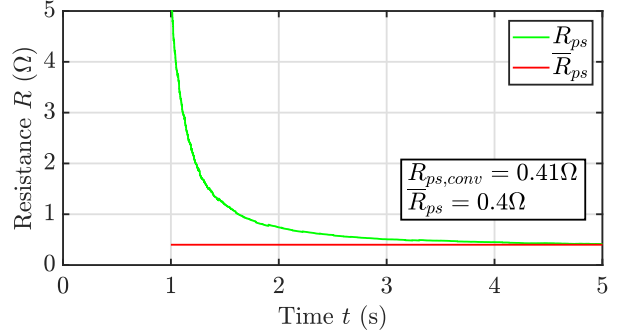
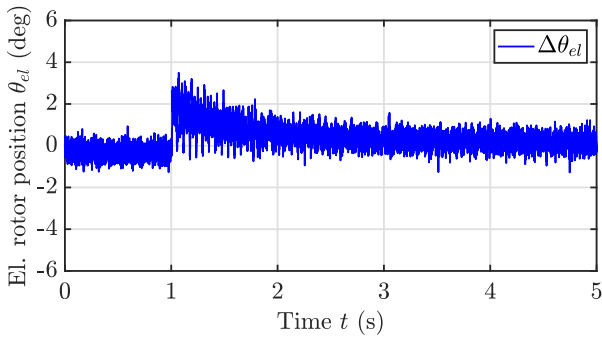
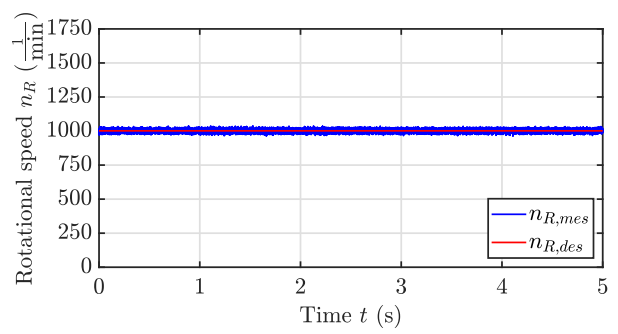
The simulation results of the overall control system are displayed in Fig 8.6³. By observing the real short circuit torque (see Fig. 8.6g and 8.6h), it can be seen that there are periodic drops in the ripple. This is caused by the demagnetized magnet each time it passes the faulty coil with phase short circuit. While this drop does influence the spectrum of the torque ripple, the harmonic $v = 2p$ still remains dominant (see Fig. 6.5). Thereby, the estimated short circuit resistance still successively adapts after fault occurrence its real value (see Fig. 8.6b), improving the prediction (Fig. 8.6g) and thus the compensation (Fig. 8.6a) of the faulty torque ripple T_{ps} . Since the demagnetization is considered in the feedforward control by the adapted Fourier coefficients, the estimated waveform of the faulty torque ripple is predicted accurately after the adaptation is converged (Fig. 8.6h). The slight increased ripple in the actual torque after convergence compared to the prefault conditions is again a result of the more complex current waveforms required under the multi-fault scenario (Fig. 8.6f), which result in higher deviations of the actual currents to the desired ones. Furthermore, demagnetization is not explicitly accounted for in the algorithm used to adapt the short circuit resistance (see section 7.1), which results in a slightly less optimal convergence of the estimated short circuit resistance compared to the scenario shown in Fig. 7.3, involving only a phase short circuit⁴.

³In the plots of this section showing the actual currents $\vec{i}_{mes}$, the phase short circuited phase is represented by the **green curve**.

⁴Since the *estimated* PM flux linkage is assumed to be higher than the *real* one (see Tab. 5.3), the estimated faulty torque ripple would be higher as well compared to the real one at the same short circuit resistance. For this reason, the optimal estimated short circuit resistance is slightly larger than the real one, as evident in Fig. 7.3b.



(a) Actual and desired torque.

(b) **Estimated** and **real** short circuit resistance.(c) Deviation of the rotor angle ($\Delta\theta_{el} = p\Delta\theta_R$).

(d) Actual and desired rotational speed.

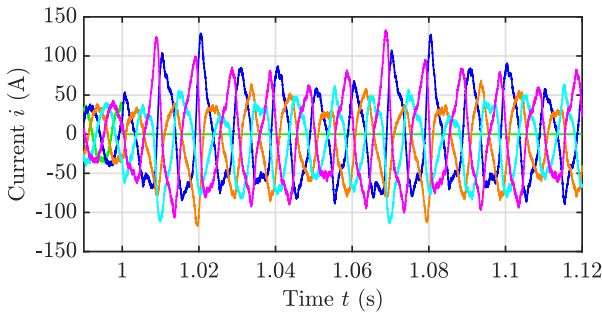
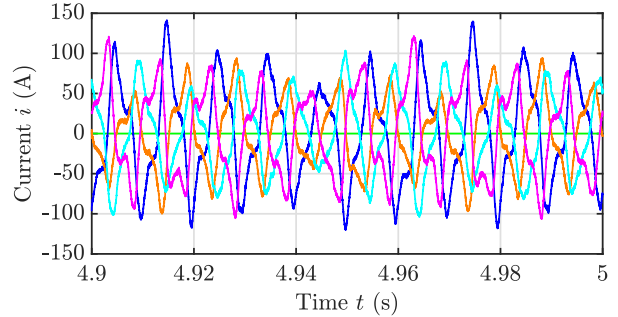
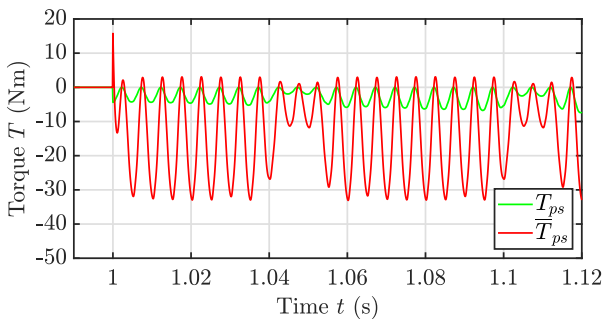
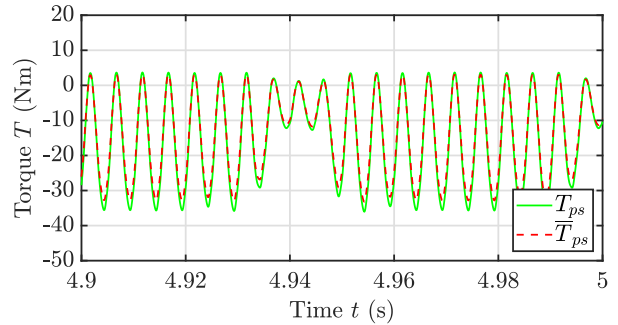
(e) Actual currents $\vec{i}_{mes}$ at the time period of fault occurrence.(f) Actual currents $\vec{i}_{mes}$ at a time period after convergence.(g) **Estimated** and **real** faulty torque ripple at the time period of fault occurrence.(h) **Estimated** and **real** faulty torque ripple at a time period after convergence.

Figure 8.6: **Phase short circuit and demagnetization** fault tolerant operation of the overall control system, including the algorithm for adapting the short circuit resistance.

8.4 Concluding discussion

In this work it was demonstrated that the developed control system is capable of handling complex fault scenarios involving both single faults (chapter 6) and multiple simultaneous faults (chapter 8). Through selected, representative simulations, it was shown that the control strategy maintains stable operation and successfully compensates open phase faults, phase short circuits, interturn short circuits, demagnetization faults, and high resistance connections by leveraging the fault-handling strategies developed in chapter 6. Furthermore, the field weakening strategy was shown to remain fully functional under consideration of the electrical limitations.

Despite its strengths, some challenges and limitations exist in the practical application of the proposed control system. The accuracy of the control relies on the knowledge of the relevant physical fault parameters. This can be challenging in practice, particularly for faults like interturn short circuits as well as combinations of simultaneous faults, where the fault parameter estimation is inherently difficult. To address this, a suitable diagnostic system would be required, which may not yet be available and would therefore need to be developed as part of future work.

Although the mathematical formulation is computationally efficient due to the ability to precalculate and store constant components of the relevant equations, the overall computational effort remains higher than that of conventional control methods. This increased complexity may lead to higher demands on the processing hardware, which must be taken into account for real-time applications. However, a major advantage is the formulation of the Subdomain Model equations using Fourier series. This allows the number of considered Fourier coefficients to be increased or decreased as needed, making it possible to optimally adjust the computational effort to the processing capabilities of the target hardware. Nevertheless it should be noted that this also directly affects the accuracy of the control.

Another challenge is the determination of the load torque, which in this work is assumed to be extracted through a direct measurement of the PMSM's produced torque. In practical applications torque measurement shafts are rarely used due to their high cost and mechanical sensitivity. An alternative approach could involve an estimation of the load torque based on a detailed model of the mechanical dynamics. However, the feasibility and accuracy of such a model depends strongly on the specific application.

A notable observation in the shown simulations is the significant increase in complexity of the desired current waveforms under multi-fault conditions. These waveforms deviate strongly from sinusoidal shapes, requiring the power electronics to operate with higher precision. This imposes additional demands on the inverter design and switching performance, which must be taken into account in practical implementations.

Overall, the proposed control strategy is most suitable for safety-critical applications, such as electrified aircraft propulsion systems, in which even unlikely multi-fault scenarios must be considered while still ensuring accurate tracking of the desired trajectory. Although the method entails increased demands on power electronics and processing hardware, these requirements are acceptable in such environments, where elevated system robustness and performance are essential anyway.

9 Conclusion and outlook

The objective of this work was to develop a control system for Permanent Magnet Synchronous Machines (PMSMs) with an explicit focus on the multi-fault tolerant operation, to compensate not only a wide variety of individual faults, but also all possible combinations of faults. An additional objective was to incorporate a robust field weakening strategy, capable of remaining functional across all considered fault conditions. To this end, the Subdomain Model was used as the foundation. This established modeling approach based on Maxwell equations enables a highly accurate computation of the magnetic flux distribution within the machine. Based on this model, a feedforward control was developed in a first step to predict optimal currents for accurately tracking a desired mechanical trajectory. It was then extended with an iterative algorithm to determine the requirement of field weakening in dependency of desired torque and rotational speed. The results were stored in a Lookup Table (LUT) for efficient online access. In a second step a feedback control was designed to compensate deviations from the desired mechanical trajectory by applying optimally distributed voltage corrections.

Overall, the strength of this approach lies in the detailed physical description of the PMSM using Maxwell equations, combined with an efficient mathematical formulation through the analytical solution of constrained minimization problems. This foundation enables systematic fault adaptation by requiring only minor adjustments in the optimization problems of the feedforward and feedback control. As a result, the control system can compensate open phase faults, phase short circuits, interturn short circuits, demagnetization faults, high resistance connections as well as all possible combinations of faults without requiring structural changes to the control architecture. Compared to the existing state of the art, the following original contributions have been achieved:

1. Control for PMSMs based on the highly accurate Subdomain Model which, by means of an efficient mathematical formulation, can be solved in real time.
2. High fault tolerance considering all relevant electrical and magnetic faults of PMSMs, including multi-fault tolerant operation (all possible combinations of faults).
3. Robust field weakening strategy that remains fully functional in the presence of faults.

In addition to these central contributions, a notable advancement is the compensation of (partial) demagnetization faults. Since existing approaches in literature on this specific topic are rare, the proposed method presents a relevant contribution. Arbitrary symmetric and asymmetric demagnetization profiles can be addressed efficiently by merely adjusting the Fourier coefficients of the magnetization considered in the control system.

In representative simulations it could be demonstrated that the control system is capable of handling all considered fault scenarios in dynamic operation while maintaining accurate tracking of the desired mechanical trajectory. Although the simulations are realistic within the scope of the modeling capabilities, experimental validation remains an essential future step to assess the 'real-world' performance of the control system. In particular, it is necessary to investigate the impact of magnetic saturation, which is not considered in the Subdomain Model, on the effectiveness of the control. Additionally, the feasibility of the complex current waveforms with actual inverter hardware and switching dynamics as well as the real-time capability of the control on the target processing hardware must be thoroughly evaluated. One further relevant aspect requiring experimental investigation is the influence of the machine geometry. Since real electrical machines can deviate from the idealized geometry assumed in the Subdomain Model, it is important to determine how these geometric discrepancies affect the model accuracy, and consequently the precision of the feedforward control. Moreover, the effectiveness of the feedback control in compensating this resulting prediction error must be evaluated to ensure reliable operation in practical implementations.

To harness the benefits of the developed multi-fault tolerant control system in practice, it is essential to establish a comprehensive diagnostic system. This system must not only detect single faults but also reliably identify combinations of faults and estimate the associated physical fault parameters with sufficient accuracy. The methods presented in this work for the algorithm to adapt the control to phase short circuits and diagnosing partial demagnetization can be regarded as some foundational components of such a comprehensive multi-fault diagnostic framework. Therefore they may provide a basis for future research and development.

A Detailed derivation of the Subdomain Model

(The following appendix is based on [60].)

The geometry of the PMSM considered in the Subdomain Model is sketched in Fig. A.2. The rotor, with radius r_R , has $2p$ magnets mounted on the surface, where p is the number of pole pairs. The magnets have a pole-arc to pole-pitch ratio of ϵ_M and are $r_M - r_R$ high. The stator, with inner radius r_S , contains n_{SL} slots, whereby each slot (a) is positioned at $\varphi_{SL}^{(a)}$, has a width of β_{SL} and a height of $r_{SL} - r_S$.

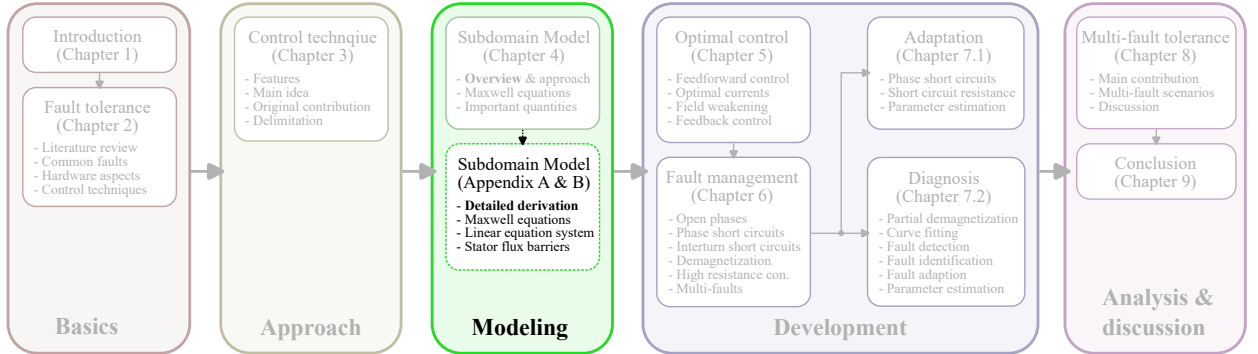


Figure A.1: Structural overview of the work, with appendices A and B highlighted.

Based on this considered geometry, the following assumptions are made for the modeling:

- **A1:** Infinite permeable ferromagnetic material (no magnetic saturation)
- **A2:** Infinite axial length of the electrical machine (side effects are neglected)
- **A3:** Rotor magnets are assumed to be radially magnetized with a relative recoil permeability equal to one
- **A4:** Each slot carries a homogeneously distributed current density oriented only in axial (z -) direction
- **A5:** Idealized slot geometry with only radial (r -) and tangential (φ -) oriented surfaces
- **A6:** No time dependencies (Maxwell equations of magnetostatics)

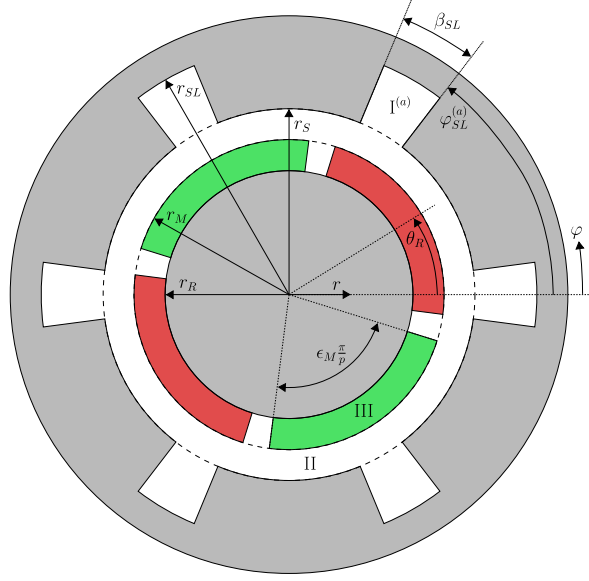


Figure A.2: Considered geometry and relevant parameters in the Subdomain Model [60].

All these assumptions made have in common that they drastically reduce the mathematical complexity and by that the computational effort. The thereby considered geometry is sketched in Fig. A.2. By neglecting the side effects at the (axial) edges of the PMSM, the magnetic flux distribution only shows a radial and tangential dependency (**A2**). This allows a two-dimensional modeling. Assuming an infinite permeability of the iron (**A1**), the magnetic flux enters or leaves orthogonally such high permeable material. This, in combination with the idealized slot geometry (**A5**), allows an efficient formulation of the relevant boundary conditions.

A.1 Derivation of Poisson's equation

Using the Maxwell equations of magnetostatics [54] (**A6**)

$$\nabla \cdot \vec{B} = 0 \quad (\text{A.1})$$

$$\nabla \times \vec{H} = \vec{J} \quad (\text{A.2})$$

and writing the relation between magnetic flux density $\vec{B}$ and magnetic field strength $\vec{H}$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) , \quad (\text{A.3})$$

where $\vec{M}$ is the magnetization, it can be rewritten based on Ampère's law (A.2) to

$$\nabla \times \vec{B} = \mu_0 (\vec{J} + \nabla \times \vec{M}) . \quad (\text{A.4})$$

Because $\vec{B}$ is always a source-free field (A.1), it can be determined by applying the differential operator $\nabla \times$ to a (still unknown) field $\vec{A}$, the magnetic vector potential, resulting in¹:

$$\vec{B} = \nabla \times \vec{A} \quad (\text{A.5})$$

Inserting equation (A.5) in (A.4), one obtains:

$$\nabla \times (\nabla \times \vec{A}) = +\mu_0 (\vec{J} + \nabla \times \vec{M}) , \quad (\text{A.6})$$

$$\nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = +\mu_0 (\vec{J} + \nabla \times \vec{M}) . \quad (\text{A.7})$$

Since the gauge fixing

$$\vec{A}' \rightarrow \vec{A} + \nabla \Lambda \quad (\text{A.8})$$

with Λ an appropriate scalar function, does not influence the magnetic field $\vec{B}$, an additional condition can be demanded, which is called Coulomb-gauge²:

$$\nabla \cdot \vec{A} = 0 . \quad (\text{A.9})$$

Thereby the Maxwell equation to be solved simplifies to Poisson's equation:

$$\nabla^2 \vec{A} = -\mu_0 (\vec{J} + \nabla \times \vec{M}) . \quad (\text{A.10})$$

With the vector potential showing only an axial dependency (assumption **A2**), the magnetization having only a radial orientation (assumption **A3**) and the current densities only an axial orientation (assumption **A4**), the following partial differential equation is obtained:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_z}{\partial \varphi^2} = -\mu_0 J_z + \frac{\mu_0}{r} \frac{\partial M_r}{\partial \varphi} . \quad (\text{A.11})$$

Using the relation between the vector potential and the magnetic field (A.5), the radial (r -) and tangential (φ -) magnetic field in 2-dimensional polar coordinates can be obtained by

$$B_r = +\frac{1}{r} \frac{\partial A_z}{\partial \varphi} , \quad (\text{A.12})$$

$$B_\varphi = -\frac{\partial A_z}{\partial r} . \quad (\text{A.13})$$

¹ $\nabla \cdot (\nabla \times \vec{V}) = 0$, for any smooth vector field $\vec{V}$.

