

Improved Open-loop Control for Rapid Power Flow Redistribution in Meshed MVDC Grids in Response to Ground Faults

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Abstract—In the event of a ground fault in a medium-voltage direct current (MVDC) grid, it is desired to rapidly drive the faulty current to zero without using any DC breaker. In a meshed MVDC grid equipped with Modular Multilevel Converters (MMCs) and Power Flow Controllers (PFCs), this can be achieved by swiftly redistributing the power flows. Based on a previously developed method, this paper presents a greatly improved control technique that solves this task. Through balanced use of the control inputs, the power flows can be redistributed much faster while minimizing the energy fluctuations in the PFCs.

Index Terms—Trajectory design, Modelling, Modular Multi-level Converter, Power flow control, Stability, State-space model, Transmission of electrical energy.

I. INTRODUCTION

In recent years, MVDC systems have gained significant attention for enhancing power distribution and improving operational flexibility in distribution grids [1]. A key challenge in implementing such DC grids is effective fault-current management [2]. Fault ride-through capability for the entire power distribution system is highly desirable, particularly in the absence of DC breakers to manage faults. This capability can be achieved in the future by fully electronically controllable DC grids [3].

Previous work [4] investigates an MVDC grid equipped with MMCs and PFCs, introducing a centralized open-loop control to enable smooth controlled transitions between different steady states of the grid. By designing the trajectory of the main variables, i.e. the currents, and obtaining the driving voltages by differentiating these current trajectories, control of the power flows is achieved, controlling the energies of the MMCs and PFCs at the same time. However, when a ground fault occurs, the current in the faulty line must be rapidly reduced to zero, enforcing a substantial redistribution of the currents in the DC grid. This entails a substantial redistribution of the magnetic energy stored in the DC grid's line inductances as well as a redistribution of power flows. Since these power flows govern the energy dynamics of the PFCs (see their

equations of motion later), this rapid redistribution induces fluctuations in their stored energy. In severe ground fault cases, the resulting change in magnetic energy may exceed the energy stored in the PFCs, pushing them to their operational limits due to their limited energy capacity, which is significantly smaller than that of the MMCs.

To address this challenge, this paper extends the DC grid presented in [4] on the hardware side by incorporating additional PFCs. The main innovation, however, is an improved control approach that utilizes the degrees of freedom offered by the PFCs in a more balanced and therefore more effective way to manage their energies. Whereas with the previous control method, managing a severe ground fault was only possible by splitting the desired transition into many partial transitions (see [5]), the new method achieves this in a very short time, avoiding major energy fluctuations in the PFCs. In addition, the presented method also makes it possible to effectively control the active and reactive powers at the interfaces to the linked AC grids.

To keep the paper self-contained, the DC grid is described in Chapter II and the trajectory design task is presented in Section III-A, similar to [4]. Section III-B enriches the previous design method with the possibility to also control the AC active and reactive powers. The main innovation of this paper, the new control approach for the PFCs, is to be found in Section III-C, where the basic idea is presented, and in the Appendix, where a calculation is carried out as an example. The effectiveness of the proposed control strategy is validated through numerical simulation results shown in Chapter IV.

II. DESCRIPTION OF THE DC GRID

A. Structure and variables

1) *Model of MMC*: Fig. 1 illustrates an MMC model connecting a star-connected three-phase AC source (grid frequency of $f_{AC} = \omega_{AC}/2\pi$) to a DC grid. The symmetric AC voltages and currents are represented using space vectors $\vec{u}_{AC}^T = (u_{AC,\alpha}, u_{AC,\beta})$ and $\vec{i}_{AC}^T = (i_{AC,\alpha}, i_{AC,\beta})$ (the

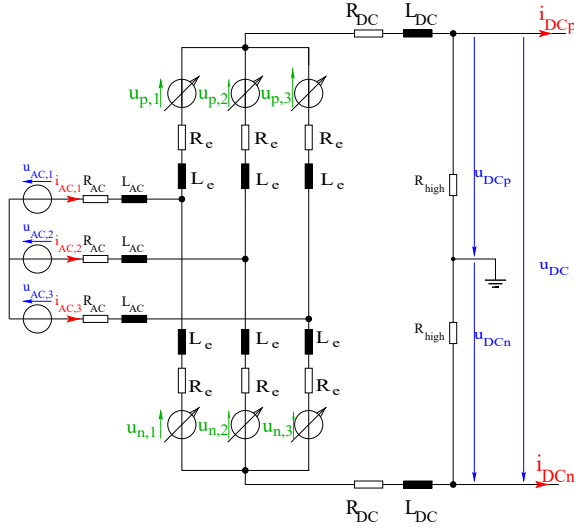


Fig. 1. Structure of MMC

components $u_{AC,0}$ and $i_{AC,0}$ are equal to zero). The DC voltages with respect to the ground at the positive and negative terminals of the MMC are correspondingly denoted by u_{DCp} and u_{DCn} . At a relatively long time scale of $\Delta t = 100\mu s$, the full-bridge submodules in each MMC arm can be approximated as a collective controllable voltage source, denoted as $u_{p/n,1/2/3}$ [6].

This study focuses on the total energy stored within all submodules, denoted as $W_{\Sigma,0}$, since the power exchange between the AC and DC grids determines its dynamics. Internal energy balancing within the MMC arms, which depends on variables such as common-mode voltage and circular currents, is outside the scope of this paper and is not addressed [7].

2) *DC grid configuration*: Figure 2 depicts the DC grid from [4], used as the example scenario in this study. It connects to three AC networks via three MMCs, labeled MMC1, MMC2, and MMC3. The positive DC terminals of these MMCs form the p-ring, while the negative terminals create the n-ring.

Each DC line is modeled with resistance R and inductance L , neglecting capacitance. The p-ring and n-ring are star-connected at their centers, ensuring redundant power flow paths. These, along with the central lines, form the internal rings, where all DC currents are termed internal currents.

MMC1 and MMC2 operate as rectifiers, supplying power to the DC grid, while MMC3 functions as an inverter, transferring power to an external AC system. Variables related to an MMC are denoted by the superscript (a) (with $a = 1, 2, 3$). At each MMC terminal, two internal currents are denoted by $i_{cw}^{(a)}$ and $i_{ccw}^{(a)}$, where cw and ccw stand for the clockwise and counterclockwise directions, respectively. The subscript p/n specifies whether the variable is associated with the p-ring or n-ring.

Unlike in [4], this work incorporates three PFCs per ring to enhance the controllability of internal power flows within the DC grid. In this study, the PFCs are symmetrically placed at

the midpoints of the transmission lines connecting the MMC terminals, ensuring that all internal lines are connected to a PFC. Further insights into the PFC placement within the DC grid are provided in [8].

3) *Model of PFC*: The PFC depicted in Fig. 3 comprises a single capacitor connected to the grid lines through six switches arranged in full-bridge configuration. The voltage across the capacitor is denoted as u_C , with the small internal resistance being neglected. Each PFC provides two independent line-to-line voltages, $u_{F\alpha}$ and $u_{F\beta}$, which act as control inputs to control the internal current and PFC energy dynamics. The switching frequency is assumed to be sufficiently high, allowing $u_{F\alpha}$ and $u_{F\beta}$ to be treated as real-valued input signals for the time scale of $\Delta t = 100\mu s$.

In Fig. 2, within the p-ring, the PFC located between MMC1 and MMC2 is labeled "PFC1 in the p-ring". Similarly, we have PFC2&3 in the p-ring. The driving voltages provided by PFC1, PFC2, and PFC3 in the p-ring are denoted by $u_{F\alpha p}^{(a=1/2/3)}$ and $u_{F\beta p}^{(a=1/2/3)}$. The same notation is used for the PFCs in the n-ring, with the subscript p replaced by n .

B. Dynamics

1) *Defining the DC grid voltage level*: In Fig. 2, considering the three DC voltages $u_{DCp}^{(a=1,2,3)}$ at the positive terminals of the MMCs, one of them can be freely selected by the grid operator, while the other two are determined by the voltage loop connecting the consecutive MMC positive terminals within the p-ring. Here, $u_{DCp}^{(a=1)}$ is chosen as the independent variable. The same consideration applies to the n-ring, where $u_{DCn}^{(a=1)}$ is selected as the independent variable.

2) *Current dynamics of the MMCs*: The equations of motion for the AC and DC currents at the MMCs ($a = 1, 2, 3$) are given by [9]:

$$\frac{d}{dt} i_{AC,\alpha/\beta}^{(a)} = -\frac{R_{AC}^{(a)}}{L_{AC}^{(a)}} i_{AC,\alpha/\beta}^{(a)} - \frac{1}{L_{AC}^{(a)}} \left(\frac{u_{\Delta,\alpha/\beta}^{(a)}}{2} - u_{AC,\alpha/\beta}^{(a)} \right), \quad (1)$$

$$\frac{d}{dt} i_{DCp}^{(a)} = -\frac{R_{DC}^{(a)}}{L_{DC}^{(a)}} i_{DCp}^{(a)} - \frac{1}{L_{DC}^{(a)}} \left(u_{\Sigma,0}^{(a)} + \frac{u_{DCp}^{(a)} + u_{DCn}^{(a)}}{2} \right), \quad (2)$$

where $R_{DC}^{(a)} = R_{DC}^{(a)} + R_e^{(a)}/3$, and $L_{DC}^{(a)} = L_{DC}^{(a)} + L_e^{(a)}/3$, $R_{AC}^{(a)} = R_{AC}^{(a)} + R_e^{(a)}/2$ and $L_{AC}^{(a)} = L_{AC}^{(a)} + L_e^{(a)}/2$. Here, $u_{AC,\alpha/\beta}^{(a)}$ are the external AC voltage sources, which are given as $u_{AC,\alpha/\beta}^{(a)} = \hat{u}_{AC}^{(a)} \begin{pmatrix} \sin(\omega_{AC}t) \\ -\cos(\omega_{AC}t) \end{pmatrix}$, with $a = 1, 2, 3$. The driving voltages of these DC and AC currents are:

$$\begin{pmatrix} u_{\Delta,\alpha}^{(a)} \\ u_{\Delta,\beta}^{(a)} \\ u_{\Sigma,0}^{(a)} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \begin{pmatrix} u_{p,1}^{(a)} \\ u_{p,2}^{(a)} \\ u_{p,3}^{(a)} \\ u_{n,1}^{(a)} \\ u_{n,2}^{(a)} \\ u_{n,3}^{(a)} \end{pmatrix}.$$

It should be noted that the DC current of MMC3 is a dependent variable: $i_{DCp}^{(3)} = -(i_{DCp}^{(1)} + i_{DCp}^{(2)})$, as the entire DC grid can

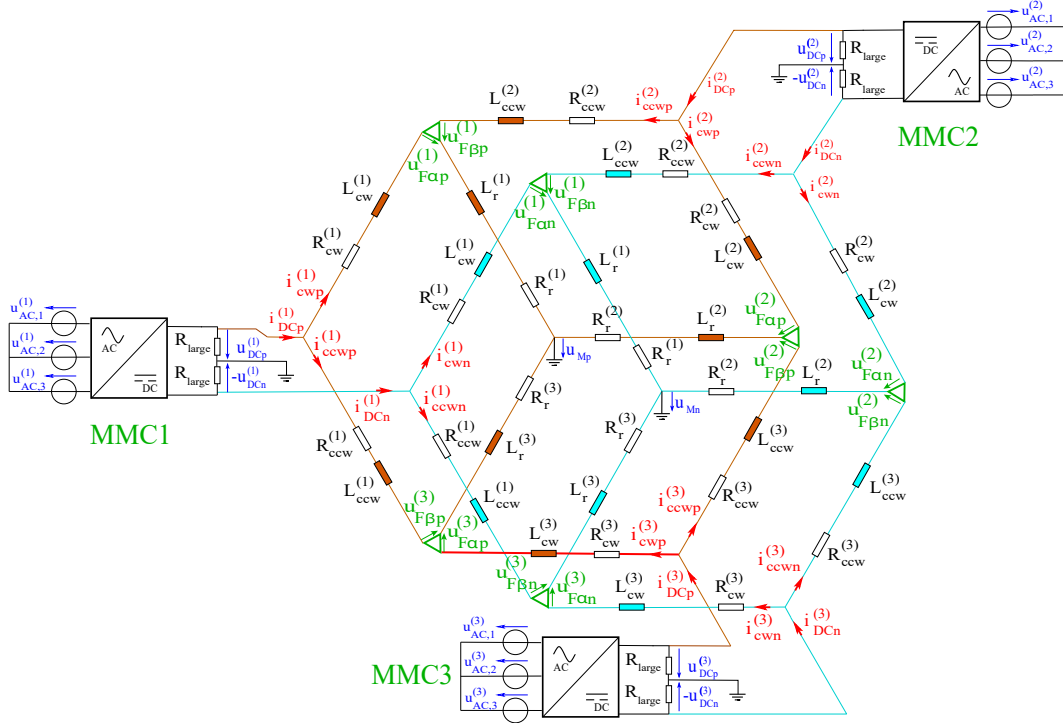


Fig. 2. Example DC grid configuration. The faulty transmission line discussed in Section IV is highlighted in red.

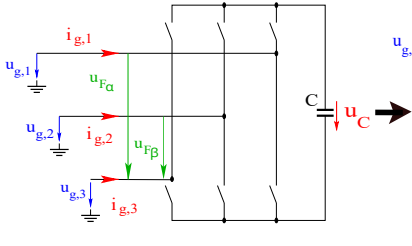


Fig. 3. Structure of PFC

be treated as a single node. Consequently, the control input $u_{\Sigma,0}^{(a=3)}$ is also dependent on $u_{\Sigma,0}^{(a=1,2)}$.