² $\nabla \times (\nabla \Lambda) = 0$, for any smooth scalar field Λ .

The solution of (A.11) is achieved by using different Fourier expansions for each of the three different subdomains, sketched in Fig. A.2: subdomain I^(a) for the stator slot with a given current density $J_z^{(a)}$ and (a) running from 1 to the total number of slots n_{SL} , subdomain II for the airgap and subdomain III for the rotor magnets with a given magnetization in radial direction. The modeling is completed with the following steps:

1. Solve Maxwell equation for each subdomain
2. Incorporate boundary conditions to the high permeable ferromagnetic material
3. Incorporate continuity condition at the interface between the subdomains
4. Create the linear equation system for calculating the unknown Fourier coefficients

Step 1) and 2) are described in the next section A.2, step 3) in section A.3 and step 4) in section A.5.

A.2 Subdomains

In this section the solution of the governing Maxwell equation for each subdomain is described and the incorporation of the boundary conditions to the high permeable ferromagnetic material.

A.2.1 Subdomain I: slots

The geometry as well as the relevant continuity and boundary conditions of the slots (subdomains I^(a)) is shown in Fig. A.3. The version of the Maxwell equation that applies here is defined by

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_z^{(I,a)}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_z^{(I,a)}}{\partial \varphi^2} = -\mu_0 J_z^{(a)} \quad \text{for} \quad \begin{cases} r_S \leq r \leq r_{SL} \\ \varphi_{SL}^{(a)} \leq \varphi \leq \varphi_{SL}^{(a)} + \beta_{SL} \end{cases} \quad (\text{A.14})$$

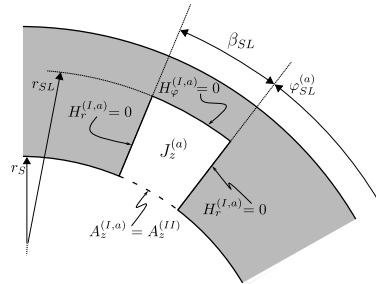


Figure A.3: Boundary and continuity conditions for subdomain I^(a) (slots).

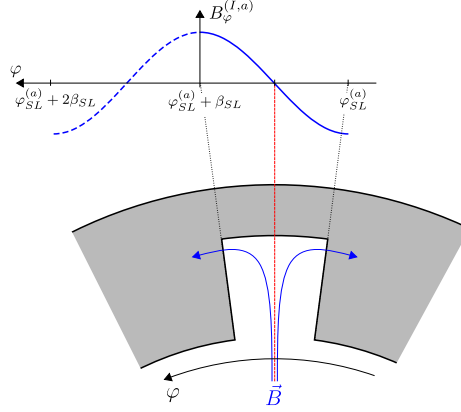


Figure A.4: Qualitative **spatial course of external flux** flowing through a slot, with the **symmetry line** highlighted. The diagram shows the tangential magnetic field component along the slot at a constant radius.

The solution of this inhomogeneous linear differential equation is based on the superposition of a homogeneous and a particular solution. Using the separation of variables technique, the most general homogeneous solution that fulfills (A.14) is equal to

$$\begin{aligned}
 A_{z,hom}^{(I,a)} = a_0^{(I,a)} + \sum_{n \in \mathbb{N}} & \left[+a_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{+\frac{n\pi}{\beta_{SL}}} \cos \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \right. \\
 & + b_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{-\frac{n\pi}{\beta_{SL}}} \cos \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \\
 & + c_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{+\frac{n\pi}{\beta_{SL}}} \sin \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \\
 & \left. + d_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{-\frac{n\pi}{\beta_{SL}}} \sin \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \right]. \quad (A.15)
 \end{aligned}$$

r_{ref} denotes some appropriate constant reference radial coordinate. As sketched in Fig. A.4, (external) magnetic flux is axissymmetrically distributed within a slot. This symmetrical distribution is mathematically incorporated by a periodicity of $2\beta_{SL}$ for the vector potential (and thereby for the magnetic field), as shown by the diagram in Fig. A.4.

Hence that homogeneous solution is the sum of four different contribution series, each one weighted by different Fourier coefficients $\{a_n^{(I,a)}, b_n^{(I,a)}, c_n^{(I,a)}, d_n^{(I,a)}\}$. Based on (A.12) and

(A.13) the magnetic field is then equal to

$$B_{r,hom}^{(I,a)} = \sum_{n \in \mathbb{N}} \frac{n\pi}{r\beta_{SL}} \left[-a_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{+\frac{n\pi}{\beta_{SL}}} \sin \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \right. \\ - b_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{-\frac{n\pi}{\beta_{SL}}} \sin \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \\ + c_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{+\frac{n\pi}{\beta_{SL}}} \cos \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \\ \left. + d_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{-\frac{n\pi}{\beta_{SL}}} \cos \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \right] \quad (A.16)$$

$$B_{\varphi,hom}^{(I,a)} = \sum_{n \in \mathbb{N}} \frac{n\pi}{r\beta_{SL}} \left[-a_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{+\frac{n\pi}{\beta_{SL}}} \cos \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \right. \\ + b_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{-\frac{n\pi}{\beta_{SL}}} \cos \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \\ - c_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{+\frac{n\pi}{\beta_{SL}}} \sin \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \\ \left. + d_n^{(I,a)} \left(\frac{r}{r_{ref}} \right)^{-\frac{n\pi}{\beta_{SL}}} \sin \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) \right] \quad (A.17)$$

One particular solution without angular dependency that fulfills (A.14) is defined by³

$$A_{z,part}^{(I,a)} = -\frac{\mu_0}{2} n_{turn} J_z^{(a)} \frac{r^2}{2} + C \ln(r) \quad (A.18)$$

with an additional factor n_{turn} for considering the number of the coil's winding turns and C the constant of integration. Due to the assumption of an infinite permeable stator iron, the magnetic flux enters or leaves the iron orthogonally. This has as result that at each surface of the slot either the radial component $B_r^{(I,a)}$ or the tangential component $B_\varphi^{(I,a)}$ of the magnetic field is equal to zero (see Fig. A.3). Thereby the following boundary conditions

$$\begin{aligned} 3 \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_{z,part}^{(I,a)}}{\partial r} \right) &= -\mu_0 n_{turn} J_z^{(a)} \\ r \frac{\partial A_{z,part}^{(I,a)}}{\partial r} &= -\mu_0 n_{turn} J_z^{(a)} \frac{r^2}{2} + C \\ A_{z,part}^{(I,a)} &= -\frac{\mu_0}{2} n_{turn} J_z^{(a)} \frac{r^2}{2} + C \ln(r) \end{aligned}$$

must be met

$$H_r^{(I,a)} \Big|_{\varphi=\varphi_{SL}^{(a)}} = 0 \quad \forall \quad r_S \leq r \leq r_{SL} \quad (\text{A.19})$$

$$H_r^{(I,a)} \Big|_{\varphi=\varphi_{SL}^{(a)}+\beta_{SL}} = 0 \quad \forall \quad r_S \leq r \leq r_{SL} \quad (\text{A.20})$$

$$H_\varphi^{(I,a)} \Big|_{r=r_{SL}} = 0 \quad \forall \quad \varphi_{SL}^{(a)} \leq \varphi \leq \varphi_{SL}^{(a)} + \beta_{SL} \quad (\text{A.21})$$

To enforce zero radial magnetic flux in (A.16) at the lateral surfaces of the slots (boundary condition (A.19) and (A.20)), Fourier coefficients $\{c_n^{(I,a)}, d_n^{(I,a)}\}$ have to be zero for all $n \in \mathbb{N}$. Based on the third boundary condition (A.21) the tangential magnetic field (A.17) at the top surface of the slots has to be zero. This is incorporated by firstly equating $b_n^{(I,a)} = a_n^{(I,a)}$ and setting $r_{ref} = r_{SL}$:

$$A_{z,hom}^{(I,a)} = a_0^{(I,a)} + \sum_{n \in \mathbb{N}} \left[a_n^{(I,a)} C_{\frac{n\pi}{\beta_{SL}}}(r, r_{SL}, +) \cos \left(\frac{n\pi}{\beta_{SL}}(\varphi - \varphi_{SL}^{(a)}) \right) \right] \quad (\text{A.22})$$

where

$$C_k(r, r_{ref}, \pm) = \left(\frac{r}{r_{ref}} \right)^{+k} \pm \left(\frac{r}{r_{ref}} \right)^{-k}. \quad (\text{A.23})$$

And secondly by defining the constant of integration in the particular solution (A.18): $C = +\frac{\mu_0}{2} n_{turn} J_z^{(a)} r_{SL}^2$ [55]. This results in

$$A_{z,part}^{(I,a)} = +\frac{\mu_0}{2} n_{turn} J_z^{(a)} \left(r_{SL}^2 \ln(r) - \frac{r^2}{2} \right). \quad (\text{A.24})$$

The general solution for subdomain I^(a) of the vector potential $A_z^{(I,a)} = A_{z,hom}^{(I,a)} + A_{z,part}^{(I,a)}$ under consideration of these boundary conditions can then be obtained

$$\begin{aligned} A_z^{(I,a)} = & a_0^{(I,a)} + \sum_{n \in \mathbb{N}} \left[a_n^{(I,a)} C_{\frac{n\pi}{\beta_{SL}}}(r, r_{SL}, +) \cos \left(\frac{n\pi}{\beta_{SL}}(\varphi - \varphi_{SL}^{(a)}) \right) \right] \\ & + \frac{\mu_0}{2} n_{turn} J_z^{(a)} \left(r_{SL}^2 \ln(r) - \frac{r^2}{2} \right). \end{aligned} \quad (\text{A.25})$$

The radial as well as tangential magnetic field is equal to

$$B_r^{(I,a)} = - \sum_{n \in \mathbb{N}} \frac{n\pi}{r\beta_{SL}} \left[a_n^{(I,a)} C_{\frac{n\pi}{\beta_{SL}}}(r, r_{SL}, +) \sin \left(\frac{n\pi}{\beta_{SL}}(\varphi - \varphi_{SL}^{(a)}) \right) \right] \quad (\text{A.26})$$

$$B_\varphi^{(I,a)} = - \sum_{n \in \mathbb{N}} \frac{n\pi}{r\beta_{SL}} \left[a_n^{(I,a)} C_{\frac{n\pi}{\beta_{SL}}}(r, r_{SL}, -) \cos\left(\frac{n\pi}{\beta_{SL}}(\varphi - \varphi_{SL}^{(a)})\right) \right] - \frac{\mu_0}{2} n_{turn} J_z^{(a)} \left(r_{SL}^2 \frac{1}{r} - r \right) \quad (\text{A.27})$$

A.2.2 Subdomain II: air gap

The geometry as well as the relevant continuity and boundary conditions of the air gap (subdomain II) is shown in Fig. A.5. The version of the Maxwell equation that applies here is defined by

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_z^{(II)}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_z^{(II)}}{\partial \varphi^2} = 0 \quad \text{for} \quad \begin{cases} r_M \leq r \leq r_S \\ 0 \leq \varphi < 2\pi \end{cases}. \quad (\text{A.28})$$

The most general solution that fulfills this Maxwell equation is equal to

$$A_z^{(II)} = a_0^{(II)} + \sum_{k \in \mathbb{N}} \left[+a_k^{(II)} \left(\frac{r}{r_{ref}} \right)^{+k} \cos(k\varphi) + b_k^{(II)} \left(\frac{r}{r_{ref}} \right)^{-k} \cos(k\varphi) + c_k^{(II)} \left(\frac{r}{r_{ref}} \right)^{+k} \sin(k\varphi) + d_k^{(II)} \left(\frac{r}{r_{ref}} \right)^{-k} \sin(k\varphi) \right] \quad (\text{A.29})$$

Again, the (homogeneous) solution is the sum of four different contribution series, this time weighted by Fourier coefficients $\{a_k^{(II)}, b_k^{(II)}, c_k^{(II)}, d_k^{(II)}\}$ and with periodicity over the full circumference of the air gap.

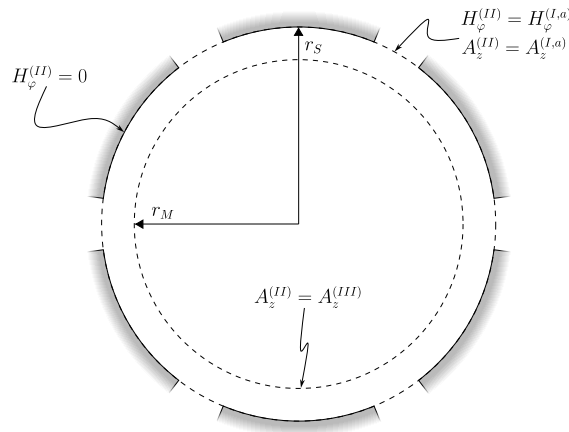


Figure A.5: Boundary and continuity conditions for subdomain II (air gap).

Setting $r_{ref} = r_S$, the magnetic field, determined based on equation (A.12) and (A.13), is

obtained by

$$B_r^{(II)} = \sum_{k \in \mathbb{N}} \frac{k}{r} \left[-a_k^{(II)} \left(\frac{r}{r_S} \right)^{+k} \sin(k\varphi) - b_k^{(II)} \left(\frac{r}{r_S} \right)^{-k} \sin(k\varphi) \right. \\ \left. + c_k^{(II)} \left(\frac{r}{r_S} \right)^{+k} \cos(k\varphi) + d_k^{(II)} \left(\frac{r}{r_S} \right)^{-k} \cos(k\varphi) \right] \quad (\text{A.30})$$

$$B_\varphi^{(II)} = \sum_{k \in \mathbb{N}} \frac{k}{r} \left[-a_k^{(II)} \left(\frac{r}{r_S} \right)^{+k} \cos(k\varphi) + b_k^{(II)} \left(\frac{r}{r_S} \right)^{-k} \cos(k\varphi) \right. \\ \left. - c_k^{(II)} \left(\frac{r}{r_S} \right)^{+k} \sin(k\varphi) + d_k^{(II)} \left(\frac{r}{r_S} \right)^{-k} \sin(k\varphi) \right] \quad (\text{A.31})$$

The boundary condition of the air gap, subdomain II, to the interface at the high permeable inner stator ($r = r_S$) is not considered yet, since it will be applied in combination with the continuous condition to the slot openings as integral expression later.

A.2.3 Subdomain III: magnets

The geometry as well as the relevant continuity and boundary conditions of the magnets (subdomain III) is shown in Fig. A.6. The version of the Maxwell equation that applies here is defined by

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_z^{(III)}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_z^{(III)}}{\partial \varphi^2} = + \frac{\mu_0}{r} \frac{\partial M_r}{\partial \varphi} \quad \text{for} \quad \begin{cases} r_R \leq r \leq r_M \\ 0 \leq \varphi < 2\pi \end{cases} \quad (\text{A.32})$$

The homogeneous solution is equal to (A.29), the solution of the airgap, besides different Fourier coefficients $\{a_k^{(III)}, b_k^{(III)}, c_k^{(III)}, d_k^{(III)}\}$ and an additional particular solution. To find such a particular solution, it is firstly required to define a Fourier expansion that describes the arrangement of the magnets over the rotor surface (as shown in Fig. A.6) in dependence of the rotor angular position θ_R :

$$\mu_0 M_r(\varphi) = \sum_{k \in \mathbb{N}} (\mu_0 M_k) \cos(k(\varphi - \theta_R)) \\ = \sum_{k \in \mathbb{N}} \frac{1}{r_R} \left[-a_k^{(M)} \sin(k\varphi) + c_k^{(M)} \cos(k\varphi) \right] \quad (\text{A.33})$$

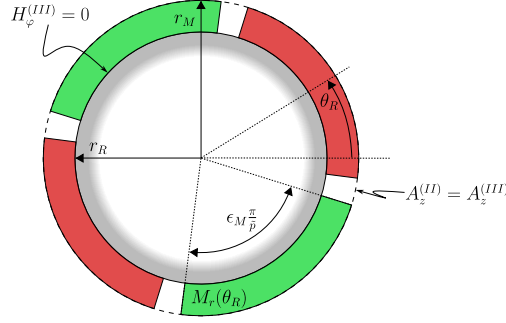


Figure A.6: Boundary and continuity conditions for subdomain III (magnets).

where

$$a_k^{(M)} = -r_R \mu_0 M_k \sin(k\theta_R) \quad (\text{A.34})$$

$$c_k^{(M)} = +r_R \mu_0 M_k \cos(k\theta_R) \quad (\text{A.35})$$

$$M_k = \frac{B_{rem}}{\mu_0} \frac{4p}{k\pi} \sin\left(\frac{k\pi}{2p} \epsilon_M\right) \quad (\text{A.36})$$

Due to the choice of the cosine function for the (symmetric) magnetization Fourier expansion, the only nonvanishing Fourier coefficients are those with index $k = (1, 3, 5, \dots)p$.