The active and reactive powers of the AC grid at each MMC with index $a = 1, 2, 3$ are formulated as:

$$\begin{aligned} P_{AC}^{(a)} &= 1.5 \left(u_{AC,\alpha}^{(a)} i_{AC,\alpha}^{(a)} + u_{AC,\beta}^{(a)} i_{AC,\beta}^{(a)} \right), \\ Q_{AC}^{(a)} &= 1.5 \left(u_{AC,\alpha}^{(a)} i_{AC,\beta}^{(a)} - u_{AC,\beta}^{(a)} i_{AC,\alpha}^{(a)} \right). \end{aligned} \quad (3)$$

C. Current dynamics of the internal DC grids

The mesh equations governing the internal closed loops within the p-ring can be expressed as follows:

$$\begin{aligned} \mathbf{L}_{rcw} \frac{d}{dt} \vec{i}_{cwp} &= \mathbf{L}_{re0} \frac{d}{dt} \vec{i}_{DCp} - \mathbf{R}_{rcw} \vec{i}_{cwp} \\ &+ \mathbf{R}_{re0} \vec{i}_{DCp} + \begin{pmatrix} -u_{F\alpha p}^{(1)} + u_{F\beta p}^{(3)} \\ -u_{F\alpha p}^{(2)} + u_{F\beta p}^{(1)} \\ -u_{F\alpha p}^{(3)} + u_{F\beta p}^{(2)} \end{pmatrix}, \end{aligned} \quad (4)$$

where $\vec{i}_{DCp}^T = (i_{DCp}^{(1)}, i_{DCp}^{(2)}, i_{DCp}^{(3)})$ and $\vec{i}_{cwp}^T = (i_{cwp}^{(1)}, i_{cwp}^{(2)}, i_{cwp}^{(3)})$. The matrices $\mathbf{L}_{re0}$, $\mathbf{L}_{rcw}$ are:

$$\begin{aligned} \mathbf{L}_{re0} &= \begin{pmatrix} L_{ccw}^{(1)} + L_r^{(3)} & -L_r^{(1)} & 0 \\ 0 & L_{ccw}^{(2)} + L_r^{(1)} & -L_r^{(2)} \\ -L_r^{(3)} & 0 & L_{ccw}^{(3)} + L_r^{(2)} \end{pmatrix}; \\ \mathbf{L}_{rcw} &= \begin{pmatrix} L_{cw}^{(1)} + L_{ccw}^{(1)} & 0 & 0 \\ 0 & L_{cw}^{(2)} + L_{ccw}^{(2)} & 0 \\ 0 & 0 & L_{cw}^{(3)} + L_{ccw}^{(3)} \end{pmatrix} \\ &+ \begin{pmatrix} L_r^{(1)} + L_r^{(3)} & -L_r^{(1)} & -L_r^{(3)} \\ -L_r^{(1)} & L_r^{(2)} + L_r^{(1)} & -L_r^{(2)} \\ -L_r^{(3)} & -L_r^{(2)} & L_r^{(3)} + L_r^{(2)} \end{pmatrix}. \end{aligned} \quad (5)$$

The matrices $\mathbf{R}_{rcw}$, $\mathbf{R}_{re0}$ are derived by substituting the inductances in $\mathbf{L}_{rcw}$, $\mathbf{L}_{re0}$ with the corresponding resistances.

D. Energy dynamics of the MMCs and the PFCs

For simplicity, all ohmic losses are neglected in the following equations, though they could be easily included if required.

Since the power flows across the MMCs determine their energy dynamics, it is crucial to consider the magnetic energy stored in the adjacent line inductances. Hence, a component energy, denoted by $W'_{\Sigma,0}^{(a)}$, is defined. This energy component encompasses the total energy stored in the capacitances of the MMC, $W_{\Sigma,0}^{(a)}$, and the magnetic energies in all adjacent lines and is written as:

$$W'_{\Sigma,0}^{(a)} = W_{\Sigma,0}^{(a)} + \frac{L'_{DC}}{6} \left(i_{DCp}^{(a)} \right)^2 + \frac{L'_{AC}}{8} \left(\vec{i}_{AC}^{(a)} \right)^T \vec{i}_{AC}^{(a)}. \quad (6)$$

The equations of motion for the component energies corresponding to three MMCs with index ($a = 1, 2, 3$) are:

$$\frac{dW'_{\Sigma,0}^{(a)}}{dt} = \underbrace{\frac{P_{AC}^{(a)}}{6}, \text{ AC active power}}_{\frac{1}{4} (\vec{u}_{AC}^{(a)})^T \vec{i}_{AC}^{(a)}} - \underbrace{\frac{i_{DCp}^{(a)}}{6} (u_{DCp}^{(a)} + u_{DCn}^{(a)})}_{\text{DC power}}. \quad (7)$$

Similarly, the energy stored in the PFCs' capacitances in the p-ring, given by $W_{Cp}^{(a)} = \frac{C}{2} (u_{Cp}^{(a)})^2$ for $a = 1, 2, 3$, along with the magnetic energy in three adjacent lines, forms a new energy component, denoted as $W_{Cp}^{\prime(a)}$. For example, the component energy for the PFC1 in the p-ring is expressed as:

$$W_{Cp}^{\prime(a=1)} = W_{Cp}^{(a=1)} + \frac{L_{cw}^{(1)}}{2} (i_{cw}^{(1)})^2 + \frac{L_{ccw}^{(2)}}{2} (i_{DCp}^{(2)} - i_{cw}^{(2)})^2 + \frac{L_r^{(1)}}{2} (i_{cw}^{(1)} + i_{DCp}^{(2)} - i_{cw}^{(2)})^2. \quad (8)$$

The power flows through PFC1 in the p-ring govern its energy dynamics, as described by:

$$\frac{dW_{Cp}^{\prime(a=1)}}{dt} = +u_{DCp}^{(1)} i_{cw}^{(1)} + u_{DCp}^{(2)} (i_{DCp}^{(2)} - i_{cw}^{(2)}) - u_{Mp} (i_{cw}^{(1)} + i_{DCp}^{(2)} - i_{cw}^{(2)}). \quad (9)$$

where u_{Mp} is the voltage at the central point of the p-ring. The component energy and the dynamic equation for other PFCs in the p- and n-ring are analogous.

E. Summary of the state-space model

The dynamics of the whole system consist of all independent AC currents, DC currents, and the extended energies of 3 MMCs and 6 PFCs. The state variables are: $i_{DCp}^{(a=1,2)}$, $i_{cw/n}^{(a=1,2,3)}$, $W_{\Sigma,0}^{\prime(a=1,2,3)}$, $W_{Cp/n}^{\prime(a=1,2,3)}$, $i_{AC,\alpha}^{(a=1,2,3)}$, $i_{AC,\beta}^{(a=1,2,3)}$ (altogether $2+6+3+6+3+3=23$ variables). These variables are driven by the control inputs: $u_{\Sigma,0}^{(a=1,2)}$, $u_{\Delta,\alpha}^{(a=1,2,3)}$, $u_{\Delta,\beta}^{(a=1,2,3)}$, $u_{F\alpha p/n}^{(a=1,2,3)}$, $u_{F\beta p/n}^{(a=1,2,3)}$ (altogether $2+3+3+6+6=20$ inputs).

III. THE OPEN-LOOP CONTROL

A. Task of designing trajectories

The open-loop control scheme ensures a smooth trajectory for all state variables, facilitating a rapid transition from the initial steady state to the desired new steady state. The transition interval is defined as $[t_0, t_0 + T_s]$, where t_0 is an arbitrary starting time and T_s is the transition time, which can be freely chosen.

The conditions for open-loop control and steady-state operation in a DC grid interconnected with multiple AC grids have been outlined in [4]. In particular, to guarantee a smooth transition between steady states and avoid any transient behavior after the transition, the following conditions for open-loop control must be satisfied:

- α) Each current trajectory in the system is driven so that its value at time $t_0 + T_s$ matches the value specified by steady state 2.
- β) The power flows during the transition interval are designed to drive the energy of the MMCs and PFCs to their desired values in steady state 2.

- γ) At time $t_0 + T_s$, all control inputs driving the currents must match the values required by steady state 2. Since equations (1), (2) and (4) relate the control inputs to the currents and their time derivatives of the currents, it follows that at $t_0 + T_s$, not only must the current trajectories reach the values specified by steady state 2, but their derivatives must also reach the required values in steady state 2.

The following subsections discuss how these three conditions can be met and how the control inputs can be derived from the desired trajectories.

B. Method for using orthogonal basis functions to control MMCs and AC grid powers

For a general state variable x , its trajectory during the transition between two steady states is described by a time-dependent function: $x(t)$, $\forall t \in [t_0, t_0 + T_s]$. The value of x in the initial steady state is denoted by $x^{(ss1)}$, while the value corresponding to the final steady state is indicated by $x^{(ss2)}$.

The DC grid should effectively support and coordinate with the AC grids without causing disturbances. It is therefore assumed that the grid operator can freely determine the active and reactive powers at the MMC1 and MMC2 without having to worry about the energy regulation of the MMCs. For each index $a = 1, 2$, the desired value of the AC active power, $P_{AC}^{(a,ss2)}$, is achieved through a smooth S-curved transition, starting from the initial value $P_{AC}^{(a,ss1)}$:

$$P_{AC}^{(a)}(t) = P_{AC}^{(a,ss1)} \tilde{s}(t) + P_{AC}^{(a,ss2)} (1 - \tilde{s}(t)),$$

where $\tilde{s}(t) = \frac{1}{2} \left(1 + \cos \left(\frac{\pi}{T_s} (t - t_0) \right) \right)^1$. Therefore, energy control for MMC1 and MMC2 must be ensured via their DC sides. However, only one input, $u_{\Sigma,0}^{(a)}$, is available, whereas two variables have to be controlled: the DC current $i_{DCp}^{(a)}$ and the energy $W_{\Sigma,0}^{\prime(a)}$, with $a = 1, 2$. To address this problem, the DC current trajectories at MMC1 and MMC2 during the transition interval $[t_0, t_0 + T_s]$ are designed to consist of two components: a S-curved smooth component and a hump-shaped component (see Fig. 4):

$$\begin{aligned} &:= i_{DCp}^{(a=1,g)}(t), \text{ smooth transition part} \\ i_{DCp}^{(a=1)}(t) &= \underbrace{i_{DCp}^{(a=1,ss1)} \tilde{s}(t) + i_{DCp}^{(a=1,ss2)} (1 - \tilde{s}(t))}_{\text{1st hump-shaped component}} \\ &+ \underbrace{\tilde{i} A^{(a=1)} \tilde{\Phi}_1(t)}_{\text{smooth transition part}}, \\ i_{DCp}^{(a=2)}(t) &= \underbrace{i_{DCp}^{(a=2,g)}(t)}_{\text{smooth transition part}} + \underbrace{\tilde{i} A^{(a=2)} \tilde{\Phi}_2(t)}_{\text{2nd hump-shaped component}}, \end{aligned} \quad (10)$$

where the smooth components are denoted by an additional superscript (g) . For each index $a = 1, 2$, the hump-shaped term in (10) includes a scaling factor $\tilde{i} = 1kA$, an initially unknown coefficient $A^{(a)}$ and a predefined basis function, $\tilde{\Phi}_a(t)$. With $\frac{d}{dt} \tilde{s}(t) = -\frac{\pi}{2T_s} \sin \left(\frac{\pi}{T_s} (t - t_0) \right)$, implying that

¹with $\tilde{s}(t_0) = 1$ and $\tilde{s}(t_0 + T_s) = 0$, implicating that $P_{AC}^{(a)}(t_0) = P_{AC}^{(a,ss1)}$ and $P_{AC}^{(a)}(t_0 + T_s) = P_{AC}^{(a,ss2)}$.

$\frac{d}{dt}\tilde{s}(t_0) = \frac{d}{dt}\tilde{s}(t_0+T_s) = 0$, the time derivative of the smooth component, i.e. $\frac{d}{dt}i_{DCp}^{(a,g)}(t) = \left(i_{DCp}^{(a,ss1)} - i_{DCp}^{(a,ss2)}\right) \frac{d}{dt}\tilde{s}(t)$, is zero at t_0 and t_0+T_s . By designing the basis function, $\tilde{\Phi}_a(t)$, with zero values and vanishing time derivatives at t_0 and t_0+T_s , the current trajectory $i_{DCp}^{(a)}(t)$ satisfies conditions α) and γ). The hump-shaped component introduces a new degree of freedom: The coefficient $A^{(a)}$ must be chosen so that condition β) is satisfied for the respective MMC.

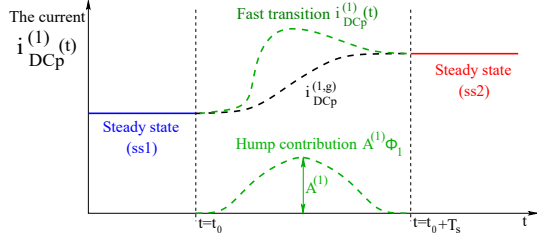


Fig. 4. The current trajectory $i_{DCp}^{(a=1)}(t)$.

The properties of the basis functions $\tilde{\Phi}_{1/2}(t)$ were detailed in [4]. Specifically, their orthogonality is defined as $\int_{t_0}^{t_0+T_s} \tilde{\Phi}_i(t) \tilde{\Phi}_j(t) dt = 0, \forall i \neq j$, which remains a crucial feature also in the improved control approach, as demonstrated in the Appendix.

The situation is different for MMC3, as the DC current $i_{DCp}^{(a=3)}$ is dependent on the other two currents $i_{DCp}^{(a=1,2)}$ and cannot be used for energy control. Consequently, energy control for MMC3 is managed through its AC active power, $P_{AC}^{(a=3)}$. The trajectory for the AC active power at the MMC3 during the transition can be designed as follows:

$$P_{AC}^{(a=3)}(t) = \underbrace{P_{AC}^{(a=3,g)}(t)}_{\text{smooth transition part}} + \underbrace{\tilde{p} C^{(a=3)} \tilde{\Phi}_1(t)}_{\text{3rd hump-shaped component}}, \quad (11)$$

where $\tilde{p} = 2MW$ is a scaling factor. Similar to equation (10), the smooth component $P_{AC}^{(a=3,g)}$ connects the values of the two steady states, while the hump-shaped component in (11) creates an additional degree of freedom, namely the correct choice of the coefficient $C^{(a=3)}$, to fulfill condition β) for MMC3.

Furthermore, it should be noted that in a general steady state, the value of $P_{AC}^{(a=3,ss)}$ is dependent on $P_{AC}^{(a=1/2,ss)}$ to maintain the power balance of the entire transmission system.