A particular solution, that fulfills (A.32) can be found by the separation of variables technique and is defined by [50, 54]

$$\begin{aligned} A_{z,part}^{(III)} = & -\frac{1}{2} \frac{r}{r_R} \ln\left(\frac{r}{r_R}\right) \left[+a_1^{(M)} \cos(\varphi) + c_1^{(M)} \sin(\varphi) \right] \\ & + \sum_{k \in \mathbb{N}, k \neq 1} \frac{r}{r_R} \frac{k}{k^2 - 1} \left[+a_k^{(M)} \cos(k\varphi) + c_k^{(M)} \sin(k\varphi) \right]. \end{aligned} \quad (\text{A.37})$$

With this particular solution the special case $k = 1$ is also considered. That is relevant for the consideration of partial demagnetization, since all Fourier coefficients $k \in \mathbb{N}$ of the asymmetric magnetization are nonzero then. The boundary condition at the rotor surface $r = r_R$

$$H_\varphi^{(III)} \Big|_{r=r_R} = 0 \quad \forall \quad 0 \leq \varphi < 2\pi \quad (\text{A.38})$$

leads with chosen $r_{ref} = r_R$ in the homogeneous solution to following relations between the Fourier coefficients of subdomain III and those of the magnetization

$$\begin{aligned} k = 1: \quad & \begin{pmatrix} b_k^{(III)} \\ d_k^{(III)} \end{pmatrix} = \begin{pmatrix} a_k^{(III)} \\ c_k^{(III)} \end{pmatrix} - \frac{1}{2} \begin{pmatrix} a_k^{(M)} \\ c_k^{(M)} \end{pmatrix} \\ k \neq 1: \quad & \begin{pmatrix} b_k^{(III)} \\ d_k^{(III)} \end{pmatrix} = \begin{pmatrix} a_k^{(III)} \\ c_k^{(III)} \end{pmatrix} + \frac{1}{k^2 - 1} \begin{pmatrix} a_k^{(M)} \\ c_k^{(M)} \end{pmatrix}. \end{aligned} \quad (\text{A.39})$$

Using these relations and setting the constant of the vector potential $a_0^{(III)} = 0$, the general solution for the vector potential $A_z^{(III)} = A_{z,hom}^{(III)} + A_{z,part}^{(III)}$ is equal to

$$\begin{aligned}
A_z^{(III)} = & + \sum_{k \in \mathbb{N}} C_k(r, r_R, +) \left[+a_k^{(III)} \cos(k\varphi) + c_k^{(III)} \sin(k\varphi) \right] \\
& - \frac{1}{2} \frac{r}{r_R} \left(\ln\left(\frac{r}{r_R}\right) + \left(\frac{r}{r_R}\right)^{-2} \right) \left[+a_1^{(M)} \cos(\varphi) + c_1^{(M)} \sin(\varphi) \right] \\
& + \sum_{k \in \mathbb{N}, k \neq 1} \frac{r}{r_R} \frac{k + \left(\frac{r}{r_R}\right)^{-k-1}}{k^2 - 1} \left[+a_k^{(M)} \cos(k\varphi) + c_k^{(M)} \sin(k\varphi) \right]. \quad (A.40)
\end{aligned}$$

The magnetic field, determined based on (A.12) and (A.13), is then defined by

$$\begin{aligned}
B_r^{(III)} = & \sum_{k \in \mathbb{N}} \frac{k}{r} C_k(r, r_R, +) \left[-a_k^{(III)} \sin(k\varphi) + c_k^{(III)} \cos(k\varphi) \right] \\
& - \frac{1}{2r_R} \left(\ln\left(\frac{r}{r_R}\right) + \left(\frac{r}{r_R}\right)^{-2} \right) \left[-a_1^{(M)} \sin(\varphi) + c_1^{(M)} \cos(\varphi) \right] \\
& + \sum_{k \in \mathbb{N}, k \neq 1} \frac{k}{r_R} \frac{k + \left(\frac{r}{r_R}\right)^{-k-1}}{k^2 - 1} \left[-a_k^{(M)} \sin(k\varphi) + c_k^{(M)} \cos(k\varphi) \right] \quad (A.41)
\end{aligned}$$

$$\begin{aligned}
B_\varphi^{(III)} = & \sum_{k \in \mathbb{N}} \frac{k}{r} C_k(r, r_R, -) \left[-a_k^{(III)} \cos(k\varphi) - c_k^{(III)} \sin(k\varphi) \right] \\
& - \frac{1}{2r_R} \left(\ln\left(\frac{r}{r_R}\right) + 1 - \left(\frac{r}{r_R}\right)^{-2} \right) \left[-a_1^{(M)} \cos(\varphi) - c_1^{(M)} \sin(\varphi) \right] \\
& + \sum_{k \in \mathbb{N}, k \neq 1} \frac{k}{r_R} \frac{1 - \left(\frac{r}{r_R}\right)^{-k-1}}{k^2 - 1} \left[-a_k^{(M)} \cos(k\varphi) - c_k^{(M)} \sin(k\varphi) \right]. \quad (A.42)
\end{aligned}$$

A.3 Continuity conditions

In the next step the aim is to impose continuity at the interfaces between two subdomains. The resulting equations can then be rearranged to a linear equation system for determining all unknown Fourier coefficients as function of the rotor angle and the current densities within each slot or coil.

A.3.1 Subdomain II and III: airgap and magnets

The continuity between the magnets and the airgap (see Fig. A.5 or A.6) can be simply imposed by equating these two subdomains at their interface $r = r_M$

$$B_r^{(II)}|_{r=r_M} = B_r^{(III)}|_{r=r_M} \quad \forall \quad 0 \leq \varphi < 2\pi \quad (\text{A.43})$$

$$H_\varphi^{(II)}|_{r=r_M} = H_\varphi^{(III)}|_{r=r_M} \quad \forall \quad 0 \leq \varphi < 2\pi \quad (\text{A.44})$$

Both continuity conditions lead to the following four relations between the Fourier coefficients in subdomain II and those of subdomain III: two for the coefficients $\{a_k^{(III)}, a_k^{(M)}, a_k^{(II)}, b_k^{(II)}\}$ of the cosine/sine functions and two for the coefficients $\{c_k^{(III)}, c_k^{(M)}, c_k^{(II)}, d_k^{(II)}\}$ of the sine/cosine functions:

$$\begin{aligned} k = 1: & \begin{cases} \begin{pmatrix} a_k^{(II)} \\ c_k^{(II)} \end{pmatrix} = \begin{pmatrix} a_k^{(III)} \\ c_k^{(III)} \end{pmatrix} \left(\frac{r_S}{r_R}\right)^k - \begin{pmatrix} a_k^{(M)} \\ c_k^{(M)} \end{pmatrix} \frac{r_M}{2r_R} \left(\ln\left(\frac{r_M}{r_R}\right) + \frac{1}{2}\right) \left(\frac{r_S}{r_M}\right)^k \\ \begin{pmatrix} b_k^{(II)} \\ d_k^{(II)} \end{pmatrix} = \begin{pmatrix} a_k^{(III)} \\ c_k^{(III)} \end{pmatrix} \left(\frac{r_R}{r_S}\right)^k - \begin{pmatrix} a_k^{(M)} \\ c_k^{(M)} \end{pmatrix} \frac{r_M}{2r_R} \left(-\frac{1}{2} + \left(\frac{r_R}{r_M}\right)^{k+1}\right) \left(\frac{r_M}{r_S}\right)^k \end{cases} \\ k \neq 1: & \begin{cases} \begin{pmatrix} a_k^{(II)} \\ c_k^{(II)} \end{pmatrix} = \begin{pmatrix} a_k^{(III)} \\ c_k^{(III)} \end{pmatrix} \left(\frac{r_S}{r_R}\right)^k + \begin{pmatrix} a_k^{(M)} \\ c_k^{(M)} \end{pmatrix} \frac{r_M}{2r_R} \frac{1}{k-1} \left(\frac{r_S}{r_M}\right)^k \\ \begin{pmatrix} b_k^{(II)} \\ d_k^{(II)} \end{pmatrix} = \begin{pmatrix} a_k^{(III)} \\ c_k^{(III)} \end{pmatrix} \left(\frac{r_R}{r_S}\right)^k + \begin{pmatrix} a_k^{(M)} \\ c_k^{(M)} \end{pmatrix} \frac{r_M}{2r_R} \frac{\left(k-1 + 2\left(\frac{r_R}{r_M}\right)^{k+1}\right)}{k^2-1} \left(\frac{r_M}{r_S}\right)^k \end{cases} \end{aligned} \quad (\text{A.45})$$

Using these relations, the vector potential $A_z^{(II)}$ of the air gap can be expressed by the Fourier coefficients of the magnets $\{a_k^{(III)}, c_k^{(III)}\}$:

$$\begin{aligned} A_z^{(II)} = & \sum_{k \in \mathbb{N}} C_k(r, r_R, +) \left[+a_k^{(III)} \cos(k\varphi) + c_k^{(III)} \sin(k\varphi) \right] \\ & + S_1(r, +) \left[+a_1^{(M)} \cos(\varphi) + c_1^{(M)} \sin(\varphi) \right] \\ & + \sum_{k \in \mathbb{N}, k \neq 1} T_k(r, +) \left[+a_k^{(M)} \cos(k\varphi) + c_k^{(M)} \sin(k\varphi) \right] \end{aligned} \quad (\text{A.46})$$

where

$$S_1(r, \pm) = -\frac{r_M}{2r_R} \left(\left(\ln\left(\frac{r_M}{r_R}\right) + \frac{1}{2} \right) \frac{r}{r_M} \pm \left(-\frac{1}{2} + \left(\frac{r_R}{r_M}\right)^2 \right) \frac{r_M}{r} \right) \quad (\text{A.47})$$

$$T_k(r, \pm) = +\frac{r_M}{2r_R} \frac{(k+1) \left(\frac{r}{r_M}\right)^k \pm \left(k-1 + 2\left(\frac{r_R}{r_M}\right)^{k+1}\right) \left(\frac{r_M}{r}\right)^k}{k^2-1}. \quad (\text{A.48})$$

In this way the magnetic field in the air gap is determined as well

$$\begin{aligned}
B_r^{(II)} = & \sum_{k \in \mathbb{N}} \frac{k}{r} C_k(r, r_R, +) \left[-a_k^{(III)} \sin(k\varphi) + c_k^{(III)} \cos(k\varphi) \right] \\
& + \frac{1}{r} S_1(r, +) \left[-a_1^{(M)} \sin(\varphi) + c_1^{(M)} \cos(\varphi) \right] \\
& + \sum_{k \in \mathbb{N}, k \neq 1} \frac{k}{r} T_k(r, +) \left[-a_k^{(M)} \sin(k\varphi) + c_k^{(M)} \cos(k\varphi) \right]
\end{aligned} \tag{A.49}$$

$$\begin{aligned}
B_\varphi^{(II)} = & \sum_{k \in \mathbb{N}} \frac{k}{r} C_k(r, r_R, -) \left[-a_k^{(III)} \cos(k\varphi) - c_k^{(III)} \sin(k\varphi) \right] \\
& + \frac{1}{r} S_1(r, -) \left[-a_1^{(M)} \cos(\varphi) - c_1^{(M)} \sin(\varphi) \right] \\
& + \sum_{k \in \mathbb{N}, k \neq 1} \frac{k}{r} T_k(r, -) \left[-a_k^{(M)} \cos(k\varphi) - c_k^{(M)} \sin(k\varphi) \right].
\end{aligned} \tag{A.50}$$

In the next step the continuity of the magnetic field in the airgap, described by the Fourier coefficients of the magnets, with the magnetic field in the slots is incorporated. This is described in the next subsection.

A.3.2 Subdomain II and I: airgap and slots

While the boundary condition between the magnets and the air gap could be simply imposed by equating the magnetic field of both subdomains at their interface, the continuity between the airgap and slots needs to be implemented in a different way. This has the following reason: because the air gap at $r = r_S$ has not only interfaces to the slots, but also to the high permeable iron of the stator, two different conditions have to be implemented simultaneously (see Fig. A.5):

$$H_\varphi^{(II)} \Big|_{r=r_S} = \begin{cases} H_\varphi^{(I,a)} \Big|_{r=r_S} & \text{for } \varphi_{SL}^{(a)} \geq \varphi \geq \varphi_{SL}^{(a)} + \beta_{SL} \quad \forall \quad a \in [1, 2, \dots, n_{SL}] \\ 0 & \text{elsewhere} \end{cases} \tag{A.51}$$

This continuity condition is met by a 'softer' integral expression, projecting on a complete set of sine/cosine basis functions

$$\frac{1}{\mu_0} \int_0^{2\pi} B_\varphi^{(II)} \Big|_{r=r_S} \cos(k\varphi) d\varphi = \frac{1}{\mu_0} \sum_{a=1}^{n_{SL}} \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} B_\varphi^{(I,a)} \Big|_{r=r_S} \cos(k\varphi) d\varphi + 0 \tag{A.52}$$

$$\frac{1}{\mu_0} \int_0^{2\pi} B_\varphi^{(II)} \Big|_{r=r_S} \sin(k\varphi) d\varphi = \frac{1}{\mu_0} \sum_{a=1}^{n_{SL}} \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} B_\varphi^{(I,a)} \Big|_{r=r_S} \sin(k\varphi) d\varphi + 0 \tag{A.53}$$

with $B_\varphi^{(II)}$ defined in (A.50) and $B_\varphi^{(I,a)}$ defined in (A.27). These conditions are formulated in such a way to allow extracting each Fourier coefficient $a_k^{(III)}$ or $c_k^{(III)}$ as a linear superposition of all $a_n^{(I,a)}$ (see also the solution of the integral (A.61) and (A.62)). The solutions of these two equations are equal to

$$\begin{aligned} & \overbrace{+a_1^{(III)}C_1(r_S, r_R, -) + a_1^{(M)}S_1(r_S, -)}^{\text{for } k=1} + \overbrace{+a_k^{(III)}C_k(r_S, r_R, -) + a_k^{(M)}T_k(r_S, -)}^{\text{for } k \neq 1} = \\ & + \sum_{a=1}^{n_{SL}} \sum_{n \in \mathbb{N}} \frac{n}{2k} a_n^{(I,a)} C_{\frac{n\pi}{\beta_{SL}}} (r_S, r_{SL}, -) f_{SL,s}^{(a)}(k, n) + \sum_{a=1}^{n_{SL}} \frac{J_z^{(a)}}{k} \frac{\mu_0}{4\pi} (r_{SL}^2 - r_S^2) f_{SL,s}^{(a)}(k, 0) \quad (\text{A.54}) \end{aligned}$$

$$\begin{aligned} & \overbrace{+c_1^{(III)}C_1(r_S, r_R, -) + c_1^{(M)}S_1(r_S, -)}^{\text{for } k=1} + \overbrace{+c_k^{(III)}C_k(r_S, r_R, -) + c_k^{(M)}T_k(r_S, -)}^{\text{for } k \neq 1} = \\ & - \sum_{a=1}^{n_{SL}} \sum_{n \in \mathbb{N}} \frac{n}{2k} a_n^{(I,a)} C_{\frac{n\pi}{\beta_{SL}}} (r_S, r_{SL}, -) f_{SL,c}^{(a)}(k, n) - \sum_{a=1}^{n_{SL}} \frac{J_z^{(a)}}{k} \frac{\mu_0}{4\pi} (r_{SL}^2 - r_S^2) f_{SL,c}^{(a)}(k, 0) \quad (\text{A.55}) \end{aligned}$$

with the solved integrals defined in (A.61)–(A.66). Besides the continuity of the tangential magnetic field over the full circumference, the continuity of the vector potential at each slot opening needs to be imposed (see Fig. A.3):

$$A_z^{(II)} \Big|_{r=r_S} = A_z^{(I,a)} \Big|_{r=r_S} \quad \forall \quad \varphi_{SL}^{(a)} \geq \varphi \geq \varphi_{SL}^{(a)} + \beta_{SL} \quad (\text{A.56})$$

This is achieved by the following condition for each slot $a \in \{1, 2, \dots, n_{SL}\}$

$$\int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} A_z^{(I,a)} \Big|_{r=r_S} \cos \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) d\varphi = \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} A_z^{(II)} \Big|_{r=r_S} \cos \left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)}) \right) d\varphi \quad (\text{A.57})$$

extracting each Fourier coefficient $a_n^{(I,a)}$ as linear superposition of all Fourier coefficients $a_k^{(III)}$ and $c_k^{(III)}$. With the equation for the vector potential of the slots (A.25) and airgap (A.46), the solution of this condition is equal to:

$$\begin{aligned} & + a_n^{(I,a)} C_{\frac{n\pi}{\beta_{SL}}} (r_S, r_{SL}, +) = \\ & + \sum_{k \in \mathbb{N}} C_k(r_S, r_R, +) \left(a_k^{(III)} f_{SL,s}^{(a)}(k, n) - c_k^{(III)} f_{SL,c}^{(a)}(k, n) \right) \\ & + S_1(r_S, +) \left(a_1^{(M)} f_{SL,s}^{(a)}(1, n) - c_1^{(M)} f_{SL,c}^{(a)}(1, n) \right) \\ & + \sum_{k \in \mathbb{N}, k \neq 1} T_k(r_S, +) \left(a_k^{(M)} f_{SL,s}^{(a)}(k, n) - c_k^{(M)} f_{SL,c}^{(a)}(k, n) \right). \quad (\text{A.58}) \end{aligned}$$

The solutions of the (non-trivial) integrals are defined in (A.63)–(A.64). With the additional condition

$$\int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)}+\beta_{SL}} A_z^{(I,a)} \Big|_{r=r_S} d\varphi = \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)}+\beta_{SL}} A_z^{(II)} \Big|_{r=r_S} d\varphi \quad (\text{A.59})$$

the constant $a_0^{(I,a)}$ of each slot's vector potential is extracted:

$$\begin{aligned} & + a_0^{(I,a)} + \frac{\mu_0}{2} n_{turn} J_z^{(a)} \left(r_{SL}^2 \ln(r_S) - \frac{r_S^2}{2} \right) = \\ & + \sum_{k \in \mathbb{N}} \frac{C_k(r_S, r_R, +)}{2} \left(a_k^{(III)} f_{SL,s}^{(a)}(k, 0) - c_k^{(III)} f_{SL,c}^{(a)}(k, 0) \right) \\ & + S_1(r_S, +) \left(a_1^{(M)} f_{SL,s}^{(a)}(1, 0) - c_1^{(M)} f_{SL,c}^{(a)}(1, 0) \right) \\ & + \sum_{k \in \mathbb{N}, k \neq 1} \frac{T_k(r_S, +)}{2} \left(a_k^{(M)} f_{SL,s}^{(a)}(k, 0) - c_k^{(M)} f_{SL,c}^{(a)}(k, 0) \right). \end{aligned} \quad (\text{A.60})$$

The solutions of the (non-trivial) integrals are defined in (A.65)–(A.66). While this constant $a_0^{(I,a)}$ does not contribute to the magnetic field due to (A.12) and (A.13), it is useful for calculating the inductances, PM flux linkages and back-EMFs of the coils [57].

A.4 Important Integrals

The following integrals are of relevance for incorporating the continuity conditions, for $\{k, n\} \in \mathbb{N}$:

$$\int_0^{2\pi} \sin(k\varphi) \cos(k\varphi) d\varphi = 0 \quad (\text{A.61})$$

$$\int_0^{2\pi} \sin^2(k\varphi) d\varphi = \int_0^{2\pi} \cos^2(k\varphi) d\varphi = +\pi \quad (\text{A.62})$$

$$\begin{aligned} & \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} \cos(k\varphi) \cos\left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)})\right) d\varphi = \\ & + \frac{\beta_{SL}}{2} \underbrace{\left(\frac{1}{k\beta_{SL} - n\pi} + \frac{1}{k\beta_{SL} + n\pi} \right) \left((-1)^n \sin(k\varphi_{SL}^{(a)} + k\beta_{SL}) - \sin(k\varphi_{SL}^{(a)}) \right)}_{f_{SL,s}^{(a)}(k, n)} \end{aligned} \quad (\text{A.63})$$

$$\begin{aligned} & \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} \sin(k\varphi) \cos\left(\frac{n\pi}{\beta_{SL}} (\varphi - \varphi_{SL}^{(a)})\right) d\varphi = \\ & - \frac{\beta_{SL}}{2} \underbrace{\left(\frac{1}{k\beta_{SL} - n\pi} + \frac{1}{k\beta_{SL} + n\pi} \right) \left((-1)^n \cos(k\varphi_{SL}^{(a)} + k\beta_{SL}) - \cos(k\varphi_{SL}^{(a)}) \right)}_{f_{SL,c}^{(a)}(k, n)} \end{aligned} \quad (\text{A.64})$$

$$\int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} \cos(k\varphi) d\varphi = + \frac{\beta_{SL}}{2} \underbrace{\left(\frac{2}{k\beta_{SL}} \right) \left(\sin(k\varphi_{SL}^{(a)} + k\beta_{SL}) - \sin(k\varphi_{SL}^{(a)}) \right)}_{f_{SL,s}^{(a)}(k, 0)} \quad (\text{A.65})$$

$$\int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} \sin(k\varphi) d\varphi = - \frac{\beta_{SL}}{2} \underbrace{\left(\frac{2}{k\beta_{SL}} \right) \left(\cos(k\varphi_{SL}^{(a)} + k\beta_{SL}) - \cos(k\varphi_{SL}^{(a)}) \right)}_{f_{SL,c}^{(a)}(k, 0)} \quad (\text{A.66})$$

A.5 Resulting equation system

All linearly independent equations of the solved continuous conditions (A.54), (A.55), (A.58), (A.60) derived in subsection A.3 can be rearranged for calculating the remaining, unknown Fourier coefficients. This results in a linear equation system of the form

$$\mathbf{o} \begin{pmatrix} \vec{a}^{(I)} \\ \vec{a}^{(III)} \\ \vec{c}^{(III)} \end{pmatrix} = n_{turn} \tilde{\mathbf{p}} \vec{J}_z + \mathbf{q} \begin{pmatrix} \vec{a}^{(M)} \\ \vec{c}^{(M)} \end{pmatrix}, \quad (\text{A.67})$$

where vector $\vec{a}^{(I)}$ contains all Fourier coefficients $n \in \mathbb{N}^0$ respectively of all subdomains $I^{(a)} \forall a \in \{1, 2, \dots, n_{SL}\}$, vectors $\{\vec{a}^{(III)}, \vec{c}^{(III)}\}$ respectively all Fourier coefficients $k \in \mathbb{N}$ and $\vec{J}_z = \left(i_z^{(a=1)} \ i_z^{(a=2)} \ \dots \ i_z^{(a=n_{SL})} \right)^T \frac{1}{S_{SL}}$ the current densities flowing through each slot.

Equation system (A.67) can then be solved by inverting matrix $\mathbf{o}$:

$$\begin{pmatrix} \vec{a}^{(I)} \\ \vec{a}^{(III)} \\ \vec{c}^{(III)} \end{pmatrix} = n_{turn} \underbrace{\mathbf{o}^{-1} \tilde{\mathbf{p}}}_{\tilde{\mathbf{P}}} \vec{J}_z + \underbrace{\mathbf{o}^{-1} \mathbf{q}}_{\mathbf{Q}} \begin{pmatrix} \vec{a}^{(M)} \\ \vec{c}^{(M)} \end{pmatrix}. \quad (\text{A.68})$$

Assuming single layer concentrated windings, matrix $\mathbf{P}_{n_{eq} \times n_{coil}}$ is built based on $\tilde{\mathbf{P}}_{n_{eq} \times n_{SL}}$:

$$\mathbf{P} = \begin{pmatrix} \tilde{P}_{n=1}^{(a=1)} & \tilde{P}_{n=1}^{(a=3)} & \dots & \tilde{P}_{n=1}^{(a=n_{SL}-1)} \\ \tilde{P}_{n=2}^{(a=1)} & \tilde{P}_{n=2}^{(a=3)} & \dots & \tilde{P}_{n=2}^{(a=n_{SL}-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{P}_{n=n_{eq}}^{(a=1)} & \tilde{P}_{n=n_{eq}}^{(a=3)} & \dots & \tilde{P}_{n=n_{eq}}^{(a=n_{SL}-1)} \end{pmatrix} - \begin{pmatrix} \tilde{P}_{n=1}^{(a=2)} & \tilde{P}_{n=1}^{(a=4)} & \dots & \tilde{P}_{n=1}^{(a=n_{SL})} \\ \tilde{P}_{n=2}^{(a=2)} & \tilde{P}_{n=2}^{(a=4)} & \dots & \tilde{P}_{n=2}^{(a=n_{SL})} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{P}_{n=n_{eq}}^{(a=2)} & \tilde{P}_{n=n_{eq}}^{(a=4)} & \dots & \tilde{P}_{n=n_{eq}}^{(a=n_{SL})} \end{pmatrix} \quad (\text{A.69})$$

where n_{eq} is the number of equations for calculating the same number of unknown Fourier coefficients. Thereby $\vec{J}_z$ contains the current density of each coil:

$$\vec{J}_z = \left(i_z^{(c=1)} \ i_z^{(c=2)} \ \dots \ i_z^{(c=n_{coil})} \right)^T \frac{1}{S_{SL}}. \quad (\text{A.70})$$

Finally, the solved equation system for obtaining all unknown Fourier coefficients is defined as follows:

$$\begin{pmatrix} \vec{a}^{(I)} \\ \vec{a}^{(III)} \\ \vec{c}^{(III)} \end{pmatrix} = n_{turn} \mathbf{P} \vec{J}_z + \mathbf{Q} \begin{pmatrix} \vec{a}^{(M)} \\ \vec{c}^{(M)} \end{pmatrix}. \quad (\text{A.71})$$

Since matrices $\mathbf{P}$ and $\mathbf{Q}$ only contain geometry depending parts of the equations, they can be stored as constant during initialization to drastically decrease the computational effort. For the calculation of the torque, only the magnetic flux in the airgap is of relevance, and thereby the according parts of the solved equation system (A.71) for calculating Fourier coefficients $\{\vec{a}^{(III)}, \vec{c}^{(III)}\}$:

$$\begin{pmatrix} \vdots \\ \vec{a}^{(III)} \\ \vec{c}^{(III)} \end{pmatrix} = n_{turn} \underbrace{\begin{pmatrix} \vdots \\ \mathbf{P}_A^{(III)} \\ \mathbf{P}_C^{(III)} \end{pmatrix}}_{\mathbf{P}} \vec{J}_z + \underbrace{\begin{pmatrix} \vdots & \vdots \\ \mathbf{Q}_{AA}^{(III)} & \mathbf{Q}_{AC}^{(III)} \\ \mathbf{Q}_{CA}^{(III)} & \mathbf{Q}_{CC}^{(III)} \end{pmatrix}}_{\mathbf{Q}} \begin{pmatrix} \vec{a}^{(M)} \\ \vec{c}^{(M)} \end{pmatrix}. \quad (\text{A.72})$$

The inductances, PM flux linkages and back-EMFs can be efficiently calculated based on $a_0^{(I,a)}$, the constant of the slot's (a) vector potential. For determining $a_0^{(I,a)}$ the corresponding part of the solved equation system (A.71) needs to be utilized:

$$\begin{pmatrix} a_0^{(I,a)} \\ \vdots \end{pmatrix} = n_{turn} \underbrace{\begin{pmatrix} (\vec{P}_0^{(I,a)})^T \\ \vdots \end{pmatrix}}_{\mathbf{P}} \vec{J}_z + \underbrace{\begin{pmatrix} (\vec{Q}_{0A}^{(I,a)})^T & (\vec{Q}_{0C}^{(I,a)})^T \\ \vdots & \vdots \end{pmatrix}}_{\mathbf{Q}} \begin{pmatrix} \vec{a}^{(M)} \\ \vec{c}^{(M)} \end{pmatrix} \quad (\text{A.73})$$

where

$$\vec{P}_0^{(I,a)} = \left(P_0^{(I,a)}(c=1) \quad P_0^{(I,a)}(c=2) \quad \dots \quad P_0^{(I,a)}(c) \quad \dots \quad P_0^{(I,a)}(c=n_{coil}) \right)^T. \quad (\text{A.74})$$

A.6 Further definitions

Some of the geometry depending parts in the equation (4.14) for calculating the torque can be summarized in vector $\vec{G}$, which is defined as follows

$$\vec{G} = (G_1 \quad G_2 \dots G_k \dots G_{k_{max}})^T, \quad (\text{A.75})$$

$$G_k = \begin{cases} +C_1(r_S, r_R, +)S_1(r_S, -) - C_1(r_S, r_R, -)S_1(r_S, +) & \forall \quad k=1 \\ +k^2 C_k(r_S, r_R, +)T_k(r_S, -) - k^2 C_k(r_S, r_R, -)T_k(r_S, +) & \forall \quad k=\{2, 3, \dots, k_{max}\} \end{cases} \quad (\text{A.76})$$

with auxiliary functions C_k , S_1 and T_k defined by (A.23), (A.47) and (A.48).

B Considering flux barriers in the Subdomain Model

(The following appendix is based on [61].)

This appendix is intended to extend the Subdomain Model derived in appendix A for considering stator flux barriers (see Fig. B.1). To incorporate these flux barriers, a similar machine topology can be used as reference: flux switching permanent magnet machines. Using [84] as basis, combining it with the rotor geometry of PMSMs [54] and replacing the stator magnets by air, the flux barriers are implemented efficiently. The resulting derivation is described in the following.

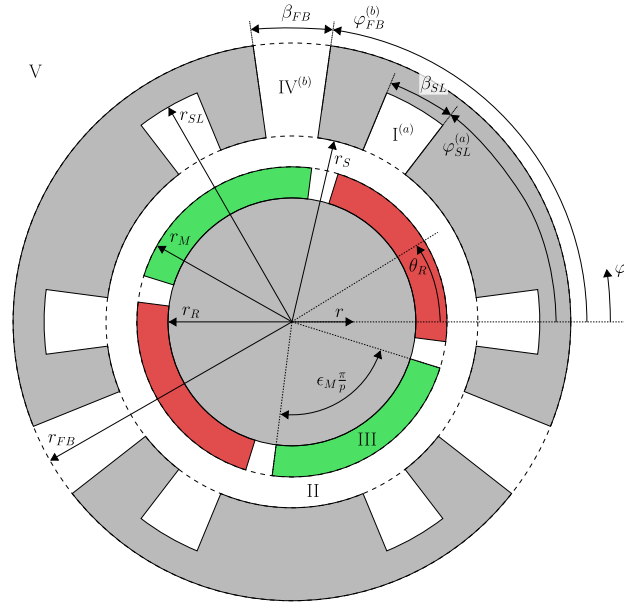


Figure B.1: Considered geometry and relevant parameters in the Subdomain Model for a PMSM with stator flux barriers [61].

B.1 Subdomains

To incorporate stator flux barriers, two additional subdomain types are required: one for the flux barriers by themselves and one for the surrounding air. Analogously to the previous discussed subdomains, all relevant boundary and continuity conditions must be considered.

B.1.1 Subdomain IV: flux barriers

The geometry as well as the relevant continuity and boundary conditions of the flux barriers (subdomains $IV^{(b)}$) are shown in Fig. B.2. The version of the Maxwell equation that applies here is defined by

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_z^{(IV,b)}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_z^{(IV,b)}}{\partial \varphi^2} = 0 \quad \text{for} \quad \begin{cases} r_S \leq r \leq r_{FB} \\ \varphi_{FB}^{(b)} \leq \varphi < \varphi_{FB}^{(b)} + \beta_{FB} \end{cases}. \quad (\text{B.1})$$

A general solution that fulfills this Maxwell equation is equal to

$$\begin{aligned} A_z^{(IV,b)} = & a_0^{(IV,b)} + b_0^{(IV,b)} \ln(r) + \sum_{m \in \mathbb{N}} \left[+a_m^{(IV,b)} \left(\frac{r}{r_{ref}} \right)^{+\frac{m\pi}{\beta_{FB}}} \cos \left(\frac{m\pi}{\beta_{FB}} (\varphi - \varphi_{FB}^{(b)}) \right) \right. \\ & + b_m^{(IV,b)} \left(\frac{r}{r_{ref}} \right)^{-\frac{m\pi}{\beta_{FB}}} \cos \left(\frac{m\pi}{\beta_{FB}} (\varphi - \varphi_{FB}^{(b)}) \right) \\ & + c_m^{(IV,b)} \left(\frac{r}{r_{ref}} \right)^{+\frac{m\pi}{\beta_{FB}}} \sin \left(\frac{m\pi}{\beta_{FB}} (\varphi - \varphi_{FB}^{(b)}) \right) \\ & \left. + d_m^{(IV,b)} \left(\frac{r}{r_{ref}} \right)^{-\frac{m\pi}{\beta_{FB}}} \sin \left(\frac{m\pi}{\beta_{FB}} (\varphi - \varphi_{FB}^{(b)}) \right) \right], \quad (\text{B.2}) \end{aligned}$$

which is similar to the homogeneous solution of the slots (A.15). To consider the boundary condition at the surfaces of the flux barriers to the high permeable material (see Fig. B.2)

$$H_r^{(IV,b)} \Big|_{\varphi=\varphi_{FB}^{(b)}} = 0 \quad \forall \quad r_S \leq r \leq r_{FB} \quad (\text{B.3})$$

$$H_r^{(IV,b)} \Big|_{\varphi=\varphi_{FB}^{(b)}+\beta_{FB}} = 0 \quad \forall \quad r_S \leq r \leq r_{FB} \quad (\text{B.4})$$

Fourier coefficients $\{c_m^{(IV,b)}, d_m^{(IV,b)}\}$ need to be zero for all $m \in \mathbb{N}$ (similar to the subdomains of the slots). Thereby, only two of the four Fourier coefficients remain unknown. Setting r_{ref} to r_{FB} and r_S respectively in one of the remaining terms, the vector potential as well

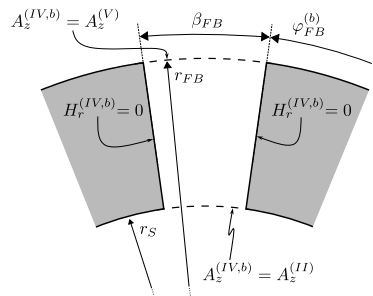


Figure B.2: Boundary and continuity conditions for subdomain $IV^{(b)}$ (flux barriers).

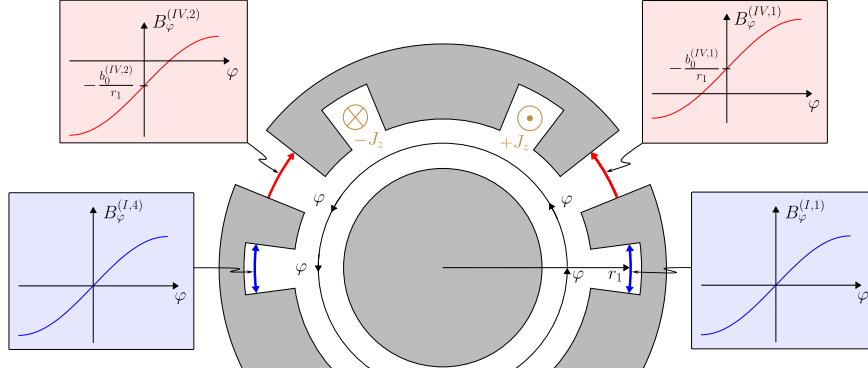


Figure B.3: Qualitative curves of the tangential magnetic field within the **flux barriers** and **slots** in the case of one active coil (without influence of the magnets).

as the magnetic field is equal to:

$$A_z^{(IV,b)} = +a_0^{(IV,b)} + b_0^{(IV,b)} \log(r) + \sum_{m \in \mathbb{N}} \left[+a_m^{(IV,b)} \left(\frac{r}{r_{FB}} \right)^{+\frac{m\pi}{\beta_{FB}}} + b_m^{(IV,b)} \left(\frac{r}{r_S} \right)^{-\frac{m\pi}{\beta_{FB}}} \right] \cos \left(\frac{m\pi}{\beta_{FB}} (\varphi - \varphi_{FB}^{(b)}) \right) \quad (\text{B.5})$$

$$B_r^{(IV,b)} = - \sum_{m \in \mathbb{N}} \frac{m\pi}{\beta_{FB}} \left[+\frac{a_m^{(IV,b)}}{r_{FB}} \left(\frac{r}{r_{FB}} \right)^{+\frac{m\pi}{\beta_{FB}}-1} + \frac{b_m^{(IV,b)}}{r_S} \left(\frac{r}{r_S} \right)^{-\frac{m\pi}{\beta_{FB}}-1} \right] \sin \left(\frac{m\pi}{\beta_{FB}} (\varphi - \varphi_{FB}^{(b)}) \right) \quad (\text{B.6})$$

$$B_\varphi^{(IV,b)} = - \frac{b_0^{(IV,b)}}{r} - \sum_{m \in \mathbb{N}} \frac{m\pi}{\beta_{FB}} \left[+\frac{a_m^{(IV,b)}}{r_{FB}} \left(\frac{r}{r_{FB}} \right)^{+\frac{m\pi}{\beta_{FB}}-1} - \frac{b_m^{(IV,b)}}{r_S} \left(\frac{r}{r_S} \right)^{-\frac{m\pi}{\beta_{FB}}-1} \right] \cos \left(\frac{m\pi}{\beta_{FB}} (\varphi - \varphi_{FB}^{(b)}) \right) \quad (\text{B.7})$$

It can be observed that besides constant $a_0^{(IV,b)}$ of the vector potential a second constant $b_0^{(IV,b)} \ln(r)$ associated with a logarithmic radial dependency is considered. This constant is part of a more general homogeneous solution [50]. To better understand why it is relevant in this case, the tangential magnetic field within the slots and flux barriers for the case of one active coil is drawn in Fig. B.3. Here it can be seen that the flux within the slots is, as already described, symmetrically distributed (see also Fig. A.4). In the left half of the slot, the tangential magnetic field has a positive sign ('left direction') and in the right side it has a negative sign ('right direction'). In contrast to that, the flux within the flux barriers shows an asymmetrical distribution, since it tends to prefer the path back to the segment with the active coil due to a lower magnetic resistance. As a result, the flux barriers cause a flux concentration within this segment. This concentration effect is mathematically

incorporated by the term $-\frac{b_0^{(IV,b)}}{r}$ in (B.7), which shifts the symmetrical distribution from $\cos\left(\frac{m\pi}{\beta_{FB}}(\varphi - \varphi_{FB}^{(b)})\right)$ to obtain an asymmetrical one (see red diagrams in Fig. B.3). For this reason the consideration of the additional constant $b_0^{(IV,b)}$ is important to correctly incorporate the flux barriers.