Since the AC reactive powers at 3 MMCs, $Q_{AC}^{(a=1,2,3)}$, do not contribute to the energy dynamics, their trajectories during the transition can be given as simple smooth functions, with the index $a = 1, 2, 3$, as follows:

$$Q_{AC}^{(a)}(t) = Q_{AC}^{(a,ss1)} \tilde{s}(t) + Q_{AC}^{(a,ss2)} (1 - \tilde{s}(t)).$$

C. Enhanced method of employing the orthogonal basis functions for the PFCs

In [4], the control strategy used for the MMCs was also applied to the PFCs. Hump-shaped components were incorporated into the internal DC current trajectories to manage the energies of the PFCs. The control inputs provided by the

PFCs were primarily responsible for realizing such current trajectories. This earlier control approach was tailored to the case where a greater number of variables had to be controlled with a limited set of control inputs: In each ring of the DC grid described in [4], only four control inputs provided by two PFCs had to regulate five variables: three independent internal currents corresponding to three internal closed loops and the energies of two PFCs. However, this earlier control approach has disadvantages, particularly when the highest currents in the DC grid must be driven to zero in the event of a ground fault. In such cases, the fluctuations in the PFCs' energy become substantial, requiring longer transition times, even when additional PFCs are incorporated into the DC grid, as demonstrated in [5].

An alternative control approach is developed in the following. In the DC grid configuration presented in this paper, each internal ring is equipped with three PFCs, yielding six control inputs. These six inputs are employed to control six variables: three independent internal currents and the energies of the three PFCs in each ring. Since the number of control inputs equals the number of variables to be controlled, this suggests an alternative control strategy in which the energies of the PFCs are directly managed through their driving voltages, rather than indirectly via the design of current trajectories.

Due to the symmetry of the p- and n-rings, the new control approach is described only for the p-ring for simplicity. Without loss of generality, three independent internal DC currents in the p-ring, controlled by the PFCs, are selected as $i_{cwp}^{(a=1/2/3)}$. The trajectories of these currents are no longer responsible for the energy control of the PFCs. Instead, they are designed to be simple smooth transitions, satisfying condition α) on $i_{cwp}^{(a=1/2/3)}$:

$$\vec{i}_{cwp}(t) = \vec{i}_{cwp}^{(ss1)} \tilde{s}(t) + \vec{i}_{cwp}^{(ss2)} (1 - \tilde{s}(t)). \quad (12)$$

In equation (4), the dynamics of the three internal currents, $i_{cwp}^{(a=1/2/3)}$, are governed by the differences of three pairs of the six available control inputs provided by the PFCs in the p-ring, specifically $(-u_{F\alpha p}^{(1)} + u_{F\beta p}^{(3)})$, $(-u_{F\alpha p}^{(2)} + u_{F\beta p}^{(1)})$ and $(-u_{F\alpha p}^{(3)} + u_{F\beta p}^{(2)})$. To satisfy condition β) imposed on the PFCs, the trajectories for three sums of the corresponding control inputs provided by these PFCs are designed to consist of a smooth transition and a hump contribution:

$$\begin{pmatrix} u_{F\alpha p}^{(1)}(t) + u_{F\beta p}^{(3)}(t) \\ u_{F\alpha p}^{(2)}(t) + u_{F\beta p}^{(1)}(t) \\ u_{F\alpha p}^{(3)}(t) + u_{F\beta p}^{(2)}(t) \end{pmatrix} = \vec{U}_{Fp}^{(g)} + \tilde{u} \underbrace{\begin{pmatrix} X_p^{(a=1)} \\ X_p^{(a=2)} \\ X_p^{(a=3)} \end{pmatrix}}_{:= \vec{X}_p} \tilde{\Phi}_1(t), \quad (13)$$

where $\tilde{u} = 2kV$ is a scaling factor, $X_p^{(a=1,2,3)}$ are three unknown coefficients. Moreover, the vector $\vec{U}_{Fp}^{(g)}$ contains three entries, representing the smooth transition parts connecting two values of the sums $(u_{F\alpha p}^{(1)} + u_{F\beta p}^{(3)})$, $(u_{F\alpha p}^{(2)} + u_{F\beta p}^{(1)})$ and $(u_{F\alpha p}^{(3)} + u_{F\beta p}^{(2)})$, which are required at time t_0 and t_0+T_s .

Since both the sums and differences of these pairs of the PFCs' control inputs are explicitly designed to precisely meet the values required by the desired steady state, condition γ) imposed on the PFCs is inherently satisfied.

In this way, all six control inputs provided by the PFCs contribute to managing the internal currents and the energies of the PFCs. This balanced input utilization ensures a more equitable distribution of power flows within the internal DC grid and minimizes energy fluctuations in the PFCs caused by the rapid redistribution of power flows during transitions.

For the PFCs in the n-ring, it is entirely analogous. The trajectories for the sums of the corresponding control inputs during the transition are composed of a smooth component $\vec{U}_{Fn}^{(g)}$ and a hump-shaped component with the unknown coefficients $\vec{X}_n^T = (X_n^{(a=1)}, X_n^{(a=2)}, X_n^{(a=3)})$.

D. Determining the unknown coefficients of the hump-shaped components

The calculation of the unknown coefficients follows a procedure analogous to that outlined in [4], with the sole distinctions being the vector of unknown coefficients and the inclusion of additional energy equations resulting from the increased number of PFCs in the DC grid (see later). This process comprises the following two tasks:

- Task 1: Since the external voltages, $u_{DCp/n}^{(a=1)}$, and the AC active power trajectory, $P_{AC}^{(a=1)}(t)$, corresponding to MMC1 during the transition are given by the grid operator, the coefficient $A^{(1)}$ of the hump-shaped component in $i_{DCp}^{(a=1)}(t)$ can be determined independently by applying condition β) to the MMC1.
- Task 2: Once the DC current trajectory $i_{DCp}^{(a=1)}(t)$ has been determined, applying condition β) to MMC2, MMC3, and six PFCs in the p-and n-rings yields a linear equation system for the unknown coefficients. The linear character of the algebraic equation system is ensured by the orthogonality of the basis functions, an example of which is discussed in detail in the Appendix. The linear equation system has the following form:

$$\mathbf{D}_{8 \times 8} \vec{x} = \vec{b}_{8 \times 1} \Rightarrow \vec{x} = \mathbf{D}_{8 \times 8}^{-1} \vec{b}_{8 \times 1}, \quad (14)$$

where $\vec{x}^T = (A^{(a=2)}, X_p^{(a=1/2/3)}, X_n^{(a=1/2/3)}, C^{(a=3)})$ contains all 8 unknown coefficients of the hump-shaped components. The matrix $\mathbf{D}_{8 \times 8}$ relates to the energy of the converters as follows:

$$\begin{array}{lcl} \text{MMC2} & \leftarrow & (d_{1,1} \mid d_{1,2} \mid d_{1,3} \mid d_{1,4} \mid d_{1,5} \mid d_{1,6} \mid d_{1,7} \mid 0) \\ \text{MMC3} & \leftarrow & (d_{2,1} \mid d_{2,2} \mid d_{2,3} \mid d_{2,4} \mid d_{2,5} \mid d_{2,6} \mid d_{2,7} \mid d_{2,8}) \\ \text{PFC1, p-ring} & \leftarrow & (d_{3,1} \mid d_{3,2} \mid d_{3,3} \mid d_{3,4} \mid 0 \mid 0 \mid 0 \mid 0) \\ \text{PFC2, p-ring} & \leftarrow & (d_{4,1} \mid d_{4,2} \mid d_{4,3} \mid d_{4,4} \mid 0 \mid 0 \mid 0 \mid 0) \\ \text{PFC3, p-ring} & \leftarrow & (d_{5,1} \mid d_{5,2} \mid d_{5,3} \mid d_{5,4} \mid 0 \mid 0 \mid 0 \mid 0) \\ \text{PFC1, n-ring} & \leftarrow & (d_{6,1} \mid 0 \mid 0 \mid 0 \mid d_{6,5} \mid d_{6,6} \mid d_{6,7} \mid 0) \\ \text{PFC2, n-ring} & \leftarrow & (d_{7,1} \mid 0 \mid 0 \mid 0 \mid d_{7,5} \mid d_{7,6} \mid d_{7,7} \mid 0) \\ \text{PFC3, n-ring} & \leftarrow & (d_{8,1} \mid 0 \mid 0 \mid 0 \mid d_{8,5} \mid d_{8,6} \mid d_{8,7} \mid 0) \end{array}$$

Solving this linear equation system requires minimal computational effort, making the process of determining the trajectories very fast.