B.1.2 Subdomain V: surrounding air

The geometry as well as the relevant continuity and boundary conditions of the surrounding air (subdomain V) are shown in Fig. B.4. The version of the Maxwell equation that applies here is defined by

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_z^{(V)}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_z^{(V)}}{\partial \varphi^2} = 0 \quad \text{for} \quad \begin{cases} r_{FB} \leq r \rightarrow \infty \\ 0 \leq \varphi < 2\pi \end{cases} \quad (\text{B.8})$$

Its general solution is similar to the one for the airgap (A.29), but with Fourier coefficients $\{a_t^{(V)}, b_t^{(V)}, c_t^{(V)}, d_t^{(V)}\}$. Since the magnetic field vanishes for $r \rightarrow \infty$, only the Fourier coefficients which are accompanied by a r^{-t} -dependency are nonzero. This leads for $r_{ref} = r_{FB}$ to the following expressions for the vector potential and the radial as well as tangential magnetic field

$$A_z^{(V)} = a_0^{(V)} + \sum_{t \in \mathbb{N}} \left[+b_t^{(V)} \left(\frac{r}{r_{FB}} \right)^{-t} \cos(t\varphi) + d_t^{(V)} \left(\frac{r}{r_{FB}} \right)^{-t} \sin(t\varphi) \right] \quad (\text{B.9})$$

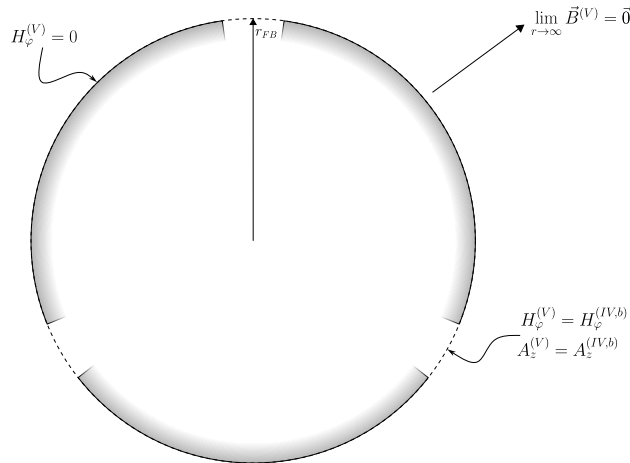


Figure B.4: Boundary and continuity conditions for subdomain V (surrounding air).

$$B_r^{(V)} = \sum_{t \in \mathbb{N}} t \left[-\frac{b_t^{(V)}}{r_{FB}} \left(\frac{r}{r_{FB}} \right)^{-t-1} \sin(t\varphi) + \frac{d_t^{(V)}}{r_{FB}} \left(\frac{r}{r_{FB}} \right)^{-t-1} \cos(t\varphi) \right] \quad (\text{B.10})$$

$$B_\varphi^{(V)} = \sum_{t \in \mathbb{N}} t \left[+\frac{b_t^{(V)}}{r_{FB}} \left(\frac{r}{r_{FB}} \right)^{-t-1} \cos(t\varphi) + \frac{d_t^{(V)}}{r_{FB}} \left(\frac{r}{r_{FB}} \right)^{-t-1} \sin(t\varphi) \right] \quad (\text{B.11})$$

Similar to the airgap (subdomain II), the boundary condition of the surrounding air to the interface at the high permeable outer stator ($r = r_{FB}$) is not considered yet, since it will be applied in combination with the continuous condition to the flux barriers as integral expression in the following section.

B.2 Continuity conditions

In this section the incorporation of the relevant continuity conditions for considering the stator flux barriers are described.

B.2.1 Subdomain IV and V: flux barriers and surrounding air

The following boundary condition needs to be fulfilled at the interface between the flux barriers and the surrounding air:

$$H_\varphi^{(V)}|_{r=r_{FB}} = \begin{cases} H_\varphi^{(IV,b)}|_{r=r_{FB}} & \text{for } \varphi_{FB}^{(b)} \leq \varphi \leq \varphi_{FB}^{(b)} + \beta_{FB} \quad \forall \quad b \in [1, 2, \dots, n_{FB}] \\ 0 & \text{elsewhere} \end{cases} \quad (\text{B.12})$$

which is met by the following integral expressions

$$\frac{1}{\mu_0} \int_0^{2\pi} B_\varphi^{(V)}|_{r=r_{FB}} \cos(t\varphi) d\varphi = \frac{1}{\mu_0} \sum_{b=1}^{n_{FB}} \int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} B_\varphi^{(IV,b)}|_{r=r_{FB}} \cos(t\varphi) d\varphi + 0 \quad (\text{B.13})$$

$$\frac{1}{\mu_0} \int_0^{2\pi} B_\varphi^{(V)}|_{r=r_{FB}} \sin(t\varphi) d\varphi = \frac{1}{\mu_0} \sum_{b=1}^{n_{FB}} \int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} B_\varphi^{(IV,b)}|_{r=r_{FB}} \sin(t\varphi) d\varphi + 0 \quad (\text{B.14})$$

and with solution procedure similar to (A.52) and (A.53).

The continuity of the vector potential

$$A_z^{(V)}|_{r=r_{FB}} = A_z^{(IV,b)}|_{r=r_{FB}} \quad \forall \quad \varphi_{FB}^{(b)} \leq \varphi \leq \varphi_{FB}^{(b)} + \beta_{FB} \quad (\text{B.15})$$

is incorporated by the following expression for each flux barrier $b \in \{1, 2, \dots, n_{FB}\}$

$$\int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} A_z^{(V)} \Big|_{r=r_{FB}} \cos\left(\frac{m\pi}{\beta_{FB}}(\varphi - \varphi_{FB}^{(b)})\right) d\varphi = \int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} A_z^{(IV,b)} \Big|_{r=r_{FB}} \cos\left(\frac{m\pi}{\beta_{FB}}(\varphi - \varphi_{FB}^{(b)})\right) d\varphi \quad (\text{B.16})$$

with the solution procedure similar to (A.57).

The constants $a_0^{(IV,b)}$ and $b_0^{(IV,b)}$ of a flux barrier's vector potential are extracted for each $b \in \{1, 2, \dots, n_{FB}\}$ by

$$\int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} A_z^{(V)} \Big|_{r=r_{FB}} d\varphi = \int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} A_z^{(IV,b)} \Big|_{r=r_{FB}} d\varphi \quad (\text{B.17})$$

with the solution procedure similar to (A.60).

B.2.2 Subdomain IV, II and I: flux barriers, air gap and slots

For the continuity condition of the tangential magnetic field within the air gap now the flux barriers need to be considered additionally:

$$H_\varphi^{(II)} \Big|_{r=r_S} = \begin{cases} H_\varphi^{(I,a)} \Big|_{r=r_S} & \text{for } \varphi_{SL}^{(a)} \leq \varphi \leq \varphi_{SL}^{(a)} + \beta_{SL} \quad \forall \quad a \in [1, 2, \dots, n_{SL}] \\ H_\varphi^{(IV,b)} \Big|_{r=r_S} & \text{for } \varphi_{FB}^{(b)} \leq \varphi \leq \varphi_{FB}^{(b)} + \beta_{FB} \quad \forall \quad b \in [1, 2, \dots, n_{FB}] \\ 0 & \text{elsewhere} \end{cases} \quad (\text{B.18})$$

This condition is met with the following integral expressions

$$\begin{aligned} \frac{1}{\mu_0} \int_0^{2\pi} B_\varphi^{(II)} \Big|_{r=r_S} \cos(k\varphi) d\varphi &= + \frac{1}{\mu_0} \sum_{a=1}^{n_{SL}} \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} B_\varphi^{(I,a)} \Big|_{r=r_S} \cos(k\varphi) d\varphi \\ &+ \frac{1}{\mu_0} \sum_{b=1}^{n_{FB}} \int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} B_\varphi^{(IV,b)} \Big|_{r=r_S} \cos(k\varphi) d\varphi \end{aligned} \quad (\text{B.19})$$

$$\begin{aligned} \frac{1}{\mu_0} \int_0^{2\pi} B_\varphi^{(II)} \Big|_{r=r_S} \sin(k\varphi) d\varphi &= + \frac{1}{\mu_0} \sum_{a=1}^{n_{SL}} \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} B_\varphi^{(I,a)} \Big|_{r=r_S} \sin(k\varphi) d\varphi \\ &+ \frac{1}{\mu_0} \sum_{b=1}^{n_{FB}} \int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} B_\varphi^{(IV,b)} \Big|_{r=r_S} \sin(k\varphi) d\varphi \end{aligned} \quad (\text{B.20})$$

The solutions are equal to (A.54) and (A.55) in combination with the additional terms for the flux barriers.

The continuity of the vector potential at the opening of a flux barriers

$$A_z^{(II)}|_{r=r_S} = A_z^{(IV,b)}|_{r=r_S} \quad \forall \quad \varphi_{FB}^{(b)} \geq \varphi \geq \varphi_{FB}^{(b)} + \beta_{FB} \quad (\text{B.21})$$

is achieved by the following integral expression for each flux barrier $b \in \{1, 2, \dots, n_{FB}\}$

$$\int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} A_z^{(II)}|_{r=r_S} \cos\left(\frac{m\pi}{\beta_{FB}}(\varphi - \varphi_{FB}^{(b)})\right) d\varphi = \int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} A_z^{(IV,b)}|_{r=r_S} \cos\left(\frac{m\pi}{\beta_{FB}}(\varphi - \varphi_{FB}^{(b)})\right) d\varphi \quad (\text{B.22})$$

with solution procedure similar to (A.57).

The constants $a_0^{(IV,b)}$ and $b_0^{(IV,b)}$ of a flux barrier's vector potential are extracted for each flux barrier $b \in \{1, 2, \dots, n_{FB}\}$ by

$$\int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} A_z^{(II)}|_{r=r_S} d\varphi = \int_{\varphi_{FB}^{(b)}}^{\varphi_{FB}^{(b)} + \beta_{FB}} A_z^{(IV,b)}|_{r=r_S} d\varphi \quad (\text{B.23})$$

with solution procedure similar to (A.59).

B.3 Resulting equation system

All resulting equations based on the continuous conditions defined in the subsection above can be rearranged for calculating the unknown Fourier coefficients. This results in a linear equation system of the following form

$$\mathbf{O}_{\mathbf{FB}} \begin{pmatrix} \vec{a}^{(I)} \\ \vec{a}^{(III)} \\ \vec{c}^{(III)} \\ \vec{a}^{(IV)} \\ \vec{b}^{(IV)} \\ \vec{b}^{(V)} \\ \vec{d}^{(V)} \end{pmatrix} = n_{turn} \tilde{\mathbf{P}}_{\mathbf{FB}} \vec{J}_z + \mathbf{Q}_{\mathbf{FB}} \begin{pmatrix} \vec{a}^{(M)} \\ \vec{c}^{(M)} \end{pmatrix} \quad (\text{B.24})$$

where vector $\vec{a}^{(I)}$ contains all Fourier coefficients $n \in \mathbb{N}^0$ of all subdomains $I^{(a)} \forall a \in \{1, 2, \dots, n_{SL}\}$ and vectors $\{\vec{a}^{(IV)}, \vec{b}^{(IV)}\}$ all Fourier coefficients $m \in \mathbb{N}^0$ of all subdomains $IV^{(b)} \forall a \in \{1, 2, \dots, n_{FB}\}$. Vectors $\{\vec{a}^{(III)}, \vec{c}^{(III)}, \vec{b}^{(V)}, \vec{d}^{(V)}, \}$ all Fourier coefficients $k, t \in \mathbb{N}$ respectively for each subdomain. Vector $\vec{J}_z = \left(i_z^{(a=1)} i_z^{(a=2)} \dots i_z^{(a=n_{SL})}\right)^T \frac{1}{S_{SL}}$ contains the current density flowing through each slot.

Equation system (B.24) can be solved by inverting matrix $\mathbf{O}_{\mathbf{FB}}$:

$$\begin{pmatrix} \vec{a}^{(I)} \\ \vec{a}^{(III)} \\ \vec{c}^{(III)} \\ \vec{a}^{(IV)} \\ \vec{b}^{(IV)} \\ \vec{b}^{(V)} \\ \vec{d}^{(V)} \end{pmatrix} = n_{turn} \underbrace{\mathbf{O}_{\mathbf{FB}}^{-1} \tilde{\mathbf{P}}_{\mathbf{FB}}}_{\tilde{\mathbf{P}}_{\mathbf{FB}}} \vec{J}_z + \underbrace{\mathbf{O}_{\mathbf{FB}}^{-1} \mathbf{Q}_{\mathbf{FB}}}_{\mathbf{Q}_{\mathbf{FB}}} \begin{pmatrix} \vec{a}^{(M)} \\ \vec{c}^{(M)} \end{pmatrix} \quad (\text{B.25})$$

For considering concentrated windings, matrix $\mathbf{P}_{\mathbf{FB}}$ can be built based on $\tilde{\mathbf{P}}_{\mathbf{FB}}$, analogously to (A.69). The solved equation system for obtaining all unknown Fourier coefficients is then defined as follows:

$$\begin{pmatrix} \vec{a}^{(I)} \\ \vec{a}^{(III)} \\ \vec{c}^{(III)} \\ \vec{a}^{(IV)} \\ \vec{b}^{(IV)} \\ \vec{b}^{(V)} \\ \vec{d}^{(V)} \end{pmatrix} = n_{turn} \mathbf{P}_{\mathbf{FB}} \vec{J}_z + \mathbf{Q}_{\mathbf{FB}} \begin{pmatrix} \vec{a}^{(M)} \\ \vec{c}^{(M)} \end{pmatrix} \quad (\text{B.26})$$

For the calculation of the torque, only the magnetic flux in the airgap is of relevance, and thereby the according parts of the solved equation system (B.26) for calculating Fourier coefficients $\{a_k^{(III)}, c_k^{(III)}\}$:

$$\begin{pmatrix} \vdots \\ \vec{a}^{(III)} \\ \vec{c}^{(III)} \\ \vdots \end{pmatrix} = n_{turn} \underbrace{\begin{pmatrix} \vdots \\ \mathbf{P}_{\mathbf{A}}^{(III)} \\ \mathbf{P}_{\mathbf{C}}^{(III)} \\ \vdots \end{pmatrix}}_{\mathbf{P}_{\mathbf{FB}}} \vec{J}_z + \underbrace{\begin{pmatrix} \vdots & \vdots \\ \mathbf{Q}_{\mathbf{AA}}^{(III)} & \mathbf{Q}_{\mathbf{AC}}^{(III)} \\ \mathbf{Q}_{\mathbf{CA}}^{(III)} & \mathbf{Q}_{\mathbf{CC}}^{(III)} \\ \vdots & \vdots \end{pmatrix}}_{\mathbf{Q}_{\mathbf{FB}}} \begin{pmatrix} \vec{a}^{(M)} \\ \vec{c}^{(M)} \end{pmatrix} \quad (\text{B.27})$$

The inductances, PM flux linkages and back-EMFs can be efficiently calculated based on $a_0^{(I,a)}$, the constant of the slot's (a) vector potential. For determining $a_0^{(I,a)}$ the according

part of the solved equation system (B.26) can be used:

$$\begin{pmatrix} a_0^{(I,a)} \\ \vdots \end{pmatrix} = n_{turn} \underbrace{\begin{pmatrix} (\vec{P}_0^{(I,a)})^T \\ \vdots \end{pmatrix}}_{\mathbf{P}_{\text{FB}}} \vec{J}_z + \underbrace{\begin{pmatrix} (\vec{Q}_{0A}^{(I,a)})^T & (\vec{Q}_{0C}^{(I,a)})^T \\ \vdots & \vdots \end{pmatrix}}_{\mathbf{Q}_{\text{FB}}} \begin{pmatrix} \vec{a}^{(M)} \\ \vec{c}^{(M)} \end{pmatrix} \quad (\text{B.28})$$

where

$$\vec{P}_0^{(I,a)} = \left(P_0^{(I,a)}(c=1) \quad P_0^{(I,a)}(c=2) \quad \dots \quad P_0^{(I,a)}(c) \quad \dots \quad P_0^{(I,a)}(c=n_{coil}) \right)^T. \quad (\text{B.29})$$

C Considering short circuits in the Subdomain Model

In this appendix the required adjustments for considering phase short circuits and interturn short circuits in the Subdomain Model are described.

C.1 Phase short circuits

(The following section is based on [72].)

C.1.1 Coefficients of the back-EMF

The equation for calculating the back-EMF (4.7) can be rearranged into the following Fourier series

$$e^{(f)}(\theta_R) = \omega_R \sum_{k \in \mathbb{N}} (\alpha_k \sin(k\theta_R) + \beta_k \cos(k\theta_R)) . \quad (\text{C.1})$$

Its coefficients $\{\alpha_k, \beta_k\}$ are calculated based on vector $\vec{Q}_{0A}^{(I,a)}$ and $\vec{Q}_{0C}^{(I,a)}$ from equation system (A.73) for a common PMSM and from (B.28) for a PMSM with flux barriers. They have the following structure

$$\vec{Q}_{0A}^{(I,a)} = \left(Q_{0A}^{(I,a)}(1) \quad Q_{0A}^{(I,a)}(2) \quad \dots \quad Q_{0A}^{(I,a)}(k) \dots \quad Q_{0A}^{(I,a)}(k = k_{max}) \right)^T , \quad (\text{C.2})$$

$\vec{Q}_{0C}^{(I,a)}$ is defined analogously. Fourier coefficients $\{\alpha_k, \beta_k\}$ can be obtained by multiplying vector element $Q_{0A}^{(I,a)}(k)$ or $Q_{0C}^{(I,a)}(k)$ with the derivatives $\{\frac{\partial}{\partial \theta_R} a_k^{(M)}, \frac{\partial}{\partial \theta_R} c_k^{(M)}\}$ of the magnetization Fourier coefficients, excluding the trigonometric functions ($\sin(k\theta_R)$ or $\cos(k\theta_R)$):

$$\begin{aligned} \alpha_k &= -\ell n_{turn} r_R M_k \mu_0 \left(Q_{0C}^{(I,fi)}(k) - Q_{0C}^{(I,fr)}(k) \right) , \\ \beta_k &= -\ell n_{turn} r_R M_k \mu_0 \left(Q_{0A}^{(I,fi)}(k) - Q_{0A}^{(I,fr)}(k) \right) . \end{aligned}$$

C.1.2 Coefficients of the torque capability

The equation for calculating the torque (4.14) contains the torque capabilities $\vec{P}$ of the coils, which can be formulated as the following Fourier series for the faulty coil (f):

$$P^{(f)}(\theta_R) = \sum_{k \in \mathbb{N}} (\sigma_k \sin(k\theta_R) + \nu_k \cos(k\theta_R)) \quad (\text{C.3})$$

Fourier coefficients $\{\sigma_k, \nu_k\}$ are calculated based on matrices $\mathbf{P}_A^{(III)}$ and $\mathbf{P}_C^{(III)}$ from equation system (A.72) for a common PMSM and from (B.27) for a PMSM with flux barriers, which have the following structure

$$\mathbf{P}_A^{(III)} = \begin{pmatrix} P_A^{(III)}(1, 1) & P_A^{(III)}(1, 2) & \dots & P_A^{(III)}(1, c) & \dots & P_A^{(III)}(1, c = n_{coil}) \\ P_A^{(III)}(2, 1) & P_A^{(III)}(2, 2) & \dots & P_A^{(III)}(2, c) & \dots & P_A^{(III)}(2, c = n_{coil}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P_A^{(III)}(k, 1) & P_A^{(III)}(k, 2) & \dots & P_A^{(III)}(k, c) & \dots & P_A^{(III)}(k, c = n_{coil}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P_A^{(III)}(k_{max}, 1) & P_A^{(III)}(k_{max}, 2) & \dots & P_A^{(III)}(k_{max}, c) & \dots & P_A^{(III)}(k_{max}, c = n_{coil}) \end{pmatrix}, \quad (\text{C.4})$$

$\mathbf{P}_C^{(III)}$ is defined analogously. By multiplying matrix element $P_A^{(III)}(k, f)$ or $P_C^{(III)}(k, f)$ with Fourier coefficients $\{a_k^{(M)}, c_k^{(M)}\}$, but without the trigonometric (sine/cosine) function, one obtains $\{\sigma_k, \nu_k\}$:

$$\sigma_k = -r_R \mu_0 M_k n_{turn} G_k P_C^{(III)}(k, f), \quad (\text{C.5})$$

$$\nu_k = +r_R \mu_0 M_k n_{turn} G_k P_A^{(III)}(k, f). \quad (\text{C.6})$$

Since $\{\alpha_k, \beta_k, \sigma_k, \nu_k\}$ are constant over the time, they can be calculated and stored during initialization to drastically decrease the computational effort.

C.1.3 Faulty torque ripple

The product of the torque capability $P^{(f)}$ and the short circuit current density J_{ps} , defined by equation (6.11), gives the faulty torque ripple T_{ps} :

$$T_{ps}(\theta_R, R_{ps}) = \frac{\ell \pi}{\mu_0} \underbrace{\sum_{k \in \mathbb{N}} (\sigma_k \sin(k\theta_R) + \nu_k \cos(k\theta_R))}_{P^{(f)}(\theta_R)} \underbrace{\sum_{k' \in \mathbb{N}} (\zeta_{k'}(R_{ps}) \sin(k'\theta_R) + \eta_{k'}(R_{ps}) \cos(k'\theta_R))}_{J_{ps}(\theta_R, R_{ps})}. \quad (\text{C.7})$$

By multiplying these two Fourier series one obtains:

$$T_{ps}(\theta_R, R_{ps}) = \frac{\ell\pi}{\mu_0} \sum_{k \in \mathbb{N}} \sum_{k' \in \mathbb{N}} (+ \sigma_k \zeta_{k'}(R_{ps}) \sin(k\theta_R) \sin(k'\theta_R) + \sigma_k \eta_{k'}(R_{ps}) \sin(k\theta_R) \cos(k'\theta_R) \\ + \nu_k \zeta_{k'}(R_{ps}) \cos(k\theta_R) \sin(k'\theta_R) + \nu_k \eta_{k'}(R_{ps}) \cos(k\theta_R) \cos(k'\theta_R)) . \quad (C.8)$$

With the trigonometric relations

$$\sin(k\theta_R) \cos(k'\theta_R) = +\frac{1}{2} [\sin((k+k')\theta_R) + \sin((k-k')\theta_R)] \quad (C.9)$$

$$\cos(k\theta_R) \cos(k'\theta_R) = +\frac{1}{2} [\cos((k+k')\theta_R) + \cos((k-k')\theta_R)] \quad (C.10)$$

$$\sin(k\theta_R) \sin(k'\theta_R) = -\frac{1}{2} [\cos((k+k')\theta_R) - \cos((k-k')\theta_R)] \quad (C.11)$$

each torque harmonic $v' \in \mathbb{Z}$ is extracted by

$$T_{ps,v'}(\theta_R, R_{ps}) = \frac{\ell\pi}{2\mu_0} \sum_{k \in \mathbb{N}} \sum_{k' \in \mathbb{N}} [- \sigma_k \zeta_{k'}(R_{ps}) (\cos((k+k')\theta_R) \delta_{k+k',v'} - \cos((k-k')\theta_R) \delta_{k-k',v'}) \\ + \sigma_k \eta_{k'}(R_{ps}) (\sin((k+k')\theta_R) \delta_{k+k',v'} + \sin((k-k')\theta_R) \delta_{k-k',v'}) \\ + \nu_k \zeta_{k'}(R_{ps}) (\sin((k+k')\theta_R) \delta_{k+k',v'} - \sin((k-k')\theta_R) \delta_{k-k',v'}) \\ + \nu_k \eta_{k'}(R_{ps}) (\cos((k+k')\theta_R) \delta_{k+k',v'} + \cos((k-k')\theta_R) \delta_{k-k',v'})] \quad (C.12)$$

where $\delta_{a,b}$ is the Kronecker delta. Rearranging this equation and with the symmetrical properties of the trigonometric functions

$$+ \sin(-v'\theta_R) = -\sin(+v\theta_R) , \quad (C.13)$$

$$+ \cos(-v'\theta_R) = +\cos(+v\theta_R) , \quad (C.14)$$

gives for $v \in \mathbb{N}^0$ the effective Fourier series for describing the faulty torque ripple

$$T_{ps}(\theta_R, R_{ps}) = \sum_{v \in \mathbb{N}^0} T_{ps,v}(\theta_R, R_{ps}) = \sum_{v \in \mathbb{N}^0} (\xi_v(R_{ps}) \sin(v\theta_R) + \rho_v(R_{ps}) \cos(v\theta_R)) \quad (C.15)$$

with

$$\begin{aligned} \xi_v(R_{ps}) = \frac{\ell\pi}{2\mu_0} \sum_{k \in \mathbb{N}} \sum_{k' \in \mathbb{N}} [(&+\sigma_k \eta_{k'}(R_{ps}) + \nu_k \zeta_{k'}(R_{ps}))\delta_{k+k',v} + (+\sigma_k \eta_{k'}(R_{ps}) - \nu_k \zeta_{k'}(R_{ps}))\delta_{k-k',v} \\ &- (+\sigma_k \eta_{k'}(R_{ps}) - \nu_k \zeta_{k'}(R_{ps}))\delta_{k-k',-v}(1 - \delta_{k-k',v=0})] \end{aligned} \quad (\text{C.16})$$

$$\begin{aligned} \rho_v(R_{ps}) = \frac{\ell\pi}{2\mu_0} \sum_{k \in \mathbb{N}} \sum_{k' \in \mathbb{N}} [(&-\sigma_k \zeta_{k'}(R_{ps}) + \nu_k \eta_{k'}(R_{ps}))\delta_{k+k',v} + (+\sigma_k \zeta_{k'}(R_{ps}) + \nu_k \eta_{k'}(R_{ps}))\delta_{k-k',v} \\ &+ (+\sigma_k \zeta_{k'}(R_{ps}) + \nu_k \eta_{k'}(R_{ps}))\delta_{k-k',-v}(1 - \delta_{k-k',v=0})]. \end{aligned} \quad (\text{C.17})$$

C.2 Interturn short circuits

(The following section is based on [73].)

C.2.1 Particular solution

In the case of an interturn short circuit the faulty coil splits up into multiple sub-coils, leading to three different current densities within the slot (see Fig. C.1). Therefor, the particular solution (A.24) needs to be adjusted [55], such that these different current densities are considered and the governing Maxwell equation (A.14) is still fulfilled. For the three partial subdomains one obtains the following particular solutions

$$\begin{aligned} A_{z,part}^{(I,a,1)} &= \frac{\mu_0}{2} \left[n_{turn}^{(a,1)} J_z^{(a,1)} \left(r_{S,1}^2 \ln(r) - \frac{r^2}{2} \right) + n_{turn}^{(a,2)} J_z^{(a,2)} (r_{S,2}^2 - r_{S,1}^2) \ln(r) + n_{turn}^{(a,3)} J_z^{(a,3)} (r_{SL}^2 - r_{S,2}^2) \ln(r) \right] \\ A_{z,part}^{(I,a,2)} &= \frac{\mu_0}{2} \left[\left(n_{turn}^{(a,1)} J_z^{(a,1)} - n_{turn}^{(a,2)} J_z^{(a,2)} \right) r_{S,1}^2 \left(\ln(r_{S,1}) - \frac{1}{2} \right) + n_{turn}^{(a,2)} J_z^{(a,2)} \left(r_{S,2}^2 \ln(r) - \frac{r^2}{2} \right) \right. \\ &\quad \left. + n_{turn}^{(a,3)} J_z^{(a,3)} (r_{SL}^2 - r_{S,2}^2) \ln(r) \right] \\ A_{z,part}^{(I,a,3)} &= \frac{\mu_0}{2} \left[\left(n_{turn}^{(a,1)} J_z^{(a,1)} - n_{turn}^{(a,2)} J_z^{(a,2)} \right) r_{S,1}^2 \left(\ln(r_{S,1}) - \frac{1}{2} \right) + \left(n_{turn}^{(a,2)} J_z^{(a,2)} - n_{turn}^{(a,3)} J_z^{(a,3)} \right) r_{S,2}^2 \left(\ln(r_{S,2}) - \frac{1}{2} \right) \right. \\ &\quad \left. + n_{turn}^{(a,3)} J_z^{(a,3)} \left(r_{SL}^2 \ln(r) - \frac{r^2}{2} \right) \right] \end{aligned} \quad (\text{C.18})$$

For each affected slot or coil with interturn short circuit, the vector potential for the complete subdomain $I^{(a)}$ is thereby equal to

$$A_z^{(I,a)} = A_{z,hom}^{(I,a)} + \begin{cases} A_{z,part}^{(I,a,1)} & \forall \quad r \in [r_S, r_{S,1}] \\ A_{z,part}^{(I,a,2)} & \forall \quad r \in [r_{S,1}, r_{S,2}] \\ A_{z,part}^{(I,a,3)} & \forall \quad r \in [r_{S,2}, r_{SL}] \end{cases} \quad (\text{C.19})$$

where $A_{z,hom}^{(I,a)}$ is defined by (A.22). For the continuous conditions to the airgap (see section A.3 and B.2) only $A_{z,part}^{(I,a,1)}$ is of relevance, since this is the only area which has an interface

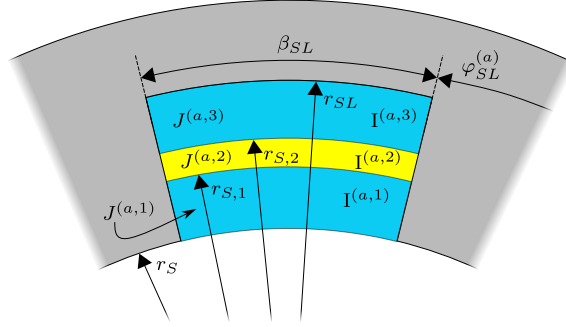


Figure C.1: Current densities of a coil with interturn short circuit, with the area of the **functional parts** (L_g) as well as the area of the **short circuited part** (L_s) [73].

to the airgap at $r = r_S$.

C.2.2 Inductances

Similar to the inductance of a healthy coil, the inductance of each sub-coil can be calculated by the magnetic energy method [62]. This leads for the inductance of the functional sub-coil $L_g = L_1 + L_3 + 2M_{13}$ to:

$$L_1 = \frac{n_{turn}^{(f,1)} \ell}{i^2} \sum_{a \in S(f)} \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} \int_{r_S}^{r_{S,1}} A_z^{(I,a)} \Big|_{J_z^{(a,2)} = J_z^{(a,3)} = 0; |J_z^{(a,1)}| > 0} J_z^{(a,1)} r dr d\varphi \quad (C.20)$$

with $A_z^{(I,a)}$ defined by (C.19). The sign of $J_z^{(a,1)} = \pm \frac{i}{S_{SL,1}}$ denotes the relative current direction within the coil and $S_{SL,1}$ is the surface of partial subdomain $I^{(a,1)}$. $S(f)$ is the set of all slots from faulty coil (f). The integrals for L_3 and M_{13} are defined analogously:

$$L_3 = \frac{n_{turn}^{(f,3)} \ell}{i^2} \sum_{a \in S(f)} \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} \int_{r_{S,2}}^{r_{SL}} A_z^{(I,a)} \Big|_{J_z^{(a,1)} = J_z^{(a,2)} = 0; |J_z^{(a,3)}| > 0} J_z^{(a,3)} r dr d\varphi \quad (C.21)$$

$$M_{13} = \frac{n_{turn}^{(f,1)} \ell}{i^2} \sum_{a \in S(f)} \int_{\varphi_{SL}^{(a)}}^{\varphi_{SL}^{(a)} + \beta_{SL}} \int_{r_S}^{r_{S,1}} A_z^{(I,a)} \Big|_{J_z^{(a,1)} = J_z^{(a,2)} = 0; |J_z^{(a,3)}| > 0} J_z^{(a,1)} r dr d\varphi \quad (C.22)$$

Solving these integrals gives

$$\begin{aligned} L_1 &= \left(n_{turn}^{(f,1)}\right)^2 \ell \frac{a_0^{(I,fi)} - a_0^{(I,fr)}}{i} + \mu_0 \frac{\left(n_{turn}^{(f,1)}\right)^2 \ell \beta_{SL}}{S_{SL,1}^2} \left(\frac{r_{S,1}^4}{4} (2 \ln(r_{S,1}) - 1) - \frac{r_{S,1}^2 r_S^2}{4} (2 \ln(r_S) - 1) - \frac{r_{S,1}^4}{8} + \frac{r_S^4}{8} \right) \\ L_3 &= \left(n_{turn}^{(f,3)}\right)^2 \ell \frac{a_0^{(I,fi)} - a_0^{(I,fr)}}{i} + \mu_0 \frac{\left(n_{turn}^{(f,3)}\right)^2 \ell \beta_{SL}}{S_{SL,3}^2} \left(\frac{r_{SL}^4}{8} (4 \ln(r_{SL}) - 3) - r_{SL}^2 r_{S,2}^2 (\ln(r_{S,2}) - \frac{1}{2}) + \frac{r_{S,2}^4}{2} \left(\ln(r_{S,2}) - \frac{1}{4} \right) \right) \\ M_{13} &= n_{turn}^{(f,1)} n_{turn}^{(f,3)} \ell \frac{a_0^{(I,fi)} - a_0^{(I,fr)}}{i} + \mu_0 \frac{n_{turn}^{(f,1)} n_{turn}^{(f,3)} \ell \beta_{SL}}{S_{SL,1} S_{SL,3}} (r_{SL}^2 - r_{S,2}^2) \left(\frac{r_{S,1}^2}{4} (2 \ln(r_{S,1}) - 1) - \frac{r_S^2}{4} (2 \ln(r_S) - 1) \right) \end{aligned} \quad (C.23)$$

Inserting (A.73) for the case of a common PMSM or (B.28) for the case of a PMSM with flux barriers, one obtains

$$L_1 = \left(n_{turn}^{(f,1)}\right)^2 \ell \frac{\pm P_0^{(I,fi)}(f_l) \mp P_0^{(I,fi)}(f_r) \mp P_0^{(I,fr)}(f_l) \pm P_0^{(I,fr)}(f_r)}{S_{SL,1}} \\ + \mu_0 \frac{\left(n_{turn}^{(f,1)}\right)^2 \ell \beta_{SL}}{S_{SL,1}^2} \left(\frac{r_{S,1}^4}{4} (2 \ln(r_{S,1}) - 1) - \frac{r_{S,1}^2 r_S^2}{4} (2 \ln(r_S) - 1) - \frac{r_{S,1}^4}{8} + \frac{r_S^4}{8} \right) \quad (C.24)$$

$$L_3 = \left(n_{turn}^{(f,3)}\right)^2 \ell \frac{\pm P_0^{(I,fi)}(f_l) \mp P_0^{(I,fi)}(f_r) \mp P_0^{(I,fr)}(f_l) \pm P_0^{(I,fr)}(f_r)}{S_{SL,3}} \\ + \mu_0 \frac{\left(n_{turn}^{(f,3)}\right)^2 \ell \beta_{SL}}{S_{SL,3}^2} \left(\frac{r_{SL}^4}{8} (4 \ln(r_{SL}) - 3) - r_{SL}^2 r_{S,2}^2 (\ln(r_{S,2}) - \frac{1}{2}) + \frac{r_{S,2}^4}{2} \left(\ln(r_{S,2}) - \frac{1}{4} \right) \right) \quad (C.25)$$

$$M_{13} = n_{turn}^{(f,1)} n_{turn}^{(f,3)} \ell \frac{\pm P_0^{(I,fi)}(f_l) \mp P_0^{(I,fi)}(f_r) \mp P_0^{(I,fr)}(f_l) \pm P_0^{(I,fr)}(f_r)}{S_{SL,1} S_{SL,3}} \\ + \mu_0 \frac{n_{turn}^{(f,1)} n_{turn}^{(f,3)} \ell \beta_{SL}}{S_{SL,1} S_{SL,3}} \left(r_{SL}^2 - r_{S,2}^2 \right) \left(\frac{r_{S,1}^2}{4} (2 \ln(r_{S,1}) - 1) - \frac{r_S^2}{4} (2 \ln(r_S) - 1) \right) \quad (C.26)$$

The remaining inductances $L_s = L_2$ and $M_{sg} = M_{21} + M_{23}$ can be determined by the same procedure.

C.2.3 PM flux linkage & back-EMF

As shown in subsection 4.2.1, the PM flux linkage is effectively calculated based on the *spatial* constant offset $a_0^{(I,a)}$ of a slot's vector potential. Since $a_0^{(I,a)}$ is still constant for the whole slot in the case of an interturn short circuit (situation as shown in Fig. C.1), the only required adjustment in (4.5) for calculating the PM flux linkage of the partial subdomains in the slot is the number of considered coil turns

$$\Phi_M^{(f,x)}(\theta_R) = \ell n_{turn}^{(f,x)} \left(a_0^{(I,fi)} - a_0^{(I,fr)} \right). \quad (C.27)$$

with $x \in \{1, 2, 3\}$. Inserting (A.73) for the case of a common PMSM or (B.28) for the case of a PMSM with flux barriers (with $\vec{J}_z = \vec{0}$), yields for the PM flux linkage (4.6) and for the back-EMF (4.7), besides the adjusted number of winding turns $n_{turn}^{(f,x)}$.