E. Determining the control inputs driving the calculated trajectories

Once all unknown coefficients have been calculated, the corresponding inputs - specifically, the driving voltages provided by the MMCs and PFCs to implement the transition - must be determined.

For the MMCs, the control inputs for the DC currents, i.e. $u_{\Sigma,0}^{(a=1/2/3)}$, can be derived from the current dynamics in (2). Given the external AC voltages, $u_{AC,\alpha/\beta}^{(a)}(t)$, and the known trajectories for the active and reactive powers, $P_{AC}^{(a)}(t)$ and $Q_{AC}^{(a)}(t)$, the AC current trajectories, $i_{AC,\alpha/\beta}^{(a)}(t)$, can be derived from (3) (with $a = 1, 2, 3$):

$$\begin{pmatrix} i_{AC,\alpha}^{(a)}(t) \\ i_{AC,\beta}^{(a)}(t) \end{pmatrix} = \frac{2}{3} \frac{1}{(\hat{u}_{AC})^2} \begin{pmatrix} u_{AC,\alpha}^{(a)}(t) & -u_{AC,\beta}^{(a)}(t) \\ u_{AC,\beta}^{(a)}(t) & u_{AC,\alpha}^{(a)}(t) \end{pmatrix} \begin{pmatrix} P_{AC}^{(a)}(t) \\ Q_{AC}^{(a)}(t) \end{pmatrix}.$$

Now, the control inputs for the AC currents, $u_{\Delta,\alpha/\beta}^{(a)}$, can be derived from the dynamic equations of the AC currents in (1).

For the PFCs in the p-ring, the trajectories for the sums of the corresponding control inputs are determined directly, as the coefficients $X_p^{(a=1,2,3)}$ are calculated. The trajectories for the differences of these control inputs are derived from the internal current dynamics (4). This can be summarized as follows:

$$\begin{pmatrix} -u_{F\alpha p}^{(1)}(t) + u_{F\beta p}^{(3)}(t) \\ -u_{F\alpha p}^{(2)}(t) + u_{F\beta p}^{(1)}(t) \\ -u_{F\alpha p}^{(3)}(t) + u_{F\beta p}^{(2)}(t) \\ u_{F\alpha p}^{(1)}(t) + u_{F\beta p}^{(3)}(t) \\ u_{F\alpha p}^{(2)}(t) + u_{F\beta p}^{(1)}(t) \\ u_{F\alpha p}^{(3)}(t) + u_{F\beta p}^{(2)}(t) \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & +1 \\ 0 & +1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & -1 & 0 \\ +1 & 0 & 0 & 0 & 0 & +1 \\ 0 & +1 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & +1 & 0 \end{pmatrix} \vec{u}_{Fp}$$

$$= \left(\mathbf{L}_{rcw} \frac{d}{dt} \vec{i}_{cwp}(t) - \mathbf{L}_{re0} \frac{d}{dt} \vec{i}_{DCp}(t) + \mathbf{R}_{rcw} \vec{i}_{cwp}(t) - \mathbf{R}_{re0} \vec{i}_{DCp}(t) \right) \frac{1}{\vec{U}_{Fp}^{(g)} + \vec{u}_{Fp} \vec{\Phi}_1(t)} \quad (15)$$

where $\vec{u}_{Fp}^T = (u_{F\alpha p}^{(1)}, u_{F\beta p}^{(1)}, u_{F\alpha p}^{(2)}, u_{F\beta p}^{(2)}, u_{F\alpha p}^{(3)}, u_{F\beta p}^{(3)})$. The 6×1 matrix at the far right of (15) comprises six entries, each corresponding to the computed trajectory of a respective variable. From (15), the rule of deriving the control inputs provided by the PFCs in the p-ring reads:

$$\vec{u}_{Fp}(t) = M_1 \left[\mathbf{L}_{rcw} \frac{d}{dt} \vec{i}_{cwp}(t) - \mathbf{L}_{re0} \frac{d}{dt} \vec{i}_{DCp}(t) + \mathbf{R}_{rcw} \vec{i}_{cwp}(t) - \mathbf{R}_{re0} \vec{i}_{DCp}(t) \right] + M_2 \left(\vec{U}_{Fp}^{(g)} + \vec{u}_{Fp} \vec{\Phi}_1(t) \right), \quad (16)$$

where $M_1 = \frac{1}{2} \begin{pmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \\ 0 & 0 & -1 \\ +1 & 0 & 0 \end{pmatrix}$ and $M_2 = \frac{1}{2} \begin{pmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \\ 0 & 0 & -1 \\ +1 & 0 & 0 \end{pmatrix}$. The derivation of the control inputs provided by the PFCs in the n-ring is similar.

IV. RESULTS

To effectively illustrate the advancements of the enhanced control method, this paper utilizes the most challenging ground fault scenario presented in [5], using the same simulation parameters. In this scenario, one of the highest internal currents, i.e. $i_{cwp}^{(3)}$, must be quickly reduced to zero. The simulation

framework assumes that the AC power injected into MMC1 and MMC2 remains constant, while only the active power at MMC3 is regulated to maintain the overall power balance of the system.

TABLE I
SIMULATION PARAMETERS

Parameter	Value	Parameter	Value
L_{DC}	5 mH	$\hat{u}_{AC}^{(1/2/3)}$	[9; 10; 11] kV
R_{DC}	50 mΩ	$\hat{i}_{AC}^{(1/2/3)}$	[2; 1.6; -2.31] kA
L_{AC}	10 mH	f_{AC}	50 Hz
R_{AC}	100 mΩ	$\cos(\varphi_{i,AC}^{(1/2/3)})$	[0.61; 0.78; 0.85]
L_e	1 mH	$u_{DCp/n}^{(1)}$	10 kV
R_e	10 mΩ	C in the PFC	3.5 mF

The simulation is performed by first calculating the trajectories of all corresponding variables to achieve the control objective and then deriving the control inputs, as discussed in Subsections III-D and III-E. These inputs are applied to the system dynamics, and results are obtained through numerical integration.

Each subsequent figure illustrates the evolution of the dynamics from an initial steady state—where the grid is fault-free with no PFC voltages inserted—toward a new steady state under ground fault conditions. Two vertical dashed lines in each figure indicate the transition's start and end. In the new steady state, the control inputs provided by the PFCs are determined such that the values of the currents $i_{cwp}^{(a=1,2)}$ remain unchanged from their pre-transition values, with exception $i_{cwp}^{(a=3)}$ is zero after the transition.