Bibliography

- [1] P. Wheeler, T. S. Sirimanna, S. Bozhko, and K. S. Haran. “Electric/Hybrid-Electric Aircraft Propulsion Systems”. In: *Proceedings of the IEEE* 109.6 (June 2021), pp. 1115–1127. DOI: [10.1109/jproc.2021.3073291](https://doi.org/10.1109/jproc.2021.3073291).
- [2] T. Orlowska-Kowalska, M. Wolkiewicz, P. Pietrzak, M. Skowron, P. Ewert, G. Tarchala, M. Krzysztofiak, and C. T. Kowalski. “Fault Diagnosis and Fault-Tolerant Control of PMSM Drives—State of the Art and Future Challenges”. In: *IEEE Access* 10 (2022), pp. 59979–60024. DOI: [10.1109/access.2022.3180153](https://doi.org/10.1109/access.2022.3180153).
- [3] F. Barrero and M. J. Duran. “Recent Advances in the Design, Modeling, and Control of Multiphase Machines—Part I”. In: *IEEE Transactions on Industrial Electronics* 63.1 (Jan. 2016), pp. 449–458. DOI: [10.1109/tie.2015.2447733](https://doi.org/10.1109/tie.2015.2447733).
- [4] M. J. Duran and F. Barrero. “Recent Advances in the Design, Modeling, and Control of Multiphase Machines—Part II”. In: *IEEE Transactions on Industrial Electronics* 63.1 (Jan. 2016), pp. 459–468. DOI: [10.1109/tie.2015.2448211](https://doi.org/10.1109/tie.2015.2448211).
- [5] A. G. Yepes, O. Lopez, I. Gonzalez-Prieto, M. J. Duran, and J. Doval-Gandoy. “A Comprehensive Survey on Fault Tolerance in Multiphase AC Drives, Part 1: General Overview Considering Multiple Fault Types”. In: *Machines* 10.3 (2022), p. 208. DOI: [10.3390/machines10030208](https://doi.org/10.3390/machines10030208).
- [6] A. G. Yepes, I. Gonzalez-Prieto, O. Lopez, M. J. Duran, and J. Doval-Gandoy. “A Comprehensive Survey on Fault Tolerance in Multiphase AC Drives, Part 2: Phase and Switch Open-Circuit Faults”. In: *Machines* 10.3 (2022), p. 221. DOI: [10.3390/machines10030221](https://doi.org/10.3390/machines10030221).
- [7] M. Cheng, J. Hang, and J. Zhang. “Overview of fault diagnosis theory and method for permanent magnet machine”. In: *Chinese Journal of Electrical Engineering* 1.1 (2015), pp. 21–36. DOI: [10.23919/cjee.2015.7933135](https://doi.org/10.23919/cjee.2015.7933135).
- [8] A. Kontarcek, P. Bajec, M. Nemec, V. Ambrozic, and D. Nedeljkovic. “Cost-Effective Three-Phase PMSM Drive Tolerant to Open-Phase Fault”. In: *IEEE Transactions on Industrial Electronics* 62.11 (Nov. 2015), pp. 6708–6718. DOI: [10.1109/tie.2015.2437357](https://doi.org/10.1109/tie.2015.2437357).
- [9] J. Hang, X. Ren, C. Tang, M. Tong, and S. Ding. “Fault-Tolerant Control Strategy for Five-Phase PMSM Drive System With High-Resistance Connection”. In: *IEEE Transactions on Transportation Electrification* 7.3 (2021), pp. 1390–1400. DOI: [10.1109/tte.2020.3037894](https://doi.org/10.1109/tte.2020.3037894).

- [10] L. Zarri, M. Mengoni, Y. Gritli, A. Tani, F. Filippetti, G. Serra, and D. Casadei. “Detection and Localization of Stator Resistance Dissymmetry Based on Multiple Reference Frame Controllers in Multiphase Induction Motor Drives”. In: *IEEE Transactions on Industrial Electronics* 60.8 (Aug. 2013), pp. 3506–3518. DOI: [10.1109/tie.2012.2235393](https://doi.org/10.1109/tie.2012.2235393).
- [11] I. Jeong, B. J. Hyon, and K. Nam. “Dynamic Modeling and Control for SPMSMs With Internal Turn Short Fault”. In: *IEEE Transactions on Power Electronics* 28.7 (July 2013), pp. 3495–3508. DOI: [10.1109/tpel.2012.2222049](https://doi.org/10.1109/tpel.2012.2222049).
- [12] Y. Qi, E. Bostanci, V. Gurusamy, and B. Akin. “A Comprehensive Analysis of Short-Circuit Current Behavior in PMSM Interturn Short-Circuit Faults”. In: *IEEE Transactions on Power Electronics* 33.12 (Dec. 2018), pp. 10784–10793. DOI: [10.1109/tpel.2018.2809668](https://doi.org/10.1109/tpel.2018.2809668).
- [13] T. A. Shifat and J. W. Hur. “An Effective Stator Fault Diagnosis Framework of BLDC Motor Based on Vibration and Current Signals”. In: *IEEE Access* 8 (2020), pp. 106968–106981. DOI: [10.1109/access.2020.3000856](https://doi.org/10.1109/access.2020.3000856).
- [14] L. Vancini, M. Mengoni, G. Rizzoli, L. Zarri, and A. Tani. “Local Demagnetization Detection in Six-Phase Permanent Magnet Synchronous Machines”. In: *IEEE Transactions on Industrial Electronics* 71.6 (June 2024), pp. 5508–5518. DOI: [10.1109/tie.2023.3294603](https://doi.org/10.1109/tie.2023.3294603).
- [15] M. Mengoni, L. Vancini, A. Tani, Y. Gritli, L. Zarri, and C. Rossi. “On-line Detection of Magnet Demagnetization in Asymmetrical Six-Phase Surface-Mounted Permanent Magnet Synchronous Motor Drives”. In: *2019 IEEE 12th International Symposium on Diagnostics for Electrical Machines, Power Electronics and Drives (SDEMPED)*. IEEE, Aug. 2019, pp. 188–194. DOI: [10.1109/demped.2019.8864881](https://doi.org/10.1109/demped.2019.8864881).
- [16] J. Faiz and E. Mazaheri-Tehrani. “Demagnetization Modeling and Fault Diagnosing Techniques in Permanent Magnet Machines Under Stationary and Nonstationary Conditions: An Overview”. In: *IEEE Transactions on Industry Applications* 53.3 (May 2017), pp. 2772–2785. DOI: [10.1109/tia.2016.2608950](https://doi.org/10.1109/tia.2016.2608950).
- [17] M. J. Duran, E. Levi, and F. Barrero. *Multiphase Electric Drives: Introduction*. 2017. DOI: [10.1002/047134608x.w8364](https://doi.org/10.1002/047134608x.w8364).
- [18] J. Zhao, Y. Liu, and X. Xu. “Comparisons of Concentrated and Distributed Winding PMSM in MV Power Generation”. In: *2018 XIII International Conference on Electrical Machines (ICEM)*. IEEE, Sept. 2018, pp. 2437–2443. DOI: [10.1109/icelmach.2018.8507199](https://doi.org/10.1109/icelmach.2018.8507199).
- [19] B. Lehner and D. Gerling. “Design and comparison of concentrated and distributed winding synchronous reluctance machines”. In: *2016 IEEE Energy Conversion Congress and Exposition (ECCE)*. IEEE, Sept. 2016, pp. 1–8. DOI: [10.1109/ecce.2016.7855004](https://doi.org/10.1109/ecce.2016.7855004).
- [20] H. Qiu, Y. Zhang, C. Yang, and R. Yi. “Performance analysis and comparison of PMSM with concentrated winding and distributed winding”. In: *Archives of Electrical Engineering* 69.2 (2020).

- [21] B. Mecrow. “Fault tolerant permanent magnet machine drives”. In: *Seventh International Conference on Electrical Machines and Drives*. IEE, 1995. DOI: [10.1049/cp:19950909](https://doi.org/10.1049/cp:19950909).
- [22] D. Ishak, Z. Zhu, and D. Howe. “Comparison of PM brushless motors, having either all teeth or alternate teeth wound”. In: *IEEE Transactions on Energy Conversion* 21.1 (2006), pp. 95–103. DOI: [10.1109/tec.2005.853765](https://doi.org/10.1109/tec.2005.853765).
- [23] J. W. Gerold and D. Gerling. “An equivalent winding factor larger than 1 by using flux barriers in the stator”. In: *2019 IEEE International Electric Machines and Drives Conference (IEMDC)*. IEEE, 2019. DOI: [10.1109/iemdc.2019.8785148](https://doi.org/10.1109/iemdc.2019.8785148).
- [24] G. Dajaku and D. Gerling. “Low costs and high-efficiency electric machines”. In: *2012 2nd International Electric Drives Production Conference (EDPC)*. IEEE, 2012. DOI: [10.1109/edpc.2012.6425093](https://doi.org/10.1109/edpc.2012.6425093).
- [25] S. Spas, G. Dajaku, and D. Gerling. “Comparison of PM machines with concentrated windings for automotive application”. In: *2014 International Conference on Electrical Machines (ICEM)*. IEEE, 2014. DOI: [10.1109/icelmach.2014.6960458](https://doi.org/10.1109/icelmach.2014.6960458).
- [26] A. Nollau and D. Gerling. “Novel cooling methods using flux-barriers”. In: *2014 International Conference on Electrical Machines (ICEM)*. IEEE, 2014. DOI: [10.1109/icelmach.2014.6960354](https://doi.org/10.1109/icelmach.2014.6960354).
- [27] D. Alban, G. Dajaku, and D. Gerling. “Evaluation of Flux-barrier Stator in five-phase PMSMs for Electric Aircraft Traction”. In: *2022 25th International Conference on Electrical Machines and Systems (ICEMS)*. IEEE, Nov. 2022, pp. 1–6. DOI: [10.1109/icems56177.2022.9983393](https://doi.org/10.1109/icems56177.2022.9983393).
- [28] A. Shawier, A. S. Abdel-Khalik, R. A. Hamdy, K. H. Ahmed, and S. Ahmed. “Postfault Operation of Five-Phase Induction Machine With Minimum Total Losses Under Single Open-Phase Fault”. In: *IEEE Access* 8 (2020), pp. 208696–208706. DOI: [10.1109/access.2020.3036904](https://doi.org/10.1109/access.2020.3036904).
- [29] H. M. Eldeeb, A. S. Abdel-Khalik, and C. M. Hackl. “Postfault Full Torque–Speed Exploitation of Dual Three-Phase IPMSM Drives”. In: *IEEE Transactions on Industrial Electronics* 66.9 (Sept. 2019), pp. 6746–6756. DOI: [10.1109/tie.2018.2880698](https://doi.org/10.1109/tie.2018.2880698).
- [30] S. Dwari and L. Parsa. “Fault-Tolerant Control of Five-Phase Permanent-Magnet Motors With Trapezoidal Back EMF”. In: *IEEE Transactions on Industrial Electronics* 58.2 (Feb. 2011), pp. 476–485. DOI: [10.1109/tie.2010.2045322](https://doi.org/10.1109/tie.2010.2045322).
- [31] A. Mohammadpour and L. Parsa. “A Unified Fault-Tolerant Current Control Approach for Five-Phase PM Motors With Trapezoidal Back EMF Under Different Stator Winding Connections”. In: *IEEE Transactions on Power Electronics* 28.7 (July 2013), pp. 3517–3527. DOI: [10.1109/tpel.2012.2223717](https://doi.org/10.1109/tpel.2012.2223717).
- [32] X. Kestelyn and E. Semail. “A Vectorial Approach for Generation of Optimal Current References for Multiphase Permanent-Magnet Synchronous Machines in Real Time”. In: *IEEE Transactions on Industrial Electronics* 58.11 (Nov. 2011), pp. 5057–5065. DOI: [10.1109/tie.2011.2119454](https://doi.org/10.1109/tie.2011.2119454).

- [33] N. Bianchi, S. Bolognani, and M. Dai Pre. “Strategies for the Fault-Tolerant Current Control of a Five-Phase Permanent-Magnet Motor”. In: *IEEE Transactions on Industry Applications* 43.4 (2007), pp. 960–970. DOI: [10.1109/tia.2007.900445](https://doi.org/10.1109/tia.2007.900445).
- [34] B. Welchko, J. Wai, T. Jahns, and T. Lipo. “Magnet-flux-nulling control of interior PM Machine drives for improved steady-state response to short-circuit faults”. In: *IEEE Transactions on Industry Applications* 42.1 (Jan. 2006), pp. 113–120. DOI: [10.1109/tia.2005.861313](https://doi.org/10.1109/tia.2005.861313).
- [35] J. Ede, K. Atallah, J. Wang, and D. Howe. “Effect of optimal torque control on rotor loss of fault-tolerant permanent-magnet brushless machines”. In: *IEEE Transactions on Magnetics* 38.5 (Sept. 2002), pp. 3291–3293. DOI: [10.1109/tmag.2002.802294](https://doi.org/10.1109/tmag.2002.802294).
- [36] J. Wang, K. Atallah, and D. Howe. “Optimal torque control of fault-tolerant permanent magnet brushless machines”. In: *IEEE Transactions on Magnetics* 39.5 (Sept. 2003), pp. 2962–2964. DOI: [10.1109/tmag.2003.816707](https://doi.org/10.1109/tmag.2003.816707).
- [37] A. Mohammadpour and L. Parsa. “Global Fault-Tolerant Control Technique for Multiphase Permanent-Magnet Machines”. In: *IEEE Transactions on Industry Applications* 51.1 (Jan. 2015), pp. 178–186. DOI: [10.1109/tia.2014.2326084](https://doi.org/10.1109/tia.2014.2326084).
- [38] B. Sen and J. Wang. “Stationary Frame Fault Tolerant Current Control of Poly-phase Permanent Magnet Machines under Open-Circuit and Short-Circuit Faults”. In: *IEEE Transactions on Power Electronics* (2015), pp. 1–1. DOI: [10.1109/tpe.2015.2478337](https://doi.org/10.1109/tpe.2015.2478337).
- [39] A. Mohammadpour, S. Mishra, and L. Parsa. “Fault-Tolerant Operation of Multiphase Permanent-Magnet Machines Using Iterative Learning Control”. In: *IEEE Journal of Emerging and Selected Topics in Power Electronics* 2.2 (June 2014), pp. 201–211. DOI: [10.1109/jestpe.2013.2295537](https://doi.org/10.1109/jestpe.2013.2295537).
- [40] J. G. Cintron-Rivera, S. N. Foster, and E. G. Strangas. “Mitigation of Turn-to-Turn Faults in Fault Tolerant Permanent Magnet Synchronous Motors”. In: *IEEE Transactions on Energy Conversion* 30.2 (June 2015), pp. 465–475. DOI: [10.1109/tec.2014.2360813](https://doi.org/10.1109/tec.2014.2360813).
- [41] A. Mitcham, G. Antonopoulos, and J. Cullen. “Implications of shorted turn faults in bar wound PM machines”. In: *IEE Proceedings - Electric Power Applications* 151 (6 2004), pp. 651–657. DOI: <https://doi.org/10.1049/ip-epa:20040686>.
- [42] S.-H. Im and B.-G. Gu. “Interturn Fault Tolerant Drive Method by Limiting Copper Loss of Interior Permanent Magnet Synchronous Motor”. In: *IEEE Transactions on Industrial Electronics* 67.9 (Sept. 2020), pp. 7973–7981. DOI: [10.1109/tie.2019.2941146](https://doi.org/10.1109/tie.2019.2941146).
- [43] J. Zhang, W. Zhan, and M. Ehsani. “Diagnosis and Fault-Tolerant Control of Permanent Magnet Synchronous Motors With Interturn Short-Circuit Fault”. In: *IEEE Transactions on Control Systems Technology* 31.4 (2023), pp. 1909–1916. DOI: [10.1109/tcst.2023.3239426](https://doi.org/10.1109/tcst.2023.3239426).
- [44] R. Cui, Y. Fan, and C. Li. “On-Line Inter-Turn Short-Circuit Fault Diagnosis and Torque Ripple Minimization Control Strategy Based on OW Five-Phase BFTHE-IPM”. In: *IEEE Transactions on Energy Conversion* 33.4 (Dec. 2018), pp. 2200–2209. DOI: [10.1109/tec.2018.2851615](https://doi.org/10.1109/tec.2018.2851615).

- [45] A. Tani, Y. Gritli, M. Mengoni, L. Zarri, G. Sala, A. Bellini, and G. Serra. “Detection of magnet demagnetization and high-resistance connections in five-phase surface-mounted permanent magnet generators”. In: *2015 IEEE 10th International Symposium on Diagnostics for Electrical Machines, Power Electronics and Drives (SDEMPED)*. IEEE, Sept. 2015. DOI: [10.1109/demped.2015.7303734](https://doi.org/10.1109/demped.2015.7303734).
- [46] C. Zhang, G. Wu, F. Rong, J. Feng, L. Jia, J. He, and S. Huang. “Robust Fault-Tolerant Predictive Current Control for Permanent Magnet Synchronous Motors Considering Demagnetization Fault”. In: *IEEE Transactions on Industrial Electronics* 65.7 (2018), pp. 5324–5334. DOI: [10.1109/tie.2017.2774758](https://doi.org/10.1109/tie.2017.2774758).
- [47] C. Rossi, Y. Gritli, A. Pilati, G. Rizzoli, A. Tani, and D. Casadei. “High Resistance Fault-Detection and Fault-Tolerance for Asymmetrical Six-Phase Surface-Mounted AC Permanent Magnet Synchronous Motor Drives”. In: *Energies* 13.12 (June 2020), p. 3089. DOI: [10.3390/en13123089](https://doi.org/10.3390/en13123089).
- [48] S. Jin, W. Zhao, J. Ji, and D. Xu. “Deadbeat Fault-Tolerant Control Scheme for Dual Three-Phase PMSG With High-Resistance Connection Fault”. In: *IEEE Transactions on Power Electronics* 38.3 (Mar. 2023), pp. 4015–4026. DOI: [10.1109/tpel.2022.3219544](https://doi.org/10.1109/tpel.2022.3219544).
- [49] X. Jiang, W. Huang, R. Cao, Z. Hao, and W. Jiang. “Electric Drive System of Dual-Winding Fault-Tolerant Permanent-Magnet Motor for Aerospace Applications”. In: *IEEE Transactions on Industrial Electronics* 62.12 (Dec. 2015), pp. 7322–7330. DOI: [10.1109/tie.2015.2454483](https://doi.org/10.1109/tie.2015.2454483).
- [50] Z. Zhu, D. Howe, E. Bolte, and B. Ackermann. “Instantaneous magnetic field distribution in brushless permanent magnet DC motors. I. Open-circuit field”. In: *IEEE Transactions on Magnetics* 29.1 (1993), pp. 124–135. DOI: [10.1109/20.195557](https://doi.org/10.1109/20.195557).
- [51] Z. Zhu and D. Howe. “Instantaneous magnetic field distribution in brushless permanent magnet DC motors. II. Armature-reaction field”. In: *IEEE Transactions on Magnetics* 29.1 (Jan. 1993), pp. 136–142. DOI: [10.1109/20.195558](https://doi.org/10.1109/20.195558).
- [52] Z. Zhu and D. Howe. “Instantaneous magnetic field distribution in brushless permanent magnet DC motors. III. Effect of stator slotting”. In: *IEEE Transactions on Magnetics* 29.1 (Jan. 1993), pp. 143–151. DOI: [10.1109/20.195559](https://doi.org/10.1109/20.195559).
- [53] Z. Zhu and D. Howe. “Instantaneous magnetic field distribution in permanent magnet brushless DC motors. IV. Magnetic field on load”. In: *IEEE Transactions on Magnetics* 29.1 (Jan. 1993), pp. 152–158. DOI: [10.1109/20.195560](https://doi.org/10.1109/20.195560).
- [54] B. Ackermann and R. Sottek. “Analytical modeling of the cogging torque in permanent magnet motors”. In: *Electrical Engineering* 78.2 (1995), pp. 117–125. DOI: [10.1007/bf01245643](https://doi.org/10.1007/bf01245643).
- [55] A. Bellara, Y. Amara, G. Barakat, and B. Dakyo. “Two-Dimensional Exact Analytical Solution of Armature Reaction Field in Slotted Surface Mounted PM Radial Flux Synchronous Machines”. In: *IEEE Transactions on Magnetics* 45.10 (2009), pp. 4534–4538. DOI: [10.1109/tmag.2009.2021527](https://doi.org/10.1109/tmag.2009.2021527).
- [56] T. Lubin, S. Mezani, and A. Rezzoug. “2-D Exact Analytical Model for Surface-Mounted Permanent-Magnet Motors With Semi-Closed Slots”. In: *IEEE Transactions on Magnetics* 47.2 (2011), pp. 479–492. DOI: [10.1109/tmag.2010.2095874](https://doi.org/10.1109/tmag.2010.2095874).

- [57] T. Lubin, S. Mezani, and A. Rezzoug. “IMPROVED ANALYTICAL MODEL FOR SURFACE-MOUNTED PM MOTORS CONSIDERING SLOTTING EFFECTS AND ARMATURE REACTION”. In: *Progress In Electromagnetics Research B* 25 (2010), pp. 293–314. DOI: [10.2528/pierb10081209](https://doi.org/10.2528/pierb10081209).
- [58] L. J. Wu, Z. Li, X. Huang, Y. Zhong, Y. Fang, and Z. Q. Zhu. “A Hybrid Field Model for Open-Circuit Field Prediction in Surface-Mounted PM Machines Considering Saturation”. In: *IEEE Transactions on Magnetics* 54.6 (2018), pp. 1–12. DOI: [10.1109/tmag.2018.2817178](https://doi.org/10.1109/tmag.2018.2817178).
- [59] L. J. Wu, Z. Li, D. Wang, H. Yin, X. Huang, and Z. Q. Zhu. “On-Load Field Prediction of Surface-Mounted PM Machines Considering Nonlinearity Based on Hybrid Field Model”. In: *IEEE Transactions on Magnetics* 55.3 (2019), pp. 1–11. DOI: [10.1109/tmag.2018.2890244](https://doi.org/10.1109/tmag.2018.2890244).
- [60] M. Ackermann, J.-L. Marqués, and C. Hillermeier. “Optimal multi-fault tolerant control for PMSMs – Part I: Feedforward control”. In: *2024 IEEE 21st International Power Electronics and Motion Control Conference (PEMC)*. IEEE, Sept. 2024, pp. 1–9. DOI: [10.1109/pemc61721.2024.10726337](https://doi.org/10.1109/pemc61721.2024.10726337).
- [61] M. Ackermann, J.-L. Marques, and C. Hillermeier. “Fast Analytical Calculation of the Magnetic Field in Permanent Magnet Synchronous Machines with Flux Barriers Including Saturation”. In: *PCIM Europe 2024; International Exhibition and Conference for Power Electronics, Intelligent Motion, Renewable Energy and Energy Management*. 2024, pp. 1331–1340. DOI: [10.30420/566262182](https://doi.org/10.30420/566262182).
- [62] L. Huiyong and C. Jiwei. “Analytical Calculation of Winding Inductances of Permanent-Magnet Synchronous Motor Accounting for Stator Slots”. In: *CSAA/IET International Conference on Aircraft Utility Systems (AUS 2018)*. Institution of Engineering and Technology, 2018. DOI: [10.1049/cp.2018.0132](https://doi.org/10.1049/cp.2018.0132).
- [63] L. J. Wu, Z. Q. Zhu, D. Staton, M. Popescu, and D. Hawkins. “Comparison of analytical models for predicting electromagnetic performance in surface-mounted permanent magnet machines”. In: *2010 IEEE Vehicle Power and Propulsion Conference*. IEEE, 2010. DOI: [10.1109/vppc.2010.5729155](https://doi.org/10.1109/vppc.2010.5729155).
- [64] K.-H. Shin, H.-I. Park, H.-W. Cho, and J.-Y. Choi. “Analytical Calculation and Experimental Verification of Cogging Torque and Optimal Point in Permanent Magnet Synchronous Motors”. In: *IEEE Transactions on Magnetics* 53.6 (2017), pp. 1–4. DOI: [10.1109/tmag.2017.2667643](https://doi.org/10.1109/tmag.2017.2667643).
- [65] Y. Oner, Z. Q. Zhu, L. J. Wu, X. Ge, H. Zhan, and J. T. Chen. “Analytical On-Load Subdomain Field Model of Permanent-Magnet Vernier Machines”. In: *IEEE Transactions on Industrial Electronics* 63.7 (2016), pp. 4105–4117. DOI: [10.1109/tie.2016.2532285](https://doi.org/10.1109/tie.2016.2532285).
- [66] M. Ackermann, J.-L. Marqués, and C. Hillermeier. “Optimal multi-fault tolerant control for PMSMs – Part II: Feedback control”. In: *2024 IEEE 21st International Power Electronics and Motion Control Conference (PEMC)*. IEEE, Sept. 2024, pp. 1–7. DOI: [10.1109/pemc61721.2024.10726410](https://doi.org/10.1109/pemc61721.2024.10726410).
- [67] D. G. Luenberger and Y. Ye. *Linear and Nonlinear Programming*. Springer International Publishing, 2021. DOI: [10.1007/978-3-030-85450-8](https://doi.org/10.1007/978-3-030-85450-8).

- [68] J. Chiasson. *Modeling and High-Performance Control of Electric Machines*. John Wiley and Sons, Ltd, 2005. DOI: <https://doi.org/10.1002/0471722359>.
- [69] M. Ackermann, J.-L. Marqués, and C. Hillermeier. “Diagnosis of partial demagnetization in a multi-fault tolerant control for PMSMs”. In: *2025 IEEE 19th International Conference on Compatibility, Power Electronics and Power Engineering (CPE-POWERENG)*. IEEE, May 2025, pp. 1–8. DOI: [10.1109/cpe-powereng63314.2025.11027187](https://doi.org/10.1109/cpe-powereng63314.2025.11027187).
- [70] C. Chaisakdanugull and K. F. Hoffmann. “Fault-Tolerant Operation Analysis of a Five-Phase Three-Level TNPC Inverter for Electric Aircraft Propulsion Systems”. In: *PCIM Europe 2024; International Exhibition and Conference for Power Electronics, Intelligent Motion, Renewable Energy and Energy Management*. 2024, pp. 3062–3071. DOI: [10.30420/566262435](https://doi.org/10.30420/566262435).
- [71] O. Dordevic, M. Jones, and E. Levi. “A Comparison of Carrier-Based and Space Vector PWM Techniques for Three-Level Five-Phase Voltage Source Inverters”. In: *IEEE Transactions on Industrial Informatics* 9.2 (May 2013), pp. 609–619. DOI: [10.1109/tii.2012.2220553](https://doi.org/10.1109/tii.2012.2220553).
- [72] M. Ackermann, J.-L. Marqués, and C. Hillermeier. “Adaptation of a multi-fault tolerant control for PMSMs to phase short circuits”. In: *IECON 2025 – 51st Annual Conference of the IEEE Industrial Electronics Society*. IEEE, Oct. 2025, pp. 1–8. DOI: [10.1109/iecon58223.2025.11221390](https://doi.org/10.1109/iecon58223.2025.11221390).
- [73] M. Ackermann, J.-L. Marqués, and C. Hillermeier. “Multi-Fault Tolerant Control for PMSMs Considering Interturn Short Circuits”. In: *2025 IEEE 34th International Symposium on Industrial Electronics (ISIE)*. IEEE, June 2025, pp. 1–7. DOI: [10.1109/isie62713.2025.11124800](https://doi.org/10.1109/isie62713.2025.11124800).
- [74] Z. Sun, J. Wang, D. Howe, and G. Jewell. “Analytical Prediction of the Short-Circuit Current in Fault-Tolerant Permanent-Magnet Machines”. In: *IEEE Transactions on Industrial Electronics* 55.12 (2008), pp. 4210–4217. DOI: [10.1109/tie.2008.2005019](https://doi.org/10.1109/tie.2008.2005019).
- [75] A. Fitzgibbon, M. Pilu, and R. Fisher. “Direct least square fitting of ellipses”. In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* 21.5 (May 1999), pp. 476–480. DOI: [10.1109/34.765658](https://doi.org/10.1109/34.765658).
- [76] J. A. Rosero, J. Cusido, A. Garcia, J. Ortega, and L. Romeral. “Study on the Permanent Magnet Demagnetization Fault in Permanent Magnet Synchronous Machines”. In: *IECON 2006 - 32nd Annual Conference on IEEE Industrial Electronics*. IEEE, Nov. 2006. DOI: [10.1109/iecon.2006.347598](https://doi.org/10.1109/iecon.2006.347598).
- [77] S. Rajagopalan, W. I. Roux, T. G. Habetler, and R. G. Harley. “Dynamic Eccentricity and Demagnetized Rotor Magnet Detection in Trapezoidal Flux (Brushless DC) Motors Operating Under Different Load Conditions”. In: *IEEE Transactions on Power Electronics* 22.5 (Sept. 2007), pp. 2061–2069. DOI: [10.1109/tpe1.2007.904183](https://doi.org/10.1109/tpe1.2007.904183).
- [78] B. Wang, N. Hu, L. Chen, Y. Yang, Z. Yin, and Y. Zhou. “Demagnetization Fault Detection in Surface-Mounted Permanent Magnet Synchronous Machine”. In: *2021 Global Reliability and Prognostics and Health Management (PHM-Nanjing)*. IEEE, Oct. 2021. DOI: [10.1109/phm-nanjing52125.2021.9612845](https://doi.org/10.1109/phm-nanjing52125.2021.9612845).

- [79] D. Casadei, F. Filippetti, M. Mengoni, Y. Gritli, G. Serra, A. Tani, and L. Zarri. “Detection of magnet demagnetization in five-phase surface-mounted permanent magnet generators”. In: *2012 3rd IEEE International Symposium on Power Electronics for Distributed Generation Systems (PEDG)*. IEEE, June 2012. DOI: [10.1109/pedg.2012.6254099](https://doi.org/10.1109/pedg.2012.6254099).
- [80] Z. Yang, X. Shi, and M. Krishnamurthy. “Vibration monitoring of PM synchronous machine with partial demagnetization and inter-turn short circuit faults”. In: *2014 IEEE Transportation Electrification Conference and Expo (ITEC)*. IEEE, June 2014. DOI: [10.1109/itec.2014.6861774](https://doi.org/10.1109/itec.2014.6861774).
- [81] Y. Da, X. Shi, and M. Krishnamurthy. “A New Approach to Fault Diagnostics for Permanent Magnet Synchronous Machines Using Electromagnetic Signature Analysis”. In: *IEEE Transactions on Power Electronics* 28.8 (Aug. 2013), pp. 4104–4112. DOI: [10.1109/tpel.2012.2227808](https://doi.org/10.1109/tpel.2012.2227808).
- [82] Y. Gritli, C. Rossi, D. Casadei, L. Zarri, and F. Filippetti. “Demagnetizations diagnosis for Permanent Magnet Synchronous Motors based on advanced Wavelet Analysis”. In: *2012 XXth International Conference on Electrical Machines*. IEEE, Sept. 2012, pp. 2397–2403. DOI: [10.1109/icelmach.2012.6350219](https://doi.org/10.1109/icelmach.2012.6350219).
- [83] J.-R. Ruiz, J. Rosero, A. Espinosa, and L. Romeral. “Detection of Demagnetization Faults in Permanent-Magnet Synchronous Motors Under Nonstationary Conditions”. In: *IEEE Transactions on Magnetics* 45.7 (July 2009), pp. 2961–2969. DOI: [10.1109/tmag.2009.2015942](https://doi.org/10.1109/tmag.2009.2015942).
- [84] Y. Fang, J. Ji, and W. Zhao. “Modeling of Fault-tolerant Flux-switching Permanent-magnet Machines for Predicting Magnetic and Armature Reaction Fields”. In: *CES Transactions on Electrical Machines and Systems* 6.4 (2022), pp. 413–421. DOI: [10.30941/CESTEMS.2022.00053](https://doi.org/10.30941/CESTEMS.2022.00053).

Publications

- [60] M. Ackermann, J.-L. Marqués, and C. Hillermeier. “Optimal multi-fault tolerant control for PMSMs – Part I: Feedforward control”. In: *2024 IEEE 21st International Power Electronics and Motion Control Conference (PEMC)*. IEEE, Sept. 2024, pp. 1–9. DOI: [10.1109/pemc61721.2024.10726337](https://doi.org/10.1109/pemc61721.2024.10726337).
- [61] M. Ackermann, J.-L. Marques, and C. Hillermeier. “Fast Analytical Calculation of the Magnetic Field in Permanent Magnet Synchronous Machines with Flux Barriers Including Saturation”. In: *PCIM Europe 2024; International Exhibition and Conference for Power Electronics, Intelligent Motion, Renewable Energy and Energy Management*. 2024, pp. 1331–1340. DOI: [10.30420/566262182](https://doi.org/10.30420/566262182).
- [66] M. Ackermann, J.-L. Marqués, and C. Hillermeier. “Optimal multi-fault tolerant control for PMSMs – Part II: Feedback control”. In: *2024 IEEE 21st International Power Electronics and Motion Control Conference (PEMC)*. IEEE, Sept. 2024, pp. 1–7. DOI: [10.1109/pemc61721.2024.10726410](https://doi.org/10.1109/pemc61721.2024.10726410).
- [69] M. Ackermann, J.-L. Marqués, and C. Hillermeier. “Diagnosis of partial demagnetization in a multi-fault tolerant control for PMSMs”. In: *2025 IEEE 19th International Conference on Compatibility, Power Electronics and Power Engineering (CPE-POWERENG)*. IEEE, May 2025, pp. 1–8. DOI: [10.1109/cpe-powereng63314.2025.11027187](https://doi.org/10.1109/cpe-powereng63314.2025.11027187).
- [72] M. Ackermann, J.-L. Marqués, and C. Hillermeier. “Adaptation of a multi-fault tolerant control for PMSMs to phase short circuits”. In: *IECON 2025 – 51st Annual Conference of the IEEE Industrial Electronics Society*. IEEE, Oct. 2025, pp. 1–8. DOI: [10.1109/iecon58223.2025.11221390](https://doi.org/10.1109/iecon58223.2025.11221390).
- [73] M. Ackermann, J.-L. Marqués, and C. Hillermeier. “Multi-Fault Tolerant Control for PMSMs Considering Interturn Short Circuits”. In: *2025 IEEE 34th International Symposium on Industrial Electronics (ISIE)*. IEEE, June 2025, pp. 1–7. DOI: [10.1109/isie62713.2025.11124800](https://doi.org/10.1109/isie62713.2025.11124800).

List of Symbols

A_z	Axial magnetic vector potential	$\frac{\text{T}}{\text{m}}$
α	Field weakening coefficient	—
B_r	Radial magnetic field	T
B_φ	Tangential magnetic field	T
β_{SL}	Angular width of the slots	rad
β_{FB}	Angular width of the flux barriers	rad
$e^{(c)}$	Back-EMF of coil (c)	V
e_s	Back-EMF of the interturn short circuited coil turns	V
ϵ_M	Pole-arc to pole-pitch ratio	—
$J^{(c)}$	Current density flowing through coil (c)	$\frac{\text{A}}{\text{m}^2}$
$\mathcal{L}$	Diagonal matrix containing the inductances of all coils	H
ℓ	Active rotor length	m
$\frac{\ell\pi}{\mu_0}P^{(c)}$	Torque capability of coil (c)	$\frac{\text{Nm}^3}{\text{A}}$
$\frac{\ell\pi}{\mu_0}Q$	Cogging torque	Nm
λ	Lagrangian multiplier	—
M_r	Radial magnetization (healthy)	$\frac{\text{A}}{\text{m}}$
$\tilde{M}_r$	Radial magnetization (considering demagnetization)	$\frac{\text{A}}{\text{m}}$
$a_k^{(M)}, c_k^{(M)}$	Fourier coefficients of the healthy magnetization	T
$\tilde{a}_k^{(M)}, \tilde{c}_k^{(M)}$	Fourier coef. of the magnetization considering demagnetization	T
n_{turn}	Number of winding turns of a coil	—
$\mathcal{J}_m$	Rotor's moment of inertia	kg m^2
ω_R	Rotational speed	$\frac{\text{rad}}{\text{s}}$
n_R	Rotational speed	$\frac{1}{\text{min}}$
p	Number of pole pairs	—
$\Phi_M^{(c)}$	PM flux linkage of coil (c)	Wb
$\Phi_{M,s}$	PM flux linkage of interturn short circuited coil turns	Wb
$\varphi_{SL}^{(a)}$	Angular position of slot (a)	rad
$\varphi_{FB}^{(b)}$	Angular position of flux barrier (b)	rad
r_R	Outer radius of the rotor	m
r_M	Outer radius of the magnets	m

r_S	Inner radius of the stator	m
r_{SL}	Outer radius of the slots	m
r_{FB}	Outer radius of the flux barriers	m
$\mathcal{R}$	Diagonal matrix containing the resistances of all coils	Ω
S_{SL}	Surface of a slot	m ²
T_{ps}	Faulty torque ripple caused by a phase short circuit	Nm
T_{2ps}	Faulty torque ripple caused by a phase-to-phase short circuit	Nm
θ_R	Angular rotor position	rad
x	Dynamic variable $x \in \{\theta_R, \omega_R, J\}$	$[x]$
x_{des}	Desired value of dynamic variable x	$[x]$
x_{mes}	Actual value of dynamic variable x	$[x]$
Δx	Deviation of the actual value to its desired value	$[x]$
$\Delta \Xi'$	Required (mechanical) correction computed by the feedback control	$\frac{\text{kg}^2 \text{m}^2}{\text{s}^4 \text{A}^2}$
$\tilde{y}$	Dynamic variable considering demagnetization $\tilde{y} \in \{\tilde{\Phi}_M, \tilde{e}, \tilde{Q}, \tilde{P}\}$	$[\tilde{y}]$
$\vec{z}$	Vector containing dynamic variables of all coils $\vec{z} \in \{\vec{\Phi}_M, \vec{e}, \vec{P}, \vec{J}\}$	$[z]$
$\vec{z}_h$	Vector containing solely the variables of the healthy coils from $\vec{z}$	$[z]$
$\mathcal{Z}_h$	Diagonal matrix containing solely the constants of the healthy coils from $\mathcal{Z}_h \in \{\mathcal{L}_h, \mathcal{R}_h\}$	$[\mathcal{Z}_h]$