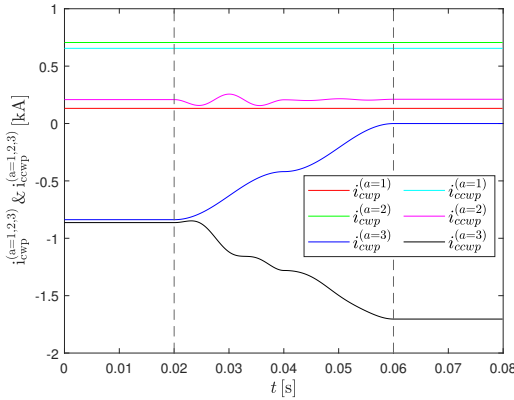


Fig. 5. The internal currents within the DC grid.

To minimize fluctuations in the PFCs' capacitor voltages even more, the faulty current $i_{cwp}^{(a=3)}$ is gradually driven to zero through two consecutively executed transitions² (see the blue curve in Fig. 5), each lasting 20ms. Consequently, the total transition duration is 40ms. This transition is **7.5 times faster**

²With the new control method, the transition can also take place in a single step, although this would lead to slightly increased energy fluctuations in the PFCs compared to those shown in Fig. 7

than the corresponding one presented in [5], using the same simulation scenario and parameters. The currents $i_{cwp}^{(a=1,2,3)}$ are selected for control, resulting in smooth transitions in each partial transition, as intended. The hump-shaped components observed in $i_{cwp}^{(a=2,3)}$ correspond to the DC currents at the MMC2 and MMC3, $i_{DCp}^{(a=2,3)}$, as discussed below.

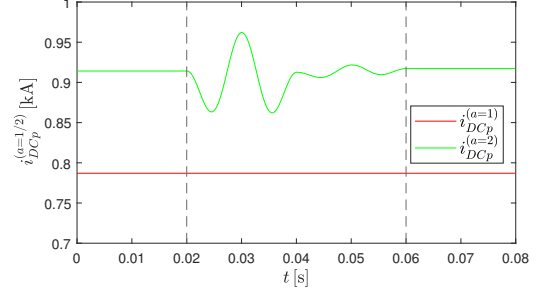


Fig. 6. The DC currents at the MMC1 and MMC2

Fig. 6 shows the DC currents at the MMC1 and MMC2, $i_{DCp}^{(a=1,2)}$. The energy of MMC1 is decoupled from the DC grid dynamics, as it is chosen to define the voltage level of the DC grid. As long as the energy of MMC1 is not actively adjusted by the grid operator, any changes on the DC grid side will not require a hump-shaped component in the current $i_{DCp}^{(a=1)}$. In contrast, the hump-shaped component in $i_{DCp}^{(a=2)}$ is designed to control the energy of the MMC2. The DC current $i_{DCp}^{(a=3)}$ at MMC3, which results from the other two, is not depicted in this figure.

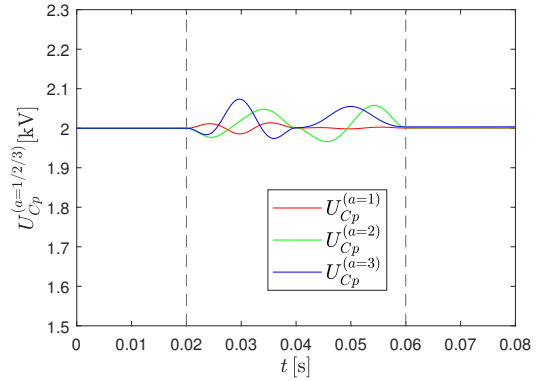


Fig. 7. The capacitor voltages of the PFCs in the p-ring.

Fig. 7 shows the fluctuations in the capacitor voltages of the PFCs in the p-ring caused by the rapid redistribution of power flows in the DC grid. The fluctuations remain **below 5%** of the initial capacitor voltages, significantly lower than the 20% peak fluctuation in the PFC capacitor voltages observed during the transition in [5]. After the transition, the PFC capacitor voltages return to their original values.

Fig. 8 depicts the evolution of the MMCs' energies over the simulation period, showing that all return to their original

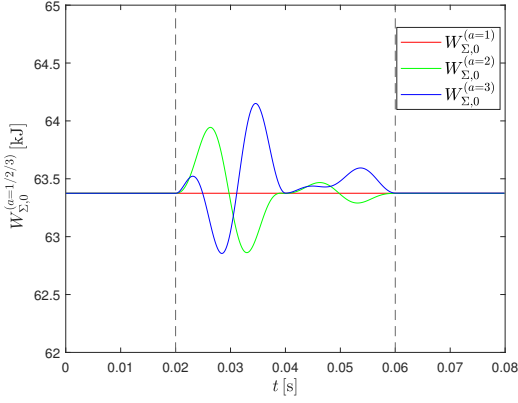


Fig. 8. The energies stored in the MMCs

values after the transition. The energy control of MMC3 is achieved by regulating the active power of the AC grid at MMC3, as illustrated in Fig. 9, along with the corresponding AC currents. The active powers of the AC grids coupled to MMC1 and MMC2 and all AC reactive powers exhibit trivial constant values, as they are kept unchanged in this study case and are therefore not shown.

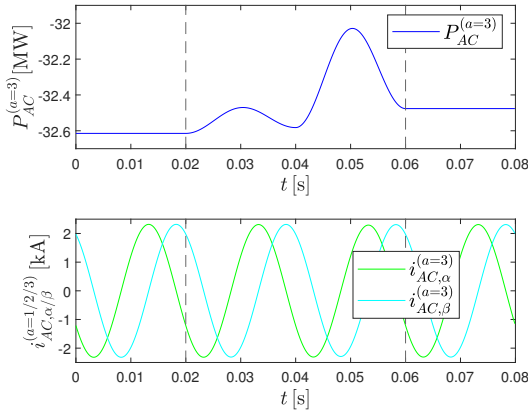


Fig. 9. The active power of the AC grid at the MMC3 and the corresponding AC currents.

V. CONCLUSION

An enhanced open-loop control method for DC grids with additional PFCs has been introduced. The main innovation of this improved control method is, in addition to an AC active and reactive power control, that it uses all PFC driving voltages in a balanced way to minimize PFC energy fluctuations, thereby enabling faster transitions to new steady states. Simulation results demonstrate that the transition in one of the most severe ground fault cases is 7.5 times faster, with drastically reduced energy fluctuations in the converters—particularly in the PFCs. Transition times are only a fraction of an AC period for less severe faults, with negligible PFC voltage fluctuations.

This enhanced method retains the other advantages of the earlier approach, including preventing oscillatory modes and low computational effort.

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APPENDIX

EXAMPLE OF DERIVING THE ENERGY EQUATION BY APPLYING CONDITION β) TO THE PFC1 IN THE P-RING

To maintain clarity, only a representative part of the derivation is shown. It is assumed that Task 1 for MMC1 has been completed and the trajectory for the DC current $i_{DCp}^{(1)}(t)$ has already been calculated. The application of condition β) to PFC1 within the p-ring is now demonstrated.

1) *Deriving the voltage u_{Mp} at the central point of the p-ring:* The voltage u_{Mp} is related to the DC voltage at the terminal of MMC1, i.e. $u_{DCp}^{(1)}$, as follows:

$$u_{Mp} = u_{DCp}^{(a=1)} - R_{cw}^{(1)} i_{cwp}^{(1)} - L_{cw}^{(1)} \frac{d}{dt} i_{cwp}^{(1)} - u_{F\alpha p}^{(1)} - R_r^{(1)} \left(i_{cwp}^{(1)} + i_{DCp}^{(2)} - i_{cwp}^{(2)} \right) - L_r^{(1)} \frac{d}{dt} \left(i_{cwp}^{(1)} + i_{DCp}^{(2)} - i_{cwp}^{(2)} \right). \quad (17)$$

Considering the trajectory for the u_{Mp} during $[t_0, t_0 + T_s]$ and substituting the control input $u_{F\alpha p}^{(1)}$ from (16) in (17) yields:

$$u_{Mp}(t) = u_{DCp}^{(a=1)}(t) + \tilde{a}_1^T \frac{d}{dt} \vec{i}_{cwp}(t) + \tilde{a}_2^T \frac{d}{dt} \vec{i}_{DCp}(t) + \tilde{a}_3^T \vec{i}_{cwp}(t) + \tilde{a}_4^T \vec{i}_{DCp}(t) - \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \vec{U}_{Fp}^{(g)}(t) - \frac{\tilde{u}}{2} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \tilde{\Phi}_1(t) \vec{X}_p, \quad (18)$$

where the vectors $\tilde{a}_{1/2}^T$ are constant, resulted from the process above as:

$$\tilde{a}_1 = \begin{pmatrix} -\left(L_{cw}^{(1)} + L_r^{(1)}\right) \\ L_r^{(1)} \\ 0 \end{pmatrix} + \frac{1}{2} \mathbf{L}_{rcw}^T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \tilde{a}_2 = \begin{pmatrix} 0 \\ -L_r^{(1)} \\ 0 \end{pmatrix} + \frac{1}{2} \mathbf{L}_{re0}^T \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}. \quad (19)$$

The vectors $\tilde{a}_3$ and $\tilde{a}_4$ are obtained by replacing the inducances in $\tilde{a}_1$ and $\tilde{a}_2$ with the corresponding resistances. Using $\vec{i}_{DCp} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} i_{DCp}^{(1)} + \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} i_{DCp}^{(2)}$ and substituting (10) in (18) yields the relation of $u_{Mp}(t)$ to the unknown coefficients $A^{(a=2)}$ and $X_p^{(a=1,2,3)}$:

$$u_{Mp}(t) = u_{DCp}^{(a=1)} + \tilde{a}_1^T \frac{d}{dt} \vec{i}_{cwp}(t) + \tilde{a}_3^T \vec{i}_{cwp}(t) + \tilde{a}_2^T \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \frac{d}{dt} i_{DCp}^{(1)}(t) + \tilde{a}_2^T \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \frac{d}{dt} i_{DCp}^{(2,g)}(t) + \tilde{a}_4^T \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} i_{DCp}^{(1)}(t) + \tilde{a}_4^T \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} i_{DCp}^{(2,g)}(t) - \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \vec{U}_{Fp}^{(g)}(t) + \left[\tilde{a}_2^T \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \frac{d}{dt} \tilde{\Phi}_1(t) + \tilde{a}_4^T \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \tilde{\Phi}_1(t) \right] A^{(a=2)} + \left[-\frac{\tilde{u}}{2} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \tilde{\Phi}_1(t) \right] \vec{X}_p, \quad (20)$$

where those terms marked with a dotted line contain no coefficients of the hump contributions.

2) *Deriving the linear energy equation:* Implementing condition β for PFC1 in the p-ring yields:

$$\int_{t_0}^{t_0+T_s} \frac{d}{dt} W_{Cp}'^{(a=1)} dt = W_{Cp}'^{(a=1,ss2)} - W_{Cp}'^{(a=1,ss1)} \approx \int_{t_0}^{t_0+T_s} u_{DCp}^{(a=1)}(t) i_{cwp}^{(1)}(t) dt + \int_{t_0}^{t_0+T_s} u_{DCp}^{(a=2)}(t) \left(i_{DCp}^{(2)} - i_{cwp}^{(2)} \right) dt - \underbrace{\int_{t_0}^{t_0+T_s} u_{Mp}(t) \left(i_{cwp}^{(1)}(t) + i_{DCp}^{(2)}(t) - i_{cwp}^{(2)}(t) \right) dt}_{:= J_1} \quad (21)$$

Considering the third integration in the right-hand side of (21), i.e. J_1 , substituting $u_{Mp}(t)$ from (20) into this integration

gives:

$$J_1 = \int_{t_0}^{t_0+T_s} \left[u_{DCp}^{(a=1)}(t) + \tilde{a}_1^T \frac{d}{dt} \vec{i}_{cwp}(t) + \tilde{a}_3^T \vec{i}_{cwp}(t) + \dots + \tilde{a}_2^T \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \frac{d}{dt} \tilde{\Phi}_1(t) A^{(a=2)} + \tilde{a}_4^T \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \tilde{\Phi}_1(t) A^{(a=2)} - \frac{\tilde{u}}{2} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \tilde{\Phi}_1(t) \vec{X}_p \right] \times \left(i_{cwp}^{(1)}(t) + i_{DCp}^{(2,g)}(t) - i_{cwp}^{(2)}(t) + \tilde{A}^{(a=2)} \tilde{\Phi}_2(t) \right) dt = \int_{t_0}^{t_0+T_s} \left(u_{DCp}^{(a=1)}(t) + \dots \right) \left(i_{cwp}^{(1)}(t) + i_{DCp}^{(2,g)}(t) - i_{cwp}^{(2)}(t) \right) dt + \left[\tilde{a}_2^T \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \int_{t_0}^{t_0+T_s} \frac{d}{dt} \tilde{\Phi}_1 \left(i_{cwp}^{(1)}(t) + i_{DCp}^{(2,g)} - i_{cwp}^{(2)}(t) \right) dt + \tilde{a}_4^T \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \int_{t_0}^{t_0+T_s} \tilde{\Phi}_1 \left(i_{cwp}^{(1)}(t) + i_{DCp}^{(2,g)} - i_{cwp}^{(2)}(t) \right) dt + \dots \right] A^{(a=2)} - \left[\frac{\tilde{u}}{2} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \int_{t_0}^{t_0+T_s} \tilde{\Phi}_1 \left(i_{cwp}^{(1)}(t) + i_{DCp}^{(2,g)} - i_{cwp}^{(2)}(t) \right) dt \right]_{3 \times 1} \vec{X}_p - \frac{\tilde{u}}{2} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \left(\underbrace{\int_{t_0}^{t_0+T_s} \tilde{\Phi}_1(t) \tilde{\Phi}_2(t) dt}_{=0, \text{ orthogonality}} \right) \times \underbrace{A^{(a=2)} \vec{X}_p}_{\text{nonlinear term}}, \quad (22)$$

where the term marked with a dotted line contains no hump-shaped component coefficients. As shown in (22), the non-linear term $A^{(a=2)} \vec{X}_p$ vanishes due to the basis function orthogonality, reducing J_1 to a linear equation in $A^{(a=2)}$ and $X_p^{(a=1,2,3)}$. Similarly, the other two integrations in (21) expand into linear equations, forming third row of $\mathbf{D}_{8 \times 8}$ and the third entry of $\vec{\mathbf{b}}_{8 \times 1}$ in (14